A PRACTICAL TREATISE

ON THE SCIENCE OF

LAND AND ENGINEERING SURVEYING,

LEVELLING, ESTIMATING QUANTITIES, &c.

WITH A

GENERAL DESCRIPTION

OF THE

SEVERAL INSTRUMENTS REQUIRED FOR SURVEYING, LEVELLING, PLOTTING, &c.

WITH ILLUSTRATIONS.

By H. S. MERRETT,
ARCHITECTURAL AND ENGINEERING SURVEYOR.

LONDON:
E. & F. N. SPON, 16, BUCKLERSBURY.

1863.

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LAND AND ENGINEERING SURVEYING.

PART I.

INTRODUCTION.

Previous authors have disagreed as to the origin of Land Surveying: it has been attributed to the annual inundation of the Nile, the deposit from which destroyed the landmarks; it therefore called forth some accurate method of ascertaining the original boundary of individual property.

This object was carried into effect in the most efficient manner by Euclid, a mathematician, who flourished three hundred and eight years before the birth of Christ; he was so highly respected in his lifetime, that King Ptolemy became one of his pupils, and from that time no mathematician was found who had not studied in the school established by him at Alexandria; even in the present day Euclid's elements are the foundation to our greatest mathematicians. Whoever considers the whole extent of geometry, will find that the main design of all its speculation is mensuration; to this the elements of Euclid are almost entirely devoted.

The latter part of this century has been remarkable for its wonderful march of scientific improvements, and has opened a

vast field for talent and enterprise in the art of land surveying, by the introduction of railroads, forming part of the duties of the civil engineer and architect.

To be a perfect surveyor, he should be well qualified in the knowledge of arithmetic, geometry, mensuration, algebra, logarithms, and decimals, and be thoroughly acquainted with the most eminent authors on mathematics.

The duties of a surveyor frequently extend beyond making a plan and giving the superficial area: such as disputed boundaries, manorial rights, exchange or the division of land, diversion and improvement of roads, measuring stone, quarries, drainage, and building materials.

Land surveying may be considered to be divided into three classes:

First, by the chain only; second, by the chain and the use of the theodolite, or other instruments for measuring angles; third, by trigonometry, which is chiefly performed by the theodolite and logarithmic tables. This branch is seldom required in ordinary surveying; it is applied to the survey of counties, kingdoms, maritime surveying, and inaccessible distances.

In all cases, whether it be a single field, estate, parish, or county, the triangle is the only figure adopted to lay down the foundation of a survey. Now, let it be remembered, a triangle itself has no proof of accuracy where the chain only is used, a fourth line must be measured from either of the two sides, or from one of its angles to its opposite side; this will also apply to the second case, or by measuring all the three angles with the instrument; the same also in the third case.

A figure of four unequal sides, called a trapezium, could not be plotted unless a line was measured from the opposite corners or angles, dividing it into two triangles. This would also require another line from either its two opposite sides, or the two opposite angles, to prove its accuracy.

To obtain the accurate quantities of fields, all the irregular fences are reduced to straight lines, and then to triangles.

Those who have not received a preparatory knowledge of geometry and the higher branches of mathematics, are recommended to apply their leisure hours to the works of any of the following authors: Hutton, Bonycastle, Keith, &c., as only such portions of geometry, mensuration, and logarithms will be here introduced as are immediately connected with the subject.

In computing the areas or contents of fields, in addition to the system shown by mensuration, another system will be described by means of a sliding scale, and will be found to be truly accurate and expeditious, particularly on very irregular boundaries.

In order that the student may more readily comprehend the system of surveying by the chain only, and by the chain and instrument, the same field will be adopted to both.

It is an erroneous opinion amongst many, that a straight line cannot be polled out, or a survey correctly made, without the use of a theodolite; the student should first make himself thoroughly master of the chain by laying out his work by large intersecting triangles. There are certain cases where a theodolite is indispensable—such as a town or village, hilly and woody country, &c.

The theodolite is the most perfect instrument for surveying; a box sextant will be found a most valuable auxiliary in filling up the details of a survey. To accompany these must be provided a protractor, to plot the angles that are taken by the instrument. These are all the instruments required for surveying—for a description of which see Part V.

The treatise of logarithms, decimal fractions, and the roots, are extended to a greater length than was intended for this work; the value of it to the surveyor will amply remunerate him for perseverance in obtaining a perfect knowledge of them from the authors before recommended.

LOGARITHMS.

Logarithms are made to facilitate troublesome calculations in numbers, reducing the labour necessary to arrive at the same results. Multiplication is performed by addition; division, by subtraction; raising of powers and extracting roots, by multiplication and division.

More generally, logarithms are the numeral exponents of ratios; or, they are a series of numbers in arithmetical progression answering to another series of numbers in geometrical progression; thus:

0,	1,	2,	3,	4,	&c.,	are indices of logarithms
1,	10,	100,	1000,	10000,	&c.,	are geometrical progression

It is evident that the same indices serve equally for any geometric series; and, consequently, there may be an endless variety of systems of logarithms to the same common numbers, by only changing the second term, 2, 3, or 10, &c., of the geometric series of whole numbers.

The logarithms most convenient for practice are such as are adapted to a geometric series, increasing in a tenfold proportion as in the above form; and are those which are found at present in most of the common tables on this subject.

The distinguishing mark of this subject of logarithms is, that the index or logarithm of 10 is 1; that of 100 is 2; that of 1000 is 3, &c. And in decimals, the logarithm of 1 —1; (placing the negative sine before the index) that of .01 is —2; that of .001 is —3, &c.

Thus the logarithm of 2705 is 3.432167, the logarithm of $\frac{1}{10}$, $\frac{1}{100}$, &c., will be thus:

NATURAL NUMBERS.	LOGARITHMS. Index.
2705	3.432167
270.5	2.432167
27.05	1.432167
2.705	0.432167
.2705	–1.432167
.02705	–2.432167

To find in the table the logarithmic number to any natural number from 1 to 999.

Seek in the left hand column the number, and against it is the logarithm required; thus:

Required the logarithm of	5	which is	0.698970
"	15	"	1.176091
"	25	"	1.397940
"	125	"	2.096910

If the number be more than 1000 and less than 10000, seek in the table for the logarithmic decimal of the first three figures 270, as before; then take out the difference between that number and the next below it 1605, multiply this difference by the last figure 5, add this product to the logarithm first found, apply the index 3, it will be the logarithm required.

-To find the logarithm of 2705:

Seek in the table for the logarithm decimal of 270 .		.	.431364
The difference next below	1605		
Multiplied by the fourth figure	5	=	803
And add thereto the proper index to the decimal found			3.432167

If the fourth figure be a cypher, the decimal will be the same as the first three figures, to which must be applied the index 3.

TO FIND THE LOGARITHM OF A VULGAR FRACTION, OR A MIXED NUMBER.

Rule. Reduce the vulgar fraction to a decimal, and find its logarithm as before—

The decimal of $\frac{7}{16}$ = .4375; and the logarithm of this decimal is = -1.640978

or, subtract the logarithm of the denominator from the logarithm of the numerator, and the remainder will be the logarithm, which, being that of a decimal fraction, must always have a negative index. If it be a mixed number, reduce it to an improper fraction, and find the difference of the logarithm of the numerator and denominator as before.

Examples.

To find the logarithm of $\frac{7}{16}$.

The log. of	7 =	-0.845098
"	16 =	-1.204120
Diff. log. of $\frac{7}{16}$	=	-1.640978

To find the logaritnm of $17\frac{14}{23}$.

First $17\frac{14}{23} = \frac{405}{23}$

Logarithm of	405 =	2.607455
"	23 =	1.361728
Diff. log. of $17\frac{14}{23}$	=	1.245727

To find the natural number to any given logarithm:

This is to be found by the reverse method to the former: find the proposed logarithm in the table and take out the corresponding number, in which the proper number of integers are to be pointed off, viz. 1 more than the index. In finding the number answering to any given logarithm, the index always shows how far the first figures must be removed from the place of units, viz. to the left hand or integers, when the index is affirmative; but to the right hand or decimals, when it is negative.

If the logarithm be exactly found in the table, the natural number is found in the first column, thus:

The natural number of logarithm	1.643453 =	44
" "	2.603144 =	401
" "	3.204120 =	1600

If the logarithm cannot be exactly found in the table, take the difference of the next greater and the next less, subtracting the one from the other, as also their natural number the one from the other, and the less logarithm from the one proposed, thus:

As the difference of the first tabular logarithm
Is to the difference of their natural number,
So is the difference of the given logarithm, and the last tabular logarithm,
To their corresponding numerical difference,
Which, being annexed to the least natural number above taken, gives the natural number sought corresponding to the proposed logarithm.

Example.

To find the natural number answering to the given logarithm 3.532708:

Take log. next greater	.532754	number	3410	given log.	.532708
next less .	.532627	number	3409	next less	.532627
	127		1		81

Then as 127 : 100 : : 81 : 64, nearly the numerical difference, which, added to 3409, gives 3409.64, the number required, marking off two integers, because the index of the given logarithm is 3.

Had the index been negative, then 3.532708, its corresponding number, would have been .340964 wholly a decimal.

MULTIPLICATION OF LOGARITHMS.

Rule. Take out the logarithms of the factors from the table, add them together, their sum will be the logarithm required. Then from the table take out the natural number answering to the same for the product sought; observing to add what is to be carried from the decimal part of the logarithm to the affirmative index, or else subtract it from the negative.*

Also adding the indices together when they are of the same kind, both affirmative, or both negative, but subtracting the less from the greater, when the one is affirmative and the other negative, and prefix the sign of the greater to the remainder.

Examples.

Multiply 23.14 by 5.062.

	Num.	Log.
The logarithm of	23.14	=1.364363
„	5.062	=0.704322
Product .	117.1347	=2.068685

Multiply 2.581926 by 3.457291.

	Num.	Log.
The logarithm of	2.581926	=0.411944
„	3.457291	=0.538736
Product .	8.92648	=0.950680

Multiply 3.902 and 597.16 and .0314728 together.

	Num.	Log.
The logarithm of	3.902 =	0.591287
„	597.16 =	2.776091
„	.0314728 =	—2.497935
Product .	73.3333 =	1.865313

Here the −2 cancels the 2, and the 1 to carry from the decimal is set down.

Multiply 3.586 and 2.1046 and 0.8372 and 0.0294 together.

	Num.	Log.
The logarithm of	3.586 =	0.554610
„	2.1046=	0.323170
„	0.8372=	−1.922829
„	0.0294=	−2.468347
Product .	0.1857618	−1.268956

Here the 2 to carry cancels the −2, and there remains −1 to set down.

DIVISION OF LOGARITHMS.

Rule. From the logarithm of the dividend subtract the logarithm of the divisor, and the number answering to the

* This rule is also applied to the areas of right lined figures.

remainder will be the quotient required; observing to change the sign of the index of the divisor from affirmative to negative, or from negative to affirmative; then take the sum of the indices, if they be of the same name, or their difference when of different signs, with the sign of the greater for the index to the logarithm of the quotient; and also when 1 is borrowed in the left hand place of the decimal part of the logarithm, add it to the index of the divisor when the index is affirmative, but subtract it when negative, then let the sign of the index arising from hence be changed, and worked with as before.

Examples.

Divide 24163 by 4567.

	Num.	Log.
Dividend .	24163	=4.383151
Divisor .	4567	=3.659631
Quotient .	5.29078	=0.723520

Divide .06314 by .007241.

	Num.	Log.
Dividend .	.06314	=−2.800305
Divisor .	.007241	=−3.859799
Quotient .	8.71979	=−0.940506

Here 1 carried from the decimals to the −3, makes it become −2, leaves 0 remaining.

Divide 37.149 by 523.76

	Num.	Log.
Dividend .	37.149	=1.569947
Divisor .	523.76	=2.719132
Quotient	.0709275	−2.850815

Divide .7438 by 12.9476

	Num.	Log.
Dividend .	.7438=	−1.871456
Divisor	12.9476=	1.112189
Quotient	.057447=	−2.759267

Here the 1 taken from the −1 makes it become −2 to set down.

PROPORTION, OR THE RULE OF THREE.

Rule. Add the logarithms of the second and third terms together, and subtract the logarithm of the first, the remainder will be the logarithm of the fourth term.

Examples.

If 8.2 be to 12 as 74 is to the fourth number, what is that number?

To the logarithm of . .	12	=1.079181
Add the logarithm of . .	74	1.869232
		2.948413
Subtract the logarithm of .	8.2	=0.913814
Answering to . .	108.29	=2.034599

Instead of subtracting one logarithm from another, add its arithmetical complement, the result will be the same, thus:

To the logarithm of . .	12=1.079181
Add the logarithm of . .	74=1.869232
And the arith. comp. of. .	8.2=9.086186
The same as before . .	108.29=2.034599

Note.—The arithmetical complement of a logarithm is what it wants of 10.000000.

To find which, begin at the left hand, and subtract each figure from 9, except the last on the right hand, which must be subtracted from 10, and for every complement which is added subtract 10 from the last sum of the indices.

The arithmetical complement is commonly used in trigonometrical calculations, when radius is not the first term in the analogy.

It is further to be noted that the arithmetical complement of any sine is the same as the co-secant of the same number of degrees.

From	=10.0000000
Subtract the log. of .	4.1= 0.6127839
The arith. comp. of .	4.1= 9.3872161

INVOLUTION.

Involution is the raising of powers, from any given number, as a root; a power is a quantity produced by multiplying any given number, called the root, a certain number of times continually by itself, thus:

2=is the root of the 1st power of 2
2×2=is the 2nd power or square of 2
$2\times2\times2$=is the 3rd power or cube of 2
$2\times2\times2\times2$=is the 4th power of 2, &c.

BY LOGARITHMS.

Take out the logarithm of the given number from the table, multiply the logarithm thus found by the index of the power proposed, find the number answering to the product, and it will be the power required.

Note.—In multiplying a logarithm with a negative index by an affirmative number, the product will be negative; but what is to be carried from the decimal part of the logarithm will always be affirmative, and therefore their difference will be the index of the product, and is always to be made of the same kind with the greater.

Examples.

To square the number 2.5791.

	Num.		Log.
The log. of	2.5791	=	0.411468
The index	. .	=	2
The power	6.65174	=	0.822936

To raise .09163 to the 4th power.

	Num.		Log.
Root	.09163	=	−2.962038
Index	. .	=	4
Power	.000070494	=	−5.848152

Here 4 times the negative index being −8, and 3 to carry the difference, −5 is the index of the product.

To find the cube of 3.07146.

	Num.		Log.
Root	3.07146	=	0.487345
Index	. . .	=	3
Power	28.9758	=	1.462035

To raise 1.0045 to the 365th power.

	Num.		Log.
Root	1.0045	=	0.001950
Index	. . .	=	365
Power	5.14932	=	0.711750

EVOLUTION, OR EXTRACTING ROOTS.

Evolution, or the reverse of involution, is the extracting or finding the roots of any given power.

Rule. Take the logarithm of the given number from the table, divide the logarithm thus formed by the index of the root, then the number answering to the quotient will be the root.

Note.—When the index to the logarithm to be divided is negative, and does not exactly contain the division without some remainder, increase the index by such a number as will make it exactly divisable, by the index carrying the unity borrowed as so many times to the left hand place of the decimal, and thus divide as in whole numbers.

Examples.

To find the square root of 365.

Power 365 .	2)2.562293
Root 19.10496 .	1.281146½

To find the $\sqrt{}$ of .093.

Power .093 .	2)−2.968483
Root .304957 .	−1.484241½

Here the divisor 2 is contained exactly once in the negative index —2, and therefore the index of the quotient is —1.

To find the 3rd root of 12345.

Power 12345 .	3)4.091491
Root 23.1116 . .	1.363830⅓

To find the $\sqrt[3]{}$ of .00048.

Power .00048 .	3)−4.681241
Root .0782973 .	−2.893747

Here the divisor 3, not being exactly contained in —4, it is augmented by 2 to make up 6, in which the divisor is contained two times; then the 2 just borrowed being carried to the decimal figure 6, &c., makes 2.68124, which divided by 3 gives .893747.

SINES AND TANGENTS.

The tables (both logarithmic and natural) of sines and tangents are calculated to every five minutes of a degree, by which all trigonometrical operations are performed.

The degrees descend from the top to the bottom, that is, from 0 to 90; the minutes are placed in the top column.

The co-sine, co-tangent, &c., reads from the bottom upwards. To find the logarithm of any sine, &c., to 5 minutes of a degree, look in the table for the degrees in the first column on the left hand, and the minutes on the top, opposite to which will be the logarithm required, thus:

The log. sine of . .	$\overset{\circ}{20}.\overset{\prime}{15}$	= 9.539223
" " . . .	140. 0	= 9.808068*
The natural sine of .	20.15	= .346117
The log. tangent of . .	20.15	= 9.566932

To find the logarithm of any intermediate minute, take the difference of the logarithm next less and next greater, multiply this difference by the number of intermediate minutes, and divide the product by 5.

Add the quotient to the foregoing less logarithm, the product will be the logarithm required, nearly.

Required the sine of 20° 18′

The tabular logarithm of	20° 15′	= 9.539223
Difference between this and the next logarithm .	1708	
Difference between minutes	3	
	5)5124	
		1025
		9.540248

To find the degrees and minutes to any logarithm, seek in the table for the logarithms, on the top for the minutes, and on the left for the degrees as before, will give the degrees and minutes, nearly.

If the logarithm should not agree with any degree and minute in the table, find the next greatest and next less, and take their difference, thus:

* The angle exceeding 90° look for 50° in the 2nd column of co-sine.

As this difference is to 5 minutes, so is the difference between the given logarithm and that next less to a fourth number of minutes.

Add this fourth number to the minutes in the less logarithm, it will be the degree and minutes required.

Required the degrees and minutes of the sine answering to 9.540248:

The next less logarithm	9.539223	= 20° 15′
The next greater	9.540931	= 20° 20′
The difference	.001708	
Given logarithm	=9.540248	
Next less logarithm . . .	=9.539223	
Difference	.001025	

Then as 1708 : 5 : : 1025 : 3 minutes, which, added to the less logarithm 20.15, will give 20° 18′ for the sine required.

DECIMAL FRACTIONS.

A decimal fraction derives its name from the Latin, *decem* (10), which denotes the nature of its numbers, representing the parts of an integral quantity, divided into a tenfold proportion.

NUMERATION.—Teaches to read and write any number proposed, either by words or characters.

In decimal fractions, the integer, or whole thing, as a gallon, a pound, a yard, an acre, &c., is supposed to be divided into ten equal parts, called tenths; those tenths into ten equal parts, called hundredths; and so on without end.

So that the denomination of a decimal being always known to consist of an unit with as many cyphers as the numerator has places, is therefore never expressed, being understood to be 10, 100, 1000, &c., according as the numerator consists of 1, 2, 3, 4, or more figures. Thus, instead of $\frac{2}{10}$, $\frac{24}{100}$, $\frac{211}{1000}$, the numbers only are written with a full point before them, thus: .2 .24 .211.

If a unit of any kind, as an acre, a gallon, &c., be divided into ten equal parts, then the decimal represents as many of

those parts as the decimal figures express; thus: .7 means seven of those parts, or seven-tenths. If the decimal consists of two figures, unity would be understood to be divided into a hundred equal parts, of which the decimal represents as many as the figures express, thus: .65 means 65 of those parts, or sixty-five hundredths; if the decimal consisted of three figures, it would be a thousand equal parts, .625 is six hundred and twenty-five of those parts; or if the decimal .0625, unity would be divided into 10,000 equal parts. The value of the figures are more clearly described by the following table:

Tenths	.5
Hundredths	.56
Thousandths	.567
Ten thousandths	.5678
Hundred thousandths	.56789

Thus: .5 is read five-tenths, .56 is read fifty-six hundredths, .567 thousandths, and so on, as in the table.

Cyphers to the right hand of decimals cause no difference in their value, as .5 .50 .500 are decimals of the same value, being each equal to $\frac{1}{2}$; that is, $.5 = \frac{5}{10}$, $.50 = \frac{50}{100}$, $.500 = \frac{500}{1000}$. If cyphers are placed on the left hand of decimals, they diminish their value in a tenfold proportion, thus: .3 .03 .003 are three tenths, three hundredths, three thousandths, and answer to the vulgar fractions, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, respectively.

A whole number and decimal are thus expressed: 8.75, 85.04, &c.

When decimals terminate after a certain number of figures, they are called finite, as $125 = \frac{125}{1000} = \frac{1}{8}$, $958 = \frac{958}{1000} = \frac{237}{250}$.

When one or more figures in the decimals become repeated, it is called a repeating or circulating decimal; as, $.33333\dot{3}$, &c., $= \frac{1}{3}$, $.66666\dot{6}$, &c., $= \frac{2}{3}$, $.42857142857\dot{1}$, &c., $= \frac{3}{7}$, and many others.

Note.—A finite decimal may be considered as infinite, by making cyphers to recur, as they do not alter the value of the decimal.

In all operations, if the result consists of several nines, reject them, and make the next superior place an unit more; thus for 26.25999, write 26.26.

In all circulating numbers dot the last figure, as $8.5466\dot{6}$.

ADDITION OF DECIMALS.

Rule. Arrange the numbers under each other, according to their several values, find the sum as in addition of whole numbers, and cut off for decimal as many figures to the right hand as there are decimals in any one of the given numbers.

Examples.

What is the sum of 23.45, 7.849, 543.2, 8.6234, 253.004.

$$\begin{array}{r} 23.450 \\ 7.849 \\ 543.200 \\ 8.6234 \\ 253.004 \\ \hline 836.1264 \end{array}$$

If any of the decimals be repetends, continue them beyond the others, and make them end together; then in adding, increase the sum of the first column by as many units as there are nines therein, as—

$$\begin{array}{r} .75 \\ .666\dot{6} \\ .8888 \\ .875 \\ .4444 \\ \hline 3.6250 \end{array}$$

Here the first sum contains 18, two nines; therefore 2 added to 18 = 20; the rest of the work is the same as usual in others.

If some of the decimals be repetends, and others circulates, continue them both beyond those that are finite, and until their periods end together; then to the sum of the first column add as many as would arise to carry on the sum, thus:

$$\begin{array}{r} 2.5 \\ 3.66666\dot{6} \\ 7.69696\dot{9} \\ 14.37237\dot{2} \\ \hline 28.236008 \end{array}$$

The repetend of $.\dot{6}$, the circulate of $.\dot{6}\dot{9}$, and $.\dot{3}7\dot{2}$, continued until their periods end together. It may easily be observed that there would be 1 to carry to the first column if it were carried any farther.

Note.—It is always necessary to attend to the rules for repetends and circulates; three or four decimal figures, according to the rule, being sufficiently near the truth for common calculations.

SUBTRACTION OF DECIMALS.

Rule. Place the numbers according to their value, subtract as in whole numbers, and cut off for decimals as in addition.

Example.

Subtract 35.87043 from 132.005.

$$\begin{array}{r} 132.005 \\ 35.87043 \\ \hline 96.13457 \end{array}$$

If both be single repetends, make them end together: and if there be occasion to borrow at the first figure, borrow 9 only, instead of 10, thus:

.8333
.6666
.1666

If both be circulates, or one a repetend and the other a circulate, continue both until their periods end together: and if there should be occasion to borrow at the following figure, where they continued that figure farther, carry 1 to the first figure, and if the numbers be in different denominations, reduce them until they are alike.

Subtract $\frac{834}{999}$ from $1\frac{2}{3}$.

$$\begin{array}{rcl} 200000 \div 3 &=& 1.66666\dot{6} \\ 834000 \div 999 &=& .83\dot{4}83\dot{4} \\ \hline & & .831831 \end{array}$$

MULTIPLICATION OF DECIMALS.

Rule. Place the factors under each other, and multiply them together as in whole numbers: then point off as many figures from the product (counting from right to left) as there are decimal places in both factors: observing if there be not enough, to annex as many cyphers to the left hand of the product as will supply the deficiency.

Example.

Multiply .2715 by .253.

.2715
.253
8145
13575
5430
.0686895

In this example the product is .686895: the number of decimals in the multiplicand are 4, and the multiplier 3, equal to 7, therefore a cypher must be placed on the left hand of the product to reduce it to its proper terms.

To multiply a repetend by a single figure, add 1 to the first product for every 9 therein, so will you have a repetend in the product; if there be several figures in the multiple, do so with each product, and continue them until they end together; then add them as so many repetends.

If the multiplicand be a circulate, consider the increase that would arise to the first product if the multiplicand was continued farther: thus do with each product, make them end together, and add them by the rule for adding circulates.

When any number of decimals are to be multiplied by 10, 100, 1000, &c., remove the separating point in the multiplicand so many places towards the right hand as there are cyphers in the multiplier; thus: $.578 \times 10 = 5.78$, $.578 \times 100 = 57.8$, $.578 \times 1000 = 578$.

DIVISION OF DECIMALS.

This rule is also worked as in whole numbers; the only difficulty is in valuing the quotient, which is done by any of the following rules:

Rule 1. The first figure of the quotient is always of the same value with that figure of the dividend which answers or stands over the place of units in the divisor.

2. The quotient must always have as many decimal places as the dividend has more than the divisor.

Note 1.—If the divisor and dividend have both the same number of decimal parts, the quotient will be a whole number.

2.—If the dividend has not so many places of decimals as are in the divisor, then so many cyphers must be annexed to the dividend as will make them equal; the quotient will then be a whole number.

3.—But if when the division is done, and the quotient has not so many figures as it should have places of decimals, then so many cyphers must be prefixed as there are places wanting.

Examples.

Divide 173.5425 by 3.75.

$173.5425 \div 3.75 = 46.278$

Divide .14856 by 2.476.

$.14856 \div 2.476 = .6000$

Divide 5.714 by 8275.

$5.714 \div 8275 = .00069$

In this example the quotient was deficient in the number of decimals required, and are placed on the left hand.

When numbers are to be divided by 10, 100, 1000, &c., it is performed by placing the separating point in the dividend so many places towards the left hand as there are cyphers in the divisor, thus:

$5784 \div 10 = 578.4$

$5784 \div 100 = 57.84$

$5784 \div 1000 = 5.784$

$5784 \div 10000 = .5784$

REDUCTION OF DECIMALS.

By reduction we change vulgar fractions, and the lesser parts of coin, weights and measures, &c., into decimals, and find the value of any decimal given.

Because decimals increase their value towards the left hand, and decrease their value towards the right hand, in the same tenfold proportion with integers, or whole numbers, they may be annexed to the whole numbers and worked in all respects as whole numbers; and if simple arithmetic be well understood, there is little more to be learned than the placing of the separating points, the rule for which ought to be well attended to.

To reduce a vulgar fraction to a decimal of an equal value.

Rule. Add a cypher or cyphers to the numerator, and divide by the denominator, the quotient will be the decimal required.

Example.

Reduce $\frac{7}{16}$ to a decimal:

$$7.0000 \div 16 = .4375$$

Thus you may take any number of cyphers at pleasure, but their number will be best ascertained when the work is finished, then you must have as many decimal figures as you have taken annexed cyphers in dividing, and if there are not so many in the quotient, you may prefix cyphers to the left hand of the product, thus: $\frac{1.00000}{32} = .03125$.

Sometimes the quotient figures will repeat continually, as $\frac{2}{3}$ thus $\frac{2000}{3} = .66\dot{6}$, then it is called the repetend, and the last figure may be dashed or marked, to distinguish it from a terminate decimal.

Sometimes two, three, or more figures will repeat, as $\frac{12}{33}$ thus $\frac{12.000}{33} = .363\dot{6}$, such are called compound repetends, or circulates, and the first and last figure may be dashed or marked.

To reduce the lesser parts of coin, weights and measures, &c., to a decimal.

Rule. Divide the least name by such number as will reduce it to the next greater; to the decimal so obtained prefix the given number of the same name, then divide by such number as will reduce it to the next greater; always annex cyphers to

the dividend as occasion may require, then proceed till it be reduced to the decimal of the required integer; or reduce the given parts to a single quantity, by reducing them to the lowest name mentioned; annex cyphers thereto, and divide by such numbers as will reduce them to the name required; or reduce the given parts to a vulgar fraction, and that fraction to a decimal.

Example 1.

Reduce 17s. $10\frac{1}{2}$d. to the decimal of a pound sterling:

$$\frac{1.0}{2} = 5 + 10\text{d.} = \frac{10.500}{12} = .875 + 17\text{s.} = \frac{17.875}{20} = .89375$$

Example 2.

Reduce 2 ft. 9 in. to the decimal of a yard:

$$\frac{33}{36}\ \frac{33.0000}{36} = .9166$$

To find the value of any given decimal.

Rule. Multiply the decimal given by the number of parts of the next inferior denomination, cutting off the decimals from the product; then multiply the remainder by the next inferior denomination; thus proceeding till you have brought the least known parts of the integer.

Example 1.

Required the value of .89375 of a pound sterling:

$$\begin{array}{r} .89375 \\ 20 \\ \hline 17.87500 \\ 12 \\ \hline 10.50000 \\ 4 \\ \hline 2.00000 \end{array} = 17\text{s. } 10\tfrac{1}{2}\text{d.}$$

Example 2.

Reduce .625 of a hundred-weight to its proper term:

.625 × 4 = 2.500 × 28 = 14.000; or 2 quarters and 14 lbs.

LOGARITHMS, SINES, AND TANGENTS.

The radius or logarithmic sine of 90 is 10. Of the natural sine is 1. The tangent of 90, whether natural or logarithmic, is infinite.

To find the logarithm of any sine, tangent, &c.:

Look for the degrees in the first column, and for the minutes on the top line, opposite to which will be the logarithm required. And in the same manner will the measure of an angle be found in degrees and minutes by referring to the table.* If the logarithm should not agree with the degree and minutes in the table, seek in the table for that which is next less and next greater, and take the difference, thus:

As this difference is to 3 minutes,
So is the difference between the given logarithm, and the next less
Is to a fourth number of minutes,
Add this fourth number to the minutes in the less logarithm, and it will be the degree and minute required.

Example.

What degree and minute of the sine answers to 9.467996:

The next less logarithm is	9.467585	= 17° . 4′
The next greater logarithm . . .	9.468407	= 17 . 6
Difference	822	
The given logarithm	9.467996	
The next less logarithm . . .	9.467585	
Difference	411	

As 822 (tab. diff.) : 3 (min.) :: 411 (other diff.) : 1 (min.) which minute add to 17° 4′ the less logarithm, will be 17° 5′, the sine required.

OF THE SQUARE ROOT.

When a number is multiplied by itself, as 6×6 or 9×9, &c., it produces the square or second power of that number, and the number itself is called the root of that square.

* The same numbers are also the co-sines and co-tangents, reading from the bottom of the table.

A root consisting of a single figure is found by inspection of the following table:

Roots .	1	2	3	4	5	6	7	8	9
Squares .	1	4	9	16	25	36	49	64	81
Cubes . .	1	8	27	64	125	216	343	512	729

To extract or find the square root of any number which consists of more figures than one, observe the following rule:

Divide the given number into periods of two figures each, by setting a point over the place of units, another over the place of hundredths, and so on over every second figure, both to the left hand in integers and to the right hand in decimals.

Find the greatest square in the first period on the left hand, and set its root on the right hand of the given number, after the manner of a quotient figure in division.

Subtract the square thus formed from the said period, and to the remainder annex the two figures of the next following period, for a dividend.

Double the root above mentioned for a divisor, and find how often it is contained in the said dividend, exclusive of its right hand figure, and set that quotient figure both in the quotient and divisor.

Multiply the whole augmented divisor by this last quotient figure, and subtract the product from the said dividend, bringing down to the next period of the given number, for a new dividend.

Repeat the same process over again—viz. find another new divisor, by doubling all the figures now found in the root; from which, and the last dividend, find the next figure of the root as before; and so on through all the periods to the last.

Note.—The best way of doubling the root, to form the new divisors, is by adding the last figure always to the last divisor, as appears in the following examples. Also, after the figures belonging to the given number are all exhausted, the operation may be continued into decimals at pleasure, by adding any number of periods of cyphers, two in each period.

Examples.

To find the square root of 17.3056.

17.3056(4.16 the root; in which the number of decimal places is the same as the number of decimal periods into which the given number was divided.

16

81) 130

1   81

826) 4956

4956

....

To find the square root of $\frac{5}{12}$.

$\frac{5}{12} = .41\dot{6}66\dot{6}$

.416666(.6454

36

124) 566

496

1285) 7066

6425

12904) 64166

51616

12550

In cases where the square roots of all the integers up to 1000 are tabulated, such an example as the above may be done more easily by a little reduction. Thus:

To find the square root of 2, to six decimals.

2,(1.414213 the root

1

24) 100

96

281) 400

281

2824) 11900

11296

28282) 60400

56564

282841) 383600

282841

2828423) 10075900

8485269

1590631

$$\sqrt{\frac{5}{12}} = \sqrt{\left(\frac{5}{12}\times\frac{12}{12}\right)} = \sqrt{\frac{60}{144}} = \frac{1}{12}\sqrt{60} = \frac{7.745967}{12} = .645497, \&c.$$

CUBE AND HIGHER ROOTS.

The rules usually given in books of arithmetic for the cube and higher roots are very tedious in practice; on which account it is advisable to work either by means of approximating rules, or by means of logarithms.

We shall merely point out here Dr. Hutton's approximating rule for the cube root, which may sometimes be serviceable when logarithmic tables are not at hand.

Rule. By trials take the nearest rational cube to the given number, whether it be greater or less, and call it the assumed cube.

Then say, by the rule of three, as the sum of the given

or value of 1*l.*, which being multiplied by the annuity 23*l.*, will give 242.42, or 242*l.* 8s. $4\frac{3}{4}$d. the value.

Rule. For finding the value of the decimal proportion of a pound in shillings, pence, and farthings (see page 18).

See Inwood's Tables on Valuing Estates, &c.

Explanation of the algebraic signs or symbols generally used to shorten productive numbers of figures, in all sorts of calculations. .

Algebra is a general computation, in which abstract quantities are represented by letters and their connexion pointed out by means of certain characters or symbols. It is one of the most important branches of mathematical science.

$=$	signifies	equality, 9 added to 4 is equal to 13.
$+$	"	plus or more, as 9 added to 4 is equal to 13.
$-$	"	minus or less, as 4 from 8 is equal to 4.
$\times$	"	plus or multiply by, as 9 by 5 is equal to 45.
$\div$	"	division, as 45 divided by 9 is equal to 5.
$\sqrt{\ }$	"	the square root, as $\sqrt{9}$ is equal to 3.
$\sqrt[3]{\ }$	"	the cube root, or 3rd power, as $\sqrt[3]{64}$ equal 4.
3^2	"	that 3 is to be squared, as 3 is equal to 9.
3^3	"	that 3 is to be cubed, as 3^3 is equal 27.
$\overline{\quad}$	"	the bar, or vinculum, denotes that two or more are to be taken together, as $4 \times \overline{6+3} = 36$, and

$\sqrt{\overline{6^2 - 3^2}} = 5$ denotes that 3 squared, subtracted from 6 squared, and the square root extracted, is equal to 5.

$\frac{45}{9}$	"	that 45 is to be divided by 9; this character is commonly used in vulgar fractions. Also the two signs may be used in like manner, thus: $\frac{6-2}{4\times 2} = \frac{4}{8} = .5$.

$4 : 6 :: 8 : 12$ called proportionals; as 4 is to 6, so is 8 to 12.

PRINCIPLES OF GEOMETRY.

GEOMETRY, originally signified according to the etymology of the name, the art of measuring the earth, but it is now the science that treats of and considers the properties of magnitude in general. It is divided into two parts, theoretical and practical.

Theoretical geometry considers and treats of first principles abstractedly. Practical geometry applies these considerations to the various purposes of life. By practical geometry many operations are performed of the utmost importance to society and the arts.

Now the whole numeration of figures may be reduced to the measure of triangles, which are always the half of a rectangle of the same base and altitude, and consequently their area is obtained by taking the half of the product of the base, multiplied by the altitude, or the whole base by half the altitude.

By dividing a polygon into triangles, and taking the value of these, that of the polygon is obtained; by considering the circle as a polygon with an infinite number of sides we obtain the measure thereof to an approximation.

The theory of triangles is, as it were, the hinge upon which all geometrical knowledge turns.

All triangles are more or less similar, according as these angles are nearer to, or more remote from, equality.

The similitude is perfect when all the angles of one are respectively equal to those of another, all the sides then are proportional.

The angles and sides determine the relative and absolute size of a triangle.

Strictly speaking, angles only determine the relative size; equiangular triangles may be of very unequal magnitudes, yet perfectly similar.

But when they are all equilateral, the one having its sides equal to the corresponding sides of the other, they are not only similar and equiangular, but are equal in every respect.

The angles, therefore, determine the relative species of the triangle, the side its absolute size, and consequently that of every other figure, as all are resolvable into triangles.

Yet the essence of a triangle seems to consist much more in the angles than the sides, for the angles are the true, precise, and determined boundaries thereof; their sum is always fixed, and equal to two right angles.

The sides have no fixed equation, but may be extended from the infinitely little to the infinitely great without the triangle changing its nature and kind.

It is from the theory of isoperimetrical figures* that we feel how efficacious angles are, and how inefficient lines, to determine not only the kind but the size of the triangle and all kinds of figures.

For the lines still subsisting the same, we see how a square decreases in proportion as it is changed into a more oblique rhomboid, and thus acquire more acute angles.

The same observation holds good in all kind of figures, whether plane or solid.

Of all isoperimetrical figures the plane triangle and solid triangle, or pyramid, are the least capacious, and amongst these those have the least capacity whose angles are most acute.

But curved surfaces and curved bodies, and among curves the circle and sphere are those whose capacities are the largest, being formed, if we may so speak, of the most obtuse angles.

The theory of geometry may, therefore, be reduced to the doctrine of angles, as it treats only of the boundary of figures, and by angles the ultimate boundary of all figures are formed; the angles give them their figure.

Angles are measured by the circle; parallel lines are the source of all geometrical similitude and comparison.

* Isoperimetrical figures are such as have equal circumferences.

Taking and measuring angles is the chief operation in practical geometry, and of great use and extent in surveying, navigation, geography, astronomy, &c.

And the instruments generally used for this purpose are quadrants, sextants, theodolites, circumferentors, &c., as described hereafter.

DEFINITIONS.

The first definition in geometry is a point, which has position, but no parts or dimension.

Note.—In surveying, a point or station is of importance, as between the beginning and end of a line there are frequently several points or stations, at which other lines commence or end; on the accuracy in fixing these points in position or in a straight line the whole survey depends.

A surface or superfices is that magnitude or quantity which is comprehended under two dimensions, length and breadth, without any regard to depth, and, therefore, bounded by lines only; so when the measure of a field, building, &c., is made, it is taken for the surface only.

A cubic or solid body is that which is contained under three dimensions; length, breadth, and thickness, or depth, is when we measure a block of stone, wood, quarry excavation, embankment, &c.

Lines are either parallel, oblique, perpendicular, or tangential; parallel lines are those which have no inclination to each other and never meet, though ever so far produced.

Note.—In surveying and plotting, to draw or pole out a straight line is of the utmost importance; however simple it may appear, it is difficult. If the base lines of a survey are not most accurately poled out straight, no matter how accurate they may be chained, it will affect the whole survey, as all other lines that terminate or intersect it will be either too short or too long. The same applies also to plotting the survey on paper; the straight edge or rule should always be tested by drawing a fine pencil line on the paper, and reversing the edge.

Angles are either right, acute, or obtuse.

A figure of three sides is called a triangle, and receives particular denominations from the relation of its sides and angles.

An equilateral triangle is that whose sides and angles are all equal.

An isosceles triangle is that which has two of its sides and angles equal, and one side greater or less.

A scalene triangle is that whose three sides and angles are all unequal.

A right angle triangle is that which has one right angle, or one line, perpendicular to another.

In a right angle triangle the side opposite to the right angle is called the hypothemese, the other sides the base and perpendicular.

An obtuse angled triangle has one obtuse and two acute angles.

An acute angled triangle has all its three angles acute.

A figure of four sides and angles is called a quadrangle or quadrilateral.

A parallelogram is a quadrilateral having both its opposite sides parallel, and all the angles right angles.

A rectangle is a parallelogram whose opposite sides are equal and parallel to one another, and all the angles right angles.

A square or tetragon is an equilateral rectangle having all its sides equal and parallel to one another, and all its angles right angles.

A rhombus is a parallelogram whose sides are parallel and equal in length to each other, and the opposite angles equal to each other, forming two obtuse angles and two acute angles.

A rhomboid is a parallelogram whose opposite sides are parallel and equal to one another, also the opposite angles are equal to each other.

A trapezoid has two sides parallel to each other, and of different lengths, both perpendicular to one side; it has two right angles, the other two angles are obtuse and acute.

A trapezium is a quadrilateral whose sides and angles are all unequal to one another.

A diagonal is a line joining any two opposite angles of a quadrilateral.

Plane figures having more than four sides are called polygons,

and receive particular names, according to the number of their sides and angles, thus:

A pentagon has 5 sides, hexagon 6 sides, heptagon 7 sides, octagon 8 sides, nonagon 9 sides, decagon 10 sides, undecagon 11 sides, duodecagon 12 sides.

A regular polygon has all its sides and angles equal to one another.

An equilateral triangle is also a regular polygon of three sides, a square is one of four sides, the former being called a trigon, the latter a tetragon.

The sum of three angles of every triangle is equal to two right angles, or 180 degrees.

The sum of two angles in any triangle taken from 180 degrees leaves the third angle.

In a right angled triangle, if either acute angle be taken from 90 degrees, the remainder will give the other acute angle.

When the sine of an acute angle is required, subtract such obtuse angle from 180 degrees, and take the sine of the remainder.

The circle is a plane figure bound by a curve line, called the circumference (it is often called the periphery), and is divided into 360 equal parts, called degrees; each degree is divided into 60 equal parts, called minutes, each minute into 60 equal parts, called seconds, the next thirds, and so on. Degrees are expressed by a small ° over the number, minutes by one dash ′, seconds by two dashes ″, thirds by three dashes ‴, and so on in progression, as thus: $45° \; 14' \; 10'' \; 4'''$ reads forty-five degrees, fourteen minutes, ten seconds, and four thirds.

The diameter is a line drawn through the centre of a circle to the circumference on both sides.

An arc of a circle is any part of the circumference.

A chord is a right line joining the extremes of an arc.

A segment is any part of a circle bounded by an arc and its chord.

A semicircle is half the circle cut off by the diameter, equal to 180 degrees.

A sector is any part of the circle bounded by an arc and two radii, drawn from the centre to its extremities.

A quadrant, or quarter of a circle, is a sector having a quarter of the circumference for its arc, and its two radii perpendicular to each other, equal to 90 degrees.

The height or altitude of a figure is a perpendicular let fall from an angle, or its vortex, to the opposite side of the base.

The complement of an arc, or an angle, is its difference from a quadrant, or what it wants of 90 degrees.

The supplement of an arc, or an angle, is its difference from a semicircle, or what it wants of 180 degrees.

The sine of an arc is a line drawn from one extremity of an arc, perpendicular to the diameter.

The versed sine of an arc is that part of the diameter intercepted between the arc and its sine.

The tangent of an arc is a line drawn perpendicular to the diameter, touching the circumference.

The secant of an arc is the line proceeding from the centre, and limiting the tangent of the same arc.

The chord of an arc is a line touching the extremities of the arc.

The chord of 60° is the sine of 90°;

The versed sine of 90°; the tangent of 45°;

The secant of 0°; are all equal to radius.

Note.—The co-sine, &c., is the abbreviation of complement.—*See* Trigonometrical Cannon, Fig. 1, Plate 27.

LINES.

Problem 1.

To draw a line parallel to another, Plate 1, Fig. 1.

Take the distance required in the compass, from two or three points on the line A B, describe arcs as *a b c*, draw a line touching those arcs will be the parallel line required.

Problem 2.

Another method when the line is to pass through a given point, C, Fig. 2.

From C draw a line at pleasure to B, with any radius describe the arc A *a*, with the same radius describe the arc *b c*; then take the distance A *a*, with the same distance from *b*, intersect the arc at *c*, and draw the line C *c*, the parallel line required.

Problem 3.

To draw lines parallel to the curve A B C, the centre being given, Fig. 3.

Bisect A B and B C, and draw the perpendiculars D E and F G; then draw the lines A D, D G, G C; with the radius A D describe the arcs A B and B C; then with the radius D E describe arc E, with the same radius and centre G describe the arc F, and with the radius D H describe the arc H, and also from G the arc I, will complete the three parallel lines required.

Problem 4.

To bisect or divide a line into two equal parts, Fig. 4.

From A and B as centres take in the compass any distance greater than half, and describe arcs intersecting each other, as at *a* and *b*; draw a line through those points, cutting A B at *c*, the point of division required.

N.B.—A perpendicular on any part of a line may be found by the same rule.

Problem 5.

To raise a perpendicular from a given point, C, on the line A B, Fig 5.

At C with any radius describe the arc *a b c*; with the same radius from *a* intersect the arc at *b*; from *b* with the same radius draw the arc *d c*; also with the same radius from *c* intersect those arcs at *b d*; draw the line *d* C, will be the perpendicular required.

Problem 6.

Another method when the point is at the end of the line A B, Fig. 6.

Take in the compass any distance, and describe the arc *a b c*; then draw a line through *a* and the centre of the arc at *d*, intersecting the same arc at *c*; then through *c*, and the point of intersection at *b*, draw the perpendicular *b c*.

Problem 7.

Another method to let fall a perpendicular from a given point near the middle of a line, Fig. 7.

From A with any radius describe the arc *b c*; with the same radius describe the arcs intersecting each other at *d*; draw the line A *d*, the perpendicular required.

Note.—Perpendiculars are more readily drawn by a protractor.—*See* Instruments, or a Right Angle Square.

Problem 8.

To find a mean proportion between two lines, as A B and B C, Fig. 8.

Draw a line, on which set off A B and B C; bisect A C in *a*; with the radius A *a* or *a* C describe the arc A *b c*; from the point B raise a perpendicular, intersecting the arc at *b*; draw the line *b* B, the length of the mean proportion required.

Problem 9.

To draw a line or scale equal in proportion to another, A B the given scale, B C the given line to be divided, Fig. 9.

From B draw the line B C at pleasure, and set off the given distance; divide A B into the required number of parts, draw the line A C, parallel to which draw lines through each division on A B, cutting C B, which will be the divided proportion required.

Problem 10.

To divide a line or scale into any number of equal parts, Fig. 10.

Given the length of the line A B, to be divided into five equal parts. Draw the line C D, on which mark off the given number of parts; with the radius C D describe arcs intersecting at *a*; from *a*, draw the lines *a* C and *a* D; then with the radius of the given length A B from *a*, set off A and B; draw lines from *a*, to the division, on C D, cutting the line A B into the required number of parts.

Problem 11.

To divide a line or scale A B into eight equal parts, Fig. 11.

From A draw the line A *b* at pleasure, and from B draw the line B *c* parallel to A *b*; from A set off seven equal divisions, and the same number on the line B *c*; draw lines from the corresponding numbers, and the intersections on A B will be the required divisions.

ANGLES.

Angles are usually expressed by three letters; that placed at the angular point is always in the middle, as E is the angle of A E D or B E D, Fig. 16*a*.

An angle is the inclination of two lines meeting in a point called the vertex, and is of a certain number of degrees, or part of the circumference of a circle. Angles are of three kinds.

A right angle is when one line is perpendicular to another equal to 90 degrees, or the fourth part of a circle, as A B C, Fig. 12, Plate 1.

An acute angle is that which is less than a right angle, as A B D.

An obtuse angle is that which is greater than a right angle, or more than 90 degrees, as D B E.

Problem 12.

To draw an angle equal to another, Fig. 13.

With any radius in the compass describe the arc B C; set off the length A B; then with the compass take the distance B C;

and from B intersect the arc at C; through C draw the line A C; then will C A B be the angle required. Or by the following rule:

Problem 13.

The measure of an angle is an arc of a circle contained between two lines which form that angle, as C B; the angular point being the centre of the circle as at A, and is estimated by the number of degrees contained in that arc.

Thus from the scale of chords (which is generally engraved on the ivory protractor marked C H) take in the compass 60 degrees; and from the angle at A, Fig. 13, Plate 1, describe the arc B C; then with the compass take the distance from B to C, and apply it to the scale of chords, which will give the measure or number of degrees and minutes contained in the angle.

Problem 14.

To measure an angle by the protractor.

Place the centre of the protractor at the point or angle A and the edge along the line A B; extend the lines sufficiently long to read where it cuts the outer edge of the protractor, and the number of degrees and minutes it reads from B to C will be the measure of the angle required, equal to 29° 5′.

Problem 15.

To bisect or divide an angle into two equal parts, Fig. 14.

From A, with any radius, describe the arc *b c*; then from *b* and *c*, as centres, describe arcs intersecting each other at *d*; draw the line *d* A, which will divide the angle as required.

Problem 16.

To trisect or divide a right angle into three equal parts, Fig. 15.

From A, with any radius, describe the arc B C; with the same radius from B and C, describe other arcs intersecting at *d* and *e*; then draw the lines A *d* and A *e*, which will divide the angle as required.

Problem 17.

Another method: To trisect a given angle A B C less than a right angle, Fig. 16.

With any radius describe a circle; draw the line A B D; from the points A and C as centres, with the radius A C, describe arcs intersecting each other at E; from E draw the line E *a* D, cutting the circle at *a*; with the distance C *a* mark off *b*, then will the angle A B C be trisected.

Problem 18.

When two lines intersect each other as at E, Fig. 16*a*, the opposite angles are equal to each other: viz. the angle A E C is equal to the angle D E B; and the angle A E D is equal to the angle C E B.

Note.—This problem is the main principle on which large surveys are conducted, for when any of the two lines are connected by another line, as, for instance, from A to C, or from B to D, the whole becomes fixed, and all other lines intersecting them must, when plotted, agree with the number or distance measured; examples of which will be hereafter given.

TRIANGLES.

Problem 19.

To describe a right angle triangle, the base and perpendicular being given, Fig. 17, Plate 2.

At the point or angle A, erect a perpendicular, and set off the required height; also the given distance on the base from A to B; then draw the hypothenuse from C to D will be the triangle required.

Note.—In any right angle triangle, the square of the hypothenuse is equal to the sum of the squares of the other two sides.

Problem 20.

To describe an equilateral triangle, Fig. 18.

From the points A and B, with the radius A B, describe arcs intersecting each other at C; draw the lines C A and C B, then will A B C be the triangle required.

Problem 21.

To describe an isosceles triangle, Fig. 18, Plate 2.

From the points A and B, with the radius C B, describe arcs intersecting each other at C; draw the lines C A and C B, then will A B C be the triangle required.

Problem 22.

To describe a scalene triangle, the three sides being given, Fig. 19.

Set off the given length of the base A B; from A, with the distance A C, describe an arc; and from B, with the distance B C, intersect the arc at C, and draw the lines A C, and C B will be the triangle required.

Note.—The longest side of any triangle is opposite the greatest angle, and the shortest side will be opposite the smallest angle.

In any triangle the sum of all three angles are equal to two right angles, or 180 degrees.

The sum of two angles in any triangle taken from 180 degrees, leaves the measure of the third angle.

All the outer angles of any triangle added together are equal to four right angles.

Every triangle has six parts—viz. three sides and three angles.

Problem 23.

Fig. 17. Two sides of a right angled triangle given to find the third.

Rule 1. To the square of the base add the square of the perpendicular; the square root of that sum will be the hypothenuse.

Rule 2. Multiply the sum of the hypothenuse and one side by their difference, and the square root of that product will be the other side.—*See* Euclid, Prob. 47, book i.

Example 1. Given the base $A\,B = 320$ and perpendicular $B\,C = 246$, required the length of the hypothenuse A C.

Ans. $320^2 + 246^2 = \sqrt{162916} = 403$ the hypothenuse

Example 2. Given the base $A\,B = 320$ and the hypothenuse $B\,C = 403$, required the length of the perpendicular B C.

Ans. $403 + 320 \times 83 = \sqrt{60009} = 245$ the perpendicular

See also Trigonometry, Case 1.

PARALLELOGRAMS.

Problem 24.

To describe a quadrilateral or square on a given base, Fig. 20, Plate 2.

From the given length A and B, draw perpendiculars; on which set off the length equal to A B; then C D will be equal to A B, and all the angles right angles.

Another method: From A and B as centres, describe the arcs A C and B D, cutting each other in *m*; bisect A *m* or B *m* in *n*; with the radius *m n*, at *m* as a centre, describe arcs cutting A C and B D; then draw the lines A D, D C, C D will be the square required.

Note.—If diagonals are drawn from the opposite angles of the square and the rhomboid, they will be at right angles to each other.

Problem 25.

To describe a rectangle whose base and perpendicular are given, Fig. 21.

From the given base raise perpendiculars at A and B; on which set off the given lengths of the sides A D and B C; then C D will be equal to A B, and all the angles right angles.

Problem 26.

To describe a rhombus on a given side, Fig. 22.

Mark off the length of the given side A B; from B, with the radius A B, describe the arc D C; from A, with the same radius, intersect the arc at D; then with the same radius from D intersect the arc at C; draw lines through the several points, and the rhombus will be complete. The opposite angles will be equal to one another, two obtuse and two acute.

Problem 27.

To describe a rhomboid whose sides and angles are given, Fig. 23.

The length of A B = 400; the length of A D = 250; the

angle D A D = 60°. Set off the line A B to the given length; by the protractor from the point A set off the given angle; draw the line B C parallel to A D, and mark off the given length of the sides A D and B C; then draw the line D C equal and parallel to A B; the angles are the same as the rhombus.

Problem 28.

To construct a trapezoid whose base and perpendiculars are given, Fig. 24, Plate 2.

From A and B raise perpendiculars, and mark off the given lengths of the sides A D and B C; draw the line D C, which gives the trapezoid; the angles A and B are right angles; the angle C is obtuse, and the angle D acute.

Problem 29.

To construct a trapezium, whose base A B and all the sides are given, Fig. 25.

Draw a line, and set off A B equal to the given length; from A, with the given length of A C, describe an arc; then from B and length of B C intersect that arc at C; draw the lines A C and C B; in like manner from A and B draw arcs equal to the given lengths of the sides A D and D B, intersecting each other at D; draw the lines A D and D B, the trapezium required.

Problem 30.

To find the side of a square that shall be any number of times the area of a given square, Fig. 26.

Let A B C D be the given square, then will the diagonal B D be the side of the square A E F G, double in area to the given square A B C D; and if the diagonal be drawn from B to G, it will be the side of the square A H K L, three times the area of the square A B C D; and the diagonal B L will be equal to the side of the square, four times the area of the square A B C D.

POLYGONS.

Problem 31.

The breadth of a polygon given to find the radius of a circle to contain that polygon, Fig. 26, Plate 2.

Rule. Multiply half the breadth of the polygon by the numbers in the sixth column of the Table No. 14, opposite its name, and the product will be the radius of a circle to contain that polygon.

If the polygon have an uneven number of sides, the half breadth is accounted from its centre to one of its sides.

Problem 32.

Required the radius of a circle to contain an octagon whose breadth A B = 36.4 feet.

Ex. A B = 36.4 ÷ 2 = 18.2 × tab. num. 1.08 = 19.656 feet

Problem 33.

Required the radius of a circle to contain a pentagon whose half breadth A B = 14.4, Fig. 27.

Ex. A B = 14.4 × tab. num. 1.238 = 17.827

Problem 34.

The radius of a circle given to find the length of the side of an octagon, Fig. 26.

Rule. Multiply the radius of any circle by the numbers in the fifth column in the table opposite the polygon required, and the product will be the length of the side nearly, that will divide that circle into the proposed number of sides.

Given the radius O C = 19.656, required the length of the side C D.

Ex. O C = 19.656 × tab. num. .76536 = 15.043916 the length of the side

Problem 35.

The length of the side C D is 15.036840 of an octagon, given to find the radius D O, Fig. 26.

Rule. Multiply the given length of side by the number on the fourth column in the table opposite the polygon required, and the product will be the radius of a circle to contain the polygon.

Ex. C D = 15.036840 × tab. num. 1.307 = 19.646 nearly

Problem 36.

To find the angle at the centre and sides of a regular polygon, Fig. 27.

Rule. Divide 360°, the degrees in a circumference, by the number of sides in the proposed polygon.

Thus: 360° ÷ 5 = 72° for the angle of the pentagon at the centre

Problem 37.

To find the angle formed by the sides.

Subtract the angle at the centre from 180°.

Thus: 180 − 72° = 108° the outside angle of a pentagon

Problem 38.

Methods of describing arcs of circles of large magnitude.

Angles in the same segment of a circle are equal to one another.

Let A B C D, Fig. 28, Plate 2, be the segment of a circle; the angles formed by lines drawn from the extremities A and B of the base of the segment to any points C and D of its arc, as the angles A C B, A D B are equal.

Note.—This problem is referred to hereafter in setting out railway curves.—*See* Euclid, Prob. 31, book iii.

Problem 39.

Of finding points in and describing large circles.

If upon the ends A B, Fig. 29, Plate 2, of a right line, A B as an axis, two circles or rollers, C D and E F, be firmly fixed, so that the said line shall pass through the centres, and at right angles to the planes of the circles, and the whole be suffered to roll upon a plane without sliding:

If the rollers C D and E F be equal in diameters, the lines described upon the plane by their circumferences will be parallel right lines, and the axis A B, and every line D F, drawn between contemporary points of contact of the rollers and plane will be parallel among themselves:

If the rollers C D and E F be unequal, these lines formed by their circumferences upon the plane will be concentric circles, and the axis A B, and also the lines D F, will in every situation tend to the centre of those circles:

Then it will be as the diameter of the wheel C D is to the difference of the diameter of C D and E F, so is the radius of the circle proposed to be described by C D to the distance A B, at which the two wheels must be asunder, measured upon the plane on which the circle is to be described.

The wheel will evidently describe simultaneously another circle, whose radius will be less than that of the former by E F.

Note.—This rule is applicable to the wheels of railway carriages rolling over a curve; instead of the outer wheel being greater in diameter than the inner one, the outer rails are gradually raised from their tangents to the proper height, according to the radius of the curve or arc of the circle.

Problem 40.

Through three points to describe an arc of a circle by means of two laths, Fig. 30, Plate 2.

On a given chord A B to describe an arc of a circle that shall contain any number of degrees, performing the operation without compasses, and without finding the centre of the circle.

Place two straight laths forming an angle A C B, equal to the supplement of half the given number of degrees, and fix them at C; fix two pins at the extremities of the given chord A B, hold a pencil at C, then move the two laths against the pins, and the pencil at C will describe the arc required.

Example. It is required to describe an arc of fifty degrees on the given chord A B, subtract 25 degrees (half the given angle) from 180°; the difference will be 155° equal to the supplement;

then form the angle A C B = to 155° with the two rules, and proceed as before described.

If required to extend the arc on either or both sides, as at D E, first mark on the edge of the two legs of the instrument at A and B; bring the centre C to A, and the edge of the right leg over the point at C, point off the distance marked off on the left leg as at D, and fix the pins as before at D A C; in like manner fix the pins at E, and describe the continuation of the arc on both sides.

The whole circle may be described in the same manner by placing the two lower pins below the diameter and describing one half the circle; then fix the lower pins in the same manner on the upper part of the circle (keeping the laths fixed), and describe the other half of the circle in like manner.

Problem 41.

A very neat instrument, called the bevel or centrolinead, for the same purpose as last described, Fig. 31.

It consists of two rules, movable on a common centre, similar to a carpenter's rule; the brass semicircle has a groove and a screw at D to fix the two legs at the required angle.

The instrument is placed against the pins at A D C, as shown in the former example, and the pencil at C; keep the rules close to the pins, holding the pencil at C, and describe the arc required.

Problem 42.

Given three points to draw a line from either of them tending to the centre of the circle, and passing through each, Fig. 32, Plate 2.

Let A B C be the given points; it is required to draw A D, so that if continued it would pass through the centre of the circle containing A B C.

Make the angle B A D equal to the complement of the angle B C A, and A D is the line required; for supposing A E a tangent to the point A, then is E A D a right angle, and E A B =

B C A; whence B A D = right angle less the angle B C A, or the complement of B C A.

Corollary 1. A D being drawn, lines from B and C, or any other points in the same circle, are easily found; thus, make A B G = B A D, which gives B G; then make B C F = C B G, which gives C F; or C F may be had without the intervention of B G, by making A C F = C A D.

Corollary 2. A tangent to a circle at any of the points, A for instance, is thus found.

Make B A E = B C A, and the line A E will touch the circle at A.

To perform the same by the bevel.

Set the bevel to the three given points A B C; lay the centre on A and the right leg on C, the other leg will give the tangent H A; from A draw A D perpendicular to H A, for the line required.

CIRCLES.

Problem 43.

To find the centre of a circle, Fig. 33, Plate 2.

Draw any chord A B, and bisect it with the line C D; bisect C D by the chord E F; the intersection at O will be the centre of the circle.

Problem 44.

Through three given points to describe the circumference, Fig. 34.

From the middle point B draw the chords B A and B C; bisect these chords and draw the lines *n* O, *m* O; from O, with the radius O A, O B, O C, describe the circle required.

Problem 45.

To draw a tangent to a given circle, to pass through a given point at A, Fig. 35.

When the point A is in the circumference of the circle.

From the centre O, draw the radius O A; through the point

A draw C D perpendicular to O A, which will be the tangent required.

Problem 46.

When the point A is without the circle, Fig. 36.

From the centre O draw O A, and bisect it in *n*; from the point *n*, with the radius *n* A or *n* O, describe the semicircle A D O, cutting the given circle in D; through the points A D draw A B, the tangent required.

Problem 47.

To cut off from a circle a segment containing any proposed angle, as 60°, Fig. 37.

Let F be the point from whence it is required to draw a chord which shall contain 60 degrees; through F draw F R, a tangent to the circle; from F draw F A, making an angle of 60 degrees with the tangent F R, and F C A is the segment required.

Problem 48.

To find the diameter of a circle that shall be any number of times the area of a given circle, Fig. 38.

Let A B C D be the given circle, draw the two diameters A B and C D at right angles to each other; the chord A D will be the radius of the circle O P, twice the area of the given circle (nearly), and half the chord will be the radius of a circle that will contain half the area, &c.

Problem 49.

To divide a given circle into any number of concentric parts, equal in proportion to each other, Fig. 39, Plate 3.

Upon the radius A B describe the semicircle A *e d* B; divide A B into the proposed number of equal parts, as 1, 2, &c.; erect perpendiculars from those points to the semicircle; then from the centre A and radius A *e* A *d*, &c., describe circles, which will be the proposed number of parts.

Problem 50.

Required the diameter of a pipe that shall be equal to three pipes of three inches diameter each, Fig. 40, Plate 3.

Rule. Multiply .7854 by the square of the given diameter; multiply that product by the number of times required to be enlarged; the square root of that product multiplied by 1.12837 will give the diameter of the pipe required.

Note.—This problem shows the error committed by an engineer on the application of a water company for the above enlargement, on which a pipe was made three times the diameter, as shown by the diagram.

Ex. $.7854 \times 3^2 = 7.0686 \times 3 = 21.2058$, the area required

And $\sqrt{21.2058} = 4.60 \times 1.12837 = 5.1905$, the dia. C D

The area of the 3 inch pipe = 7.0686
The area of the 9 inch pipe = 63.617

Therefore the pipe 9 inches in diameter = 63.617, was in excess by 58.426.

TO CONSTRUCT A PROTRACTOR, OR DIVIDE A CIRCLE INTO 360 DEGREES, MINUTES, ETC.

With a radius of 5 inches describe a circle; with the same radius mark off each length very minutely round the circle, which should be exactly 6 times, or 60° each.

Then set the compass to the natural sine of 15°, which to radius 5 will be equal to half the chord of 30°; this distance will bisect each 60°, and divide the circle into arcs of 30° each. This may be proved by setting the succeeding chords off each way from those points which they are intended to bisect; if any inaccuracy exists the bisector will not be perfect, and if the error is inconsiderable the middle point will be assumed as correct.

Each 60° may next be trisected by setting off the natural sine 10°, equal the chord of 20° to our radius, which will divide the circle to every 10 degrees.

The natural sine of 7° 30′, equal the chord of 15°, marked from the points already determined, will divide the circle to every 5th degree.

The natural sine of 3°, equal the chord of 6°, divided as before, divides 30° into 5 parts, and set off from the other divisions, divides the circle to single degrees.

Fifteen degrees bisected on the natural sine of 3° 45′, equal the chord 7° 30′, set off from the other divisions, divides the circle into half degrees.

The natural sine of 3° 20′, equal the chord of 6° 40′, divides 20° into three parts, and set off from the rest of the division, divides the whole circle to every 10 minutes, which is as minute as can possibly be obtained; smaller quantities must be subdivided by the eye.

The division should be numbered in like manner to the theodolite; that is, from 0° to 360°.

INSCRIBING AND CIRCUMSCRIBING FIGURES.

Problem 51.

To inscribe a circle in a given triangle A B C, Fig. 41, Plate 3.

Bisect the angles A and B; from the point of intersection at O, let fall a perpendicular O N, it will be the radius of the required circle.

Problem 52.

To inscribe a pentagon, or hexagon, or a decagon in a given circle, Fig. 42.

Draw the diameters A B and C E at right angles to each other; bisect D B at G, on G with the radius G C describe the arc C F; join C and F, and the line C F will be one side of the pentagon.

The two sides D C, F D, of the triangle F D C, inscribe an hexagon or a decagon in the same circle; D C is the side of the hexagon, D F that of the decagon.

Problem 53.

To inscribe a square or an octagon in a given circle, Fig. 43.

Draw the diameters A C, B D, at right angles to each other; draw the lines A D, B A, B C, C D, the square required.

For the octagon: Bisect the arc A B and A D of the square in the point F and H; draw lines from F and H through the centre, intersecting the circumference at E and G; draw lines from each point of intersection, which will be the required octagon.

Problem 54.

In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon, Fig. 44.

For the equilateral triangle: From any point A as a centre, with a distance equal to the radius A O, describe the arc F O B; draw the line B F, and make B D equal to B F; join D F, and D B F will be the equilateral triangle required.

For the hexagon: Carry the radius A O six times round the circumference it will give the hexagon required.

For the dodecagon: Bisect the arc A B of the hexagon in the point *n*; and the line A *n* being carried twelve times round the circumference will form the dodecagon required.

Problem 55.

To inscribe a dodecagon in a circle, or to divide the circumference of a given circle into twelve parts, each being equal to 30 degrees, Fig. 45.

Draw the two diameters 12, 6, 3, 9, perpendicular to each other, with the radius of the circle; on the points 12, 6, 3, 9 as centres describe arcs through the centre, intersecting the circumferences in the points required, dividing it into twelve equal parts.

Problem 56.

In a given circle to inscribe any regular polygon, Fig. 46.

Draw the diameter on A B; upon A B construct an equilateral triangle A B C; divide the diameter into as many equal parts as the polygon has sides; from C draw a line through the second division in the diameter intersecting the circumference at D; join the points A D, it will be one side the octagon.

Problem 57.

About a given triangle to circumscribe a circle, Fig. 47.

Bisect any two sides A C, B C; draw the lines D O and F O; the point of intersection at O is the centre; with the radius A O describe the circle required.

Problem 58.

About a given circle to circumscribe a pentagon, Fig. 48.

Inscribe a pentagon within a circle; through the middle of each side draw the lines O A, O C, O D, O E, and O B; through the point *n* draw the tangent A B, meeting O A and O B in A and B; through the points A and *m*, draw the line A *m* C, meeting O C in C; in like manner draw the lines C D, D E, E B; and A B C D E will be the pentagon required.

Problem 59.

On a given line A B to form a regular octagon, Fig. 49.

On the extremities of a given line A B erect perpendiculars A F, B E; produce A B both ways to *s* and *w*; bisect the angles *n* A *s* and *o* B *w* by the lines A H, B C; make A H and B C each equal to A B, and draw the line H C; make *o v* equal to *o n*, and through *v* draw G D parallel to H C; draw H G and C D parallel to A F and B E, and make *v* E equal to *v* D; through E, draw E F parallel to A B, and join the points G F and D E, and A B C D E F G H will be the octagon required.

Problem 60.

To circumscribe a circle about a given square A B C D, and construct an octagon, Fig. 50.

For the circle: Describe the square A B C D; draw the two diagonals A C and D B, intersecting each other at O; with the distance O A or O B, describe the circle required.

For the octagon: Bisect the sides A B C D; draw lines from *a* to *c* and *b* to *d*; join the lines *a d*, *d c*, *c b*, and *b a*, forming an-

other square; intersecting the sides A B C D at the points 1, 2, 3, 4, 5, 6, 7, 8, the octagon required.

Problem 61.

To inscribe a nonagon in a circle, Fig. 51.

Draw the diameters, and bisect the radius O D at *a*; draw the line *b a d* parallel to A *c*, intersecting the circumference at *b*; draw the line B *b*; divide the arc B A *b* into three equal parts, draw the line B *c*, will be one side of the nonagon.

Problem 62.

To inscribe an undecagon in a circle, Fig. 52.

Draw in the diameters and bisect the radius O D in *a*; draw the line *b a d* parallel to A C, intersecting the circumference at *b*; the distance from E to O will be one side of the undecagon.

Problem 63.

To find a right line that shall be nearly equal to any given arc of a circle, Fig. 53.

Divide the chord A B into four equal parts; set one part on the circumference from B to D; draw the line from C to the first division on the chord; and twice the length of the line C D will be the length of the arc nearly.

Problem 64.

To find the side of a square nearly equal in area to a given circle, Fig. 54.

Draw the two diameters A B and C D at right angles to each other; bisect the radius O C by a line from one end of the diameter at A, meeting the circumference in E, then will the line A E be the side of a square nearly equal in area to a given circle; and if the line E F be drawn parallel to C D, it will be $\frac{1}{4}$ of the circumference nearly, or three times the diameter, and once the versed sine Q H of the angle A O D will be the circumference nearly.

Problem 65.

To find the length of an arc of a circle, Fig. 55.

Rule 1. Subtract the chord of the whole arc from eight times the chord of half the arc, and $\frac{1}{3}$ the remainder is the length of the arc nearly.

Required the length of the arc A C B, the chord of half the arc A C = 1.30, and the chord of the whole arc A B = 2.50.

Ex. $1.30 \times 8 = 10.40 - 2.50 = 7.90 \div 3 = 2.63$ the length of the arc nearly

Rule 2. Multiply the radius by the number of degrees contained in the arc, and the product by .0174524.*

Given radius 250; angle 60°.

Ex. $2.50 \times 60° = 150.00 \times 0174524 = 2.617$ length of arc

Problem 66.

To find the length of an arc by the Table, No. 17.

Rule. Divide the height by the base, and the quotient will be the height of an arc of which the base is unity; multiply the tabular number opposite the corresponding quotient by the base of the arc, and the product equals the length required.

Problem 67.

To find the diameter of a circle by having the chord and versed sine given, Fig. 56, Plate 3.

Rule. Divide the square of half the chord by the versed sine; to the quotient add the versed sine, and the sum will be the diameter, &c.

Or: If the sum of the squares of the semichord and versed sine be divided by the versed sine, the quotient will be the diameter of the circle to which that segment corresponds.

Given the chord A B = 39.2, and the versed sine C D = 13.1; required the diameter C E.

Ex. $39.2 \div 2 = 19.6^2 = 384.16 \div 131 = 293 + 131 = 42.4$ the diameter required
Or: $19.6^2 + 13.1^2 = 55577 \div 131 = 42.4$ the diameter

* The decimal .0174524 is the natural sine of 1°.

Problem 68.

Having the diameter of a circle given to find the circumference; or the circumference given to find the diameter.

Rule 1. As 7 is to 22, so is the diameter to the circumference.

Or: As 22 is to 7, so is the circumference to the diameter.

Rule 2. As 1 is to 3.1416, so is the diameter to the circumference.

Or: As 3.1416 is to 1, so is the circumference to the diameter.

Example 1. Required the circumference of a circle whose diameter is 30.55.

$$30.55 \times 22 = 67210 \div 7 = 96.01\tfrac{3}{7} = \text{the circumference}$$

Example 2.

$$96.01\tfrac{3}{7} \times 7 = 67210 \div 22 = 30.55 \text{ the diameter}$$

Example 3. Required the circumference of a circle whose diameter is 30.55.

$$3.1416 \times 30.55 = 95.975880 \text{ the circumference}$$

Example 4. Required the diameter of a circle whose circumference is 95.975880.

$$95.975880 \div 3.1416 = 30.55 = \text{the diameter}$$

Problem 69.

To describe an ellipsis, the transverse and conjugate diameters being given, Fig. 57, Plate 3.

Draw the diameters at right angles to each other, and set off the given lengths, as at A B C D; with the difference of the two semidiameters set off O *a* and O *b*, take half the length of *a b*, and lay it off from *a* to C; then take the distance O *c*, and mark O *d*, O *e*, and O *f*, which will be the four centres of the ellipse; place the point of the compasses at *c*, with the radius *c* A, describe the arc *i* A *g*; also with the same radius from *e* describe the arc *h* B *k*; then with the radius *f c* from the point *f* describe the arc *g* C *h*, and with the same radius from *d* describe the arc *k* D *i*, which will be the ellipse required.

To find the area see problem 93.

Note.—The longest diameter is called the transverse = A B.
The shortest diameter is called the conjugate = C D.
Focii or focuses are the points on which the periphery of the ellipse is described as *c d e f*.

The most perfect method of drawing ellipses is by the trammel, or elliptic compass.

It consists of a cross with a dovetailed groove at right angles, in which two small pieces are made to run easily up and down each way, to which is a beam (similar to a beam compass) with sliding sockets, one for the pen, pencil, or point, the other two, forming the focii, fitting into the two running slides in the grooves; the instrument, when set to the two diameters, will draw the ellipse by one revolution.

To adjust it: Set the beam over the transverse A B, and slide it backwards and forwards until the pencil or ink point coincide with the point A, and tighten the screw of the slider, which moves on the conjugate axis; now turn the beam, so as to lay over the conjugate axis C D, and make the pencil or point coincide with the point C, then fix the screw, which is over the slider of the transverse axis; the compass being adjusted, move it gently round.

Ellipses may also be drawn by a string or chord; first find the two focii, then fix two pins in the focii, another in one end of the conjugate diameter, tie the string firmly, then with a pencil move round, will describe the ellipse.

Problem 70.

To describe a parabola, any ordinate to the axe and its abscissa being given, Fig. 58, Plate 3.

Let V R and R *o* be the given abscissa and ordinate, bisect R *o* in *m*, join V *m*, and draw *m n* perpendicular to it, meeting the axe in *n*, make V C and V F each equal to R *n*, then will F be the focus of the curve.

Take any number of points, *r r*, &c., in the axis, and draw the double ordinates of an indefinite length.

From F as a centre, with the radii C F, C *r*, &c., describe arcs cutting the corresponding ordinates in the points *o*, *o*, *o*, *o*, &c., and the curve *o* V *o*, drawn through all the points of intersection, will be the parabola required.

Problem 71.

To describe an egg-shape figure, Fig. 59, Plate 3.

Describe a circle A B equal to the given diameter, draw the lines H B and D F; bisect the radius A C or C B, then with half the radius draw the circle E L F K, touching the circle A B at E; then from A and B, with the distance D F, set off G H as centres to the sides A L and B K, draw the lines H I K and G I K, intersecting the circle E L F K at the tangent points K L, will be the figure required.

AREAS;

Otherwise Mensuration of Superfices.

The area or superficial content of any figure is the space contained within length and breadth, without having any regard to thickness.

As applied to land surveying in finding the areas or contents of plane figures, are calculated by the number of square links in each.

When figures are measured by yards or feet, the calculations are also made by their squares in like manner; in either case the contents are reduced to acres, roods, and perches.

All irregular or crooked fences must first be reduced to straight lines, to form the sides of the regular mathematical figures, which are chiefly confined to the triangle and trapezium.

Other methods are also introduced to obtain the accurate contents of irregular boundaries by mechanical rules, equally correct, and in some instances with greater precision.

The statute acre contains 1.00000 square links, more particu-

larly described in page 80. The area of a piece of land is calculated decimally by the number of square links contained in each piece, which are then reduced to acres, roods, and perches, by the following rules, or by the Table, No. 11. See also Tables of lineal and square measure, Nos. 27 and 28.

Problem 72.

To reduce the decimals of an acre to roods and perches.

Rule. Multiply the decimals by 4; cut off five figures to the right hand, the figure to the left (if any) will be roods; then multiply the remaining decimals by 40; cutting off five figures as before, the figures on the left will be poles.

Example. Reduce 5.90600 to acres, roods and perches.

```
 .90600
      4
 -------
3.62400
     40
 --------
24.96000 square links
```

Note.—The decimal remaining when it exceeds 5 may be considered equal to 1 pole; therefore the quantity is = 5 acres, 3 roods, 25 perches.

A more ready way of reducing decimals to roods and perches is by the Table, No. 11. (The acres or integers stand out.) The first column denotes poles; the second column, decimals; the three last columns are roods.

Example. Reduce 5.90600 to acres and decimals.

Look in the last column for the nearest number to 906; opposite to which, in the first column, is 25, or 3 roods 25 poles, equal to the quantity before calculated.

Problem 73.

To reduce acres, roods, and perches, to acres and decimals, or square links.

Rule. To the acres, annex five cyphers to the right hand; to the roods, add five cyphers and divide by four, the quotient will be the decimals of square links.

To the perches, add five cyphers and divide by sixteen, the quotient will be the decimals of square links.

Add all these sums together the product will be the decimal of square links. (See Table, No. 13.)

Example. Reduce 5A. 3R. 25P. to acres and decimals.

$$3.00000 \div 4 = .75000 \text{ and } .25000 \div 16 = .15625$$
$$\text{Then } 5.00000 + .75000 + 15620 = 5.90625 \text{ the decimal required}$$

Another method:

Rule. Multiply the number of poles by 6; reduce the roods (if any) into the 1000ths part of an acre, to which add $\frac{1}{4}$ of the perches given; add this product to the number of perches multiplied by 6; place the number of acres to the left hand of the product, it will be the required number of acres and decimals.

If there are no roods, add $\frac{1}{4}$ of the number of perches only to the amount of perches multiplied by 6.

Example 1. Reduce 5A. 3R. 24P. into decimals.

$$\begin{array}{rl} .750 & = 3 \text{ roods} \\ 6 & = \frac{1}{4} \text{ of } 25 \text{ poles} \\ \hline .756 & \end{array} \qquad \begin{array}{rl} 25 & \text{poles} \\ 6 & \\ \hline 756 & \\ \hline 5.906 & \text{the decimals required} \end{array}$$

Example 2. Reduce 4A. 0R. 27P. to decimals.

$$\begin{array}{cccl} \text{A.} & \text{R.} & \text{P.} & \\ 4 & 0 & 27 & \\ & & 6 & \\ \hline & & 162 & \\ & & 7 & = \frac{1}{4} \text{ of } 27 \text{ poles} \\ \hline & & 4.169 & \text{the decimals required} \end{array}$$

Problem 74.

To reduce links to feet, inches, and decimals; or feet, inches, and decimals into links. (See Table, No. 10.)

Rule 1. Multiply the links by 66, cutting off two figures to the right hand; if any decimals remain, multiply them by 12, cutting two decimals as before for inches, the remainder will be decimals.

Rule 2. First reduce the inches and decimals to its lowest denomination, by dividing them by 12; add this to the number of given feet, and divide by 66, the quotient will give the number of links.

Example 1. How many feet, inches, and decimals are there in 92 links?

$$92 \times 66 = 60.72 \text{ and } .72 \times 12 = 8.64$$
Or 60 ft. 8 in. = 64 dec.

Example 2. How many links are there in 60 ft. 8 in. 64 dec.?

$$8.64 \div 12 = .72. \quad \text{Then } 60.72 \div 66 = 92 \text{ links}$$

Problem 75.

To reduce square yards to acres, roods, and perches.

Rule. Divide the given quantity of square yards by 4840 (that being the number contained in one acre; see Table 28), the quotient will be acres and decimals, which are to be reduced to roods and perches.

In like manner, should the quantity given be in square feet, divide by 43560 the number of square feet in one acre.

Example. In 45678 square yards, how many acres, roods, and perches?

$$45678 \div 4840 = 9.437, \text{ or } 9\text{A. } 1\text{R. } 30\text{P.}$$

Another method:

Rule. Multiply the given quantity of yards by .0002; divide that product by $\frac{1}{30}$, or $\frac{1}{3}$ of a tenth; to which add the remainder.

Ex. $45678 \times .0002 = 9.1356 \div \frac{1}{30}$ or $\frac{1}{3}$ of a 10th $= 3045$
Then $9.1356 + 3045 = 9.4401$, or 9A. 1R. 30P.

Note.—When dimensions are taken in feet and inches and multiplied together; to reduce them to square yards, divide the product by 9.

Problem 76.

To find the area of a parallelogram A B C D, Fig. 61, Plate 4.

Rule 1. Multiply the length by the breadth, the product will be the area.

Example. Required the contents of the parallelogram, A B = .810 links, D B .156 links.

$$.810 \times .156 = 1.26360, \text{ or } 1\text{A. } 1\text{R. } 2\text{P.}$$

Problem 77.

To find the area of the triangle A B E, Fig. 61.

Rule 2. Multiply the base by half the perpendicular, or multiply the base by the perpendicular, and take half the product.

Example. Required the contents of the triangle A B = 810, E F 312 links.

A B 810 × 156 = 1.26360; or 810 × 312 = 2.52720 ÷ 2 = 1.26360

The two triangles A C G and B D H are equal to the triangle E G H.

Note.—This rule is universal in computing the quantities of land when reduced to trapeziums and triangles.

The same by logarithms.

Rule 3. Add the logarithm of the base to the logarithm of half the perpendicular; or the logarithm of half the base to the logarithm of the perpendicular.

Look in the table of logarithms of numbers for the given lengths in the first column, and against that number in the second column will be the required logarithm; add these numbers together, then seek in the second column for that or the nearest number to the product, opposite to which will be the natural number required.

Ex. The base . . .	A B =	.810 =	2.90848
Half the perpendicular	E F =	.156 =	2.19312
Square links . .		1.26360 =	5.10156

Problem 78.

By the same rule, the area of a square is obtained, Fig. 62, Plate 4.

Ex. A B = 800 × B D = 800 = 6.400, or 6A. 1R. 24P.
Or : C B = 1.130 × *a* D = 566 = 6.407, nearly

Extend the base to *b*, equal to A B, draw the line D *b*; the triangle A D *b* will be equal to the square A B C D.

Problem 79.

To find the area of a triangle, three sides being given, Fig. 63, Plate 4.

Rule 4. Add the three sides together, and take half the sum; subtract each side separately from half the sum of the

sides, and multiply the half sum by the three remainders, the square root of the sum will be the area.

Given A B = .810 links, A C = .476 links, B C = .549 links.

$$\frac{.810 + .476 + .549}{2} = .9175 \qquad \text{then } .9175 - .810 = .1075$$
$$.9175 - .476 = .4415$$
$$.9175 - .549 = .3685$$
$$.9175 \times 1075 \times 4415 \times 3685 = \sqrt{160465892984375} = 1.267$$

Note.—The same result will be found by a perpendicular demitted from the angle C to its opposite side, and multiply half the perpendicular by the side A B, as described by Case 3 in Trigonometry.

The same triangle by logarithms.

When two sides and the included angle are given.

$$\text{As Radius} \quad \ldots \quad 10.000000$$
$$: \text{sine of } \angle A = 41^\circ\ 0' \quad \ldots \quad 9.816943$$
$$:: A B = \frac{.810 \times B C = .476}{2} = .192780 = 5.285107$$
$$\text{Square links} \quad \ldots \quad 1.26490 = 5.102050$$

The same by natural sines.

Rule 5. Multiply the two given sides together, and half that sum multiplied by the number extracted from the table of natural sines, against the given angles, thus:

$$A B = .810 \times .476 = .385560 \div \tfrac{1}{2} = .192780$$

then the natural sine $\angle A = 41^\circ\ 0' = .656059 \times .192780 =$ 1.26476 square links, when reduced as before = 1A. 1R. 2P. nearly.

Problem 80.

To find the area of a trapezium A B C D, Fig. 64.

Rule 6. Divide the trapezium into two triangles by the diagonal A C, always taking the longest line; then the sum of the areas of the two triangles, found by the foregoing rules, will be the area of a trapezium.

When quantities are not given, apply the scale by which the figure was drawn, as thus: the perpendicular D a = 274 links, and b B = 160 links; then by the second rule add the two perpendiculars together, divide by two, and multiply the quotient

by the base A C = 680 links, the product will be the quantity required.

B b 160 + D a 274 = 434 ÷ 2 = 217 × A c 680 = 1.476, or 1A. 1R. 36P.

Note.—This calculation may in all cases be shortened by using the scale, Fig. 1, Plate 39. Observe it is divided one side by 40, that being the scale the plan is plotted, and in computing quantities is applied to measure the base of all triangles; on the other edge of the scale it is divided by 20, being one half, and is used to measure the perpendiculars of all triangles, as shown by the following example; it saves adding the whole perpendiculars, and taking half.

B b 80 + D a 137 = 217 × 680 = 1.476, as before

The same by natural sines, Fig. 65, Plate 4.

Rule 7. Multiply the two diagonals together, and multiply half the product by the natural sine of the included angle.

$$\frac{\text{B D } 680 \times \text{A C } 460}{2} = 156400 \times \text{nat. sine } \angle \text{ A } a \text{ D } .939693 = 1.470$$

The same by logarithms.

As Radius	. . . .	10.000000
: sine $\angle$ a 70° 0′	. . . . .	9.972986
:: A C $\frac{680 \times \text{B D } 460}{2}$ = 156400	. .	5.193792
Square links	. . 1.4682 =	5.166778

Problem 81.

To find the area of a rhombus or rhomboid, Fig. 66.

Rule 8. Multiply the base by the perpendicular height.

Given the base A B = 400 links; the perpendicular C a 342 links.

A B 400 × C a 342 = 1.368, or 1A. 1R. 19P.

The same by logarithms.

A B 400	. . . . . .	2.602060
C a 342	. . . . . .	2.534026
Square links	. . 1.36800 =	5.136086

Problem 82.

Required the area of a rhomboid, Fig. 67.

Given the base A B = 600 links; the perpendicular C a 176 links.

A B 600	. . . . . .	2.778151
C a 176	. . . . . .	2.245513
Square links	. . 1.05600 =	5.023664

Problem 83.

To find the area of a trapezoid, Fig. 68.

Rule 9. Add together the two parallel sides, and half that sum, multiplied by the height, will give the required area.

Given A B 440 links, C D 325 links, *b a* 280 links.

A B 440 + C D 325 = 762 ÷ 2 = 382 × 280 = 1.0696, or 1A. 0R. 11P.

Also draw the line *a b* through the middle of A C parallel to B D, measure the length *a* B, multiply it by the height *a b* equal to B D, the result will be the same.

Note.—For practice calculate the last four figures by Rule 6, page 58.

Problem 84.

To find the area of a pentagon, Fig. 69.

Rule 1. Multiply the perimeter or sum of its sides by a perpendicular from its centre to one of its sides, and take half the product for the area.

Rule 2. Multiply the square of the side of a polygon by the number in the third column of areas (see Table, No. 14) opposite the number of the sides required, the product will be the area required.

Required the area of a regular pentagon, each side being 210, and the perpendicular F G 144.

A B 210 × 5 = 1050 × F G 144 = 1.51200 ÷ 2 = .75600

The same by Rule 2.

210^2 = 44100 × tab. num. 1.720 = .75852, or 0A. 3R. 1P.

Problem 85.

Required the area of an octagon, each side being 150 and the perpendicular 195, Fig. 70.

A B 150 × 8 = 1200 × *a* A 195 = 234000 ÷ ½ = 1.170

The same by Rule 2.

150^2 = 22500 × 4.82843 = 1.08639 = 1A. 0R. 14P.

CIRCLES.

Problem 86.

To find the area of a circle.

Rule 1. Multiply the square of the diameter by .7854, or the diameter by the circumference, and divide by 4.

To find the area of a sphere.

Rule 2. Multiply the square of the diameter by 3.1416.

To find the solidity of a sphere.

Rule 3. Multiply the cube of the diameter by .5236.

To find the circumference of a circle.

Rule 4. Multiply the diameter by 3.1416.

To find the surface of a spherical segment or zone.

Rule 5. Multiply the diameter by the height, and then by 3.1416.

To find the solidity of a spherical segment.

Rule 6. To three times the square of the radius (or half the diameter) add the square of its height and the product by .5236.

The area being given to find the diameter.

Rule 7. Divide the area by .7854, extract the square root of the quotient for the diameter required.

The diameter given to find the area.

Rule 8. Multiply the square of the diameter by .7854.

Problem 87.

The diameter of a circle given to find the circumference, or the circumference given to find the diameter.

Rule 1. As 7 is to 22 so is the diameter to the circumference.

Or: As 22 is to 7 so is the circumference to the diameter.

Rule 2. As 1 is to 3.1416 so is the diameter to the circumference.

Or: As 3.1416 is to 1 so is the circumference to the diameter.

Example 1. Required the circumference of a circle whose diameter is 30.55.

$30.55 \times 22 = 67210 \div 7 = 96.014$ the circumference

Example 2.

$$96.014 \times 7 = 67210 \div 22 = 30.55 \text{ the diameter}$$

Example 3. Required the circumference of a circle whose diameter is 30.55.

$$3.1416 \times 30.55 = 95.975880 \text{ the circumference}$$

Example 4. Required the diameter of a circle whose circumference is 95.975880.

$$95.975880 \div 3.1416 = 30.55 \text{ the diameter}$$

AREAS AND PROPERTIES OF CIRCLES.

Problem 88.

Examples to the Table, No. 19, of the relative proportions of the circle; its equal and inscribed squares.

Example 1. The diameter of a circle is 33.25; required the side of a square equal in area to the given circle.

$$33.25 \times 8862 = 29.466150 \text{ the side of the square}$$

Example 2. The circumference of a circle being 104.5, required the side of a square equal in area.

$$104.5 \times .2821 = 29.4794 \text{ the side}$$

Example 3. The diameter of a circle being 33.25, required the side of the greatest square that can be inscribed therein.

$$33.25 \times 7071 = 23.511075 \text{ the side of inscribed square}$$

Example 4. The circumference of a circle being 104.5, required the inscribed square.

$$104.5 \times 2251 = 23.522 \text{ the inscribed square}$$

Example 5. The area of a circle being 868.309, required the area of the greatest square that can be inserted therein.

$$868.309 \times 6366 = 552.7655 \text{ the area required}$$

Example 6. The side of a square being 23.511, required the diameter of its circumscribing circle.

$$23.511 \times 1.4142 = 33.2492562$$

Example 7. Required the circumference of a circle to circumscribe a square, each side being 23.511.

$$23.511 \times 4.443 = 104.456373$$

Example 8. The side of a square being 29.4794, required the diameter of a circle equal in area to the given square.

$$29.4794 \times 1.128 = 33.25276$$

Example 9. The side of a square being 29.4794, required the circumference of a circle equal in area to the given square.

$$29.4794 \times 3.545 = 104.5044$$

SOME OF THE PROPERTIES OF A CIRCLE.

1. It is the most capacious of all plane figures, or contains the greatest area within the same perimeter or outline.

2. The areas of circles are to each other as the squares of the diameters, or of their radii.

3. Any circle whose diameter is double that of another, contains four times the area of the other.

4. The area of a circle is equal to the area of a triangle, whose base is equal to the circumference and perpendicular equal to the radius.

5. The area of a circle is equal to the rectangle of its radius, and a right line equal to half its circumference.

6. The area of a circle is to the square of the diameter as .7854 to 1; or multiply half the circumference by half the diameter, the product will be the area.

Problem 89.

To find the area of the sector of a circle when the degrees of the arc are given, Fig. 73, Plate 5.

Rule 1. The area of a sector depends on the proportion its arc bears to the whole circle, therefore divide 360° by the number of degrees in the arc of the sector for the first quotient; divide the area of the whole circle by the quotient thus found, and the last quotient will be the area of the sector.

Example 1. What is the area of a sector whose radius B O = 2.50, and the arc A B C 60°.

$$360^\circ \div 60^\circ = 6 \text{ the first quotient}$$

Then for the area of the whole circle. (See Prob. 86.)

$$5.00^2 \times .7854 = 19.6350 \div 6 = 3.2725 \text{ the area}$$

Problem 90.

When the length of the arc is given.

Rule 2. Multiply the radius of the circle by half the length of the arc of the sector, and the product will be the area.

Example 2.

B O 2.50 × A B C 60° = 15000 × .0174524 = 2.61786 length of arc (See Prob. 66)

2.61786 ÷ 2 = 1.30893 half the arc

Then 1.30893 × 2.50 = 3.2723 the area required

Another method:

Rule. Multiply the diameter of the circle by the length of the arc of the sector; and one-fourth of the product will be the area.

Ex. 5.00 × 2.61786 = 3.2723 the area

Problem 91.

To find the area of a segment of a circle, Fig. 74, Plate 5.

Rule 1. By the help of the Table of circular segments, No. 15, divide the versed sine or height C D by the diameter; with the quotient thence arising from the table take out the segment area opposite the versed sine; multiply this segment area by the square of the diameter of the circle, the product will be the area of the segment.

Example 1. Given the versed sine 7, and diameter 35, to find the area.

7 ÷ 35 = .2 the segment area = .11182 × 35^2 = 136.989 the area

Rule 2. Multiply the versed sine C D by the decimal .626; and to the square of the product add the square of half the chord A B; multiply twice the square root of the same by two-thirds of the versed sine, the product will be the area.

Example 2. Required the area of a segment whose chord A B = 52.5, and versed sine C D = 11.5.

11.5 × 626 = 7.1990^2 = 740.89810100

Then $\sqrt{740.89810100}$ = 27.2194 × 2 = 54.4388 × $\frac{2}{3}$ ver. sine 7.7 = 419.168 the area

Or the area of a segment may be found by first finding the area of the sector, having the same radius as the segment, and then deducting the area of the triangle, leaves the area of the segment.

Problem 92.

To find the area of a circular ring or space included between two concentric circles, Fig. 75, Plate 5.

Rule. Add the inside and outside diameters together, multiply the product by their difference and by .7854; the result will be the area.

Example. The diameters of two concentric circles A B = 25 and C D = 15; required the area of the ring or space contained between them.

$$25+15 = 40 \times 10 = 400 \times .7854 = 314.1600 \text{ the area}$$

Problem 93.

To find the area of an ellipse, Fig. 57, Plate 3.

Rule. Multiply the transverse diameter by the conjugate diameter, and then by .7854; the product will be the area.

Or multiply the two diameters together, divide that product by 4, which multiply by 3.1416 will be the area.

Required the area of an ellipse, the transverse diameter A B = 680, and the conjugate diameter C D = 420.

Example 1.

$$680 \times 420 = 285600 \times .7854 = 224310.24$$

Example 2.

$$680 \times 420 = 285600 \div 4 = 71400 \times 3.1416 = 224310.24$$

REDUCING AND EQUALISING PLANE FIGURES.

Problem 94.

To reduce a trapezium to a plane triangle, Fig. 76, Plate 5.

Draw the diagonal A C, parallel to it draw the line D E, draw the line A E, then will A B E be the triangle required.

Examples for Practice.

Required the area of the triangle A B C, also of the trapezium A B C D; by the rule, Problems 79, 80.

Problem 95.

To reduce a parallelogram to a square of equal area.

Fig. 77. *Rule.* Multiply the length by the breadth, and the square root of the product will be the side of the square equal to the parallelogram.

Note.—The square root of the product of any two numbers is the geometrical mean proportion of those numbers.

To construct the same geometrically.

Continue the line A B to E equal to B C; bisect A E at F, with the radius A F or F E describe the semicircle A G E, erect a perpendicular from B intersecting the semicircle at G, then will B G be the side of the square B G I H equal to the parallelogram A B C D.

Example. What is the side of a square whose area shall be equal to the parallelogram, the length A B = 440, the breadth B C = 140?

$$440 \times 140 = \sqrt{61600} \text{ the area} = 248.19 \text{ the side of the square}$$

For Practice.

The sides of a parallelogram are 600 and 300, what is the side of a square of equal area?

Problem 96.

To reduce a figure of five sides to a triangle, Fig. 78, Plate 5.

Draw the line B D, and parallel to it draw the line C F, from F draw the line B F, which forms one side of the triangle; in like manner draw B E, and parallel to it draw the line A G, from G draw the line G B the other side of the triangle; then will G B F be the triangle required.

For Practice.

Required the area of the triangle .	G B F
Also the trapezium	A B D E
And the triangle	B C D

Let these be cast by the rule as before, by the scale, Fig. 1, Plate 39; if the quantity of the triangle G B F does not

agree with the quantities of A B D E and B C D taken together, repeat the operation.

Examples for Practice.

Required the area of Figs. 71 and 72, Plate 4.

Both these figures are similar; the dotted lines show the triangles and trapeziums each figure is divided into for calculation as follows:

Fig. 71. Two triangles and one trapezium.

Fig. 72. Two trapeziums and one triangle.

Note.—If correct, both quantities should agree within a few decimals.

Problem 97.

Fig. 84, *Plate* 6, represents all the reduced lines of fences to Plan, Fig. 85, Plate 6.

Calculate these four pieces by the scale as before; then calculate Fig. 85, Plate 6, by the improved computation scale, as described in Part IV.

Note.—If there be any trifling difference in the decimals in the different modes of calculating them, take the mean between the two for the actual quantity.

Problem 98.

Examples for reducing crooked fences to straight lines by the parallel rule, Fig. 79, Plate 5.

As before stated, all crooked fences have to be reduced to straight lines to form the sides of the regular figures for computing the quantities as shown by the last example.

The following system is decidedly a most perfect one, although tedious, and was, until lately, generally adopted.

The irregular line from A to B represents the side of a field, which has to be reduced to a straight line that shall equalise the quantities on both sides, as shown by the shaded part on one side the line, and plain on the other side of the line A C.

First draw the line A B, and perpendicular to it the line B C; then lay the edge of the parallel rule at the points B and 2 on the fence, move the upper part of the rule to 1 on the

fence, with a fine needle fix it on the line B C as marked 1; holding the needle firm at that point, bring the edge of the parallel rule to the needle and to the point 3 on the fence, then move the upper part of the rule to the point 2 on the fence, and the needle on the line B C at the mark 2; holding the needle firm at that point, place the edge of the rule against the needle and point 4 on the fence line, move the rule to 3 on the fence line and fix the needle at 3 on B C; bring the rule to the needle and at 5 on the fence line, move the rule to 4 on the fence and place the needle at 4 on B C; bring the rule to 6 on the fence and against the needle, move the rule to 5 and fix the needle at 5 on B C; bring the rule to that point and 6 on the fence, move the rule to 5 on the fence, and the needle to C on the line B C; draw the line A C, it will be the equalised line of the fence from A to B, forming one side of a triangle for calculation.

Note.—This method may be applied to equalise a fence between two fields, giving to each an equal quantity.

Another method is with a piece of transparent horn perfectly straight on the edges. Place the edge of the horn over the irregular fence, shifting the horn until by the eye there appears to be an equal portion on both sides of the edge of the horn; from the point A draw the line A C; then A B C will be a triangle, and the line A B one side of a trapezium.

Proceed in like manner for practice with Figs. 80 and 81, Plate 5, as shown by the dotted lines; the quantities are marked in each figure, and collected together for the total quantity as shown.

To find the area of a field, Fig. 82, Plate 6, by the computing scales, Fig. 1, Plate 39 (which may be obtained at any of the mathematical instrument-makers).

A full description of these scales is given in Part IV.

The parallel lines drawn over the plot are supposed to be lines drawn on the tracing paper to the same scale as the plan, and laid over the piece to be calculated, Fig. 83, Plate 6.

When the area of a field or enclosure of many acres is required, it is better to cut off large portions into regular figures, and calculate them in the manner described in Figs. 64 and 65, Plate 4, and as shown in this example by the Figure A B C D.

Then all the irregular fences may be taken separately, as shown by E F G H; shifting the tracing paper accordingly, and add the several sums together.

Although the computation scale is a great improvement in calculating quantities, particularly where the fences are very irregular, it should only be applied to small enclosures, or such as are within the use of the scale without repeating.

The scale is generally made to cast 12 acres; therefore, when casting a large piece of 60 or 100 acres, which requires several repetitions, if by accident one operation is omitted to be noted, there must be an error of 12 acres; time is saved by cutting off a portion and casting up the quantity by the rule given in Fig. 64.

I have recommended these scales to be made to calculate 10 acres: that being a decimal number, is less liable to error; they are also better in practice.

MENSURATION OF SOLIDS

Comprehends the measure by length, breadth, and thickness of *all* bodies, whether solid, air, water, or gas.

A knowledge of measuring solids is indispensable to the land agent, surveyor, and civil engineer, as all earthwork, stone, brickwork, well-sinking, formation of tanks or reservoirs, timber, &c., are calculated by it; mensuration of superfices being the leading part of this branch of science.

DEFINITIONS.

A parallelopiped or parallelopipedon is a prism bounded by six parallelograms, every opposite side of which is equal, and at right angles with each other.

Fig. 1, *Plate* 7. A rectangular parallelopipedon is that whose bounding planes are all rectangles, and are perpendicular to each other.

Fig. 2. A cube is a square prism, being bounded by six equal square sides, and are perpendicular to each other.

Fig. 3. A cylinder is a round prism having the two centres of the two parallel ends, about which the figure is described.

Figs. 4, 5. A pyramid is a solid plane figure, whose base is either triangular, quadrangular, pentagonal, polygonal, &c., and its sides triangles, having all their vertices meeting in a point above the base, called the vertex of the pyramid. A pyramid, like the prism, takes its name from the base.

Fig. 6. A cone is a round pyramid, having a circular base. The axis of a cone is the right line joining the vertex or fixed point, and the centre of the circle about which it is described.

When a cone is cut obliquely, the section is then called an ellipse, as C D; when cut parallel to one of its sides, as E F, the section is then a parabola; and when cut vertical, or at right angles to the base, as D G, it is then an hyperbola.

Fig. 10. A sphere is a solid figure, bounded by one surface, that all straight lines drawn from a certain point called the centre, within the solid to the superfices, are equal to one another.

Problem 99.

To find the solid content of any rectangular body, Figs. 1 and 2.

Rule. Multiply the length by the breadth, and the product by the height.

Note.—If the dimensions are given in inches, divide by 1728, the product will be feet.

If the dimensions are given in feet, divide by 27, the product will be yards.

Fig. 1, *Plate* 7. Required the solid content of the rectangular parallelopipedon in cube yards, A B = 20, A C = 10, C D = 10 feet.

$20 \times 10 \times 10 = 2000 \div 27 = 74$ yards 2 feet

Fig. 2. Required the solid content in cubic inches of the cube A B C D, each side equal to 120 inches.

$$120 \times 120 \times 120 = 172800 \div 1728 = 1000 \text{ cube feet}$$

Problem 100.

To find the solid content and the surface of a cylinder, Fig. 3, Plate 7.

Rule. Multiply the circumference of the base by the height of the cylinder, and the product is the convex surface; and multiply the area of the base by the height of the cylinder, the product is the solid content.

Required the solid content of the cylinder A B C D, whose base A B = 36 inches, and the perpendicular height B C = 8 feet.

$$36^2 \times .7854 = 1017.8784 \text{ inches area of the base}$$
$$\text{Then } 1017.8784 \times 96 = 97716.3264 \div 1728 = 56.5486 \text{ cubic feet}$$

Required the convex surface of the cylinder A B C D in superficial feet.

$$3.1416 \times 36 = 113.0976 \text{ the circumference of the base}$$
$$\text{And } 113.0976 \times 96 = 10857.3696 \text{ superficial inches, divided}$$
$$\text{By } 144 = 75.3984 \text{ superficial feet}$$

Problem 101.

Fig. 4, *Plate* 7. To find the solid content of a triangular pyramid.*

Rule. Multiply the area of the base by $\frac{1}{3}$ the vertical height, the product will be the solid content. (For the area of the base, see Prob. 79.)

Required the solid content of the triangular pyramid A B C, each side = 10 feet, and height 40 feet.

$$10 \times 4.40 = 44 \text{ the area} \times \overset{\text{the } \frac{1}{3}}{13.334} = 586.696 \text{ cube feet}$$

Problem 102.

Fig. 5, *Plate* 7. Required the solid content of a square pyramid A B C D, each side = 10 feet, and height 40 feet.

$$10 \times 10 = 100 \text{ the area} \times \overset{\frac{1}{3}}{13.334} = 1333.40 \text{ cube feet}$$

* Every pyramid is the third of a prism having the same height and base.—Euclid, 7 of XII. Cor.

Problem 103.

To find the surface and solid content of a cone or circular pyramid, Fig. 6, Plate 7.

Rule. Multiply the circumference of the base by the slant height, and half the product will be the slant surface; to which add the area of the base, the product will be the whole surface; and multiply the area of the base by $\frac{1}{3}$ of the perpendicular height, the product is the solid content.

Required the convex surface of a cone whose base H I = 15.5 feet, and slant height A H = 50 feet.

$3.1416 \times 15.5 = 48.6948$ the circumference of base
Multiplied by $50 = 2434.740 \div 2 = 1217.37$ square feet
Divided by $9 = 135.264$ square yards

Required the solidity of the cone as above the perpendicular A B = 47.53 feet.

$15.5^2 \times .7854 = 188.69235 \times 15.843$ $\frac{1}{3}$ of the height
$= 2989.45290105 \div 27 = 110.720$ cubic yards

Problem 104.

Fig. 7, *Plate* 7. To find the surface and solid content of the frustrum of a cone or pyramid.

Rule. Multiply the sum of the perimeters of the two ends by the slant height, and half the product will be the slant surface; to which add the areas of the two ends, the product will be the whole surface of the frustrum. And to the product of the diameters of the two ends, add the sum of their squares; that multiplied by the height, and by .2618,* will be the solid content.

Required the convex surface of the frustrum of a cone A B C D, whose base A B = 30 inches, the slant height B C = 28.5 inches, and top end C D = 16.5 inches.

$3.1416 \times 30 = 94.248$ and $3.1416 \times 16.5 = 51.8364$
Added together $= 146.0844 \times 28.5 = 4163.4055 \div 2$
$= 2081.7027$ square inches $\div 144 = 14.4562$ superficial feet

Required the solidity of the same, the perpendicular height E F being 27 inches.

$30 \times 16.5 = 495 + 30^2 + 16.5^2 = 1667.25 \times 27 = 45015.75 \times$
$.2618 = 11785.1234$ cubic inches $\div 1728 = 6.82$ cubic feet

* $.2618 = \frac{1}{3}$ of .7854.

Problem 105.

Fig. 8, *Plate* 7. Required the content in imperial gallons of the inverted frustrum of a cone A B C D, whose inner dimensions are $4\frac{1}{2}$ feet deep, 22 inches diameter at bottom, and 28 inches diameter at top.

$28 \times 22 = 616 + 28^2 + 22^2 = 1884 \times 54 = 101736 \times .2618 =$
26634.4848 cubic inches $\div$ 277.274 = 96 gallons

Problem 106.

To find the solidity of a wedge, Fig. 9, Plate 7.

Rule. To the length of the wedge add twice the length of the base; multiply that sum by the height and by the breadth of the base, and $\frac{1}{6}$ of the product will be the solidity.

Required the content, in cubic inches, of the wedge A B C D E, whose base B C = 26 inches, and width A B = 15 inches, the length of the wedge D E = 22 inches, and perpendicular height A E — 20.68.

$22 + 52 = 74 \times 20.68 = 1530.32 \times 15 = 22954.80 \div 6 = 3825.8$ cubic inches

Problem 107.

To find the convex surface and solid content of a sphere or globe, Fig. 10, Plate 7.

Rule. Multiply the square of the diameter by 3.1416, the product will be the convex superfices; and multiply the cube of the diameter by .5236,* the product is the solid content.

Required the convex surface of a sphere whose diameter A B = 36 inches.

$36^2 = 1296 \times 3.1416 = 4071.5136$ square inches
$\div 144 = 28.2744$ square or superficial feet

Required the solid content of a sphere whose diameter is 36 inches.

$36^3 \times .5236 = 24429.0816$ cubic inches $\div 1728 = 14.1372$ cubic feet

Problem 108.

To find the convex surface and solid content of the segment of a sphere, Fig. 11, Plate 7.

* $.5236 = \frac{2}{3}$ of .7854.

Rule. Multiply the height of the segment by the whole circumference of the sphere, the product is the curve surface. To find the solidity, add the square of the height to three times the square of the radius of the base; multiply that sum by the height and by .5236, the product will be the solid content.

The diameter A B of the sphere A B C D = 35.813 inches; what is the convex surface of that segment whose height E D = 13 inches.

$3.1416 \times 35.813 = 112.5101 \times 13 = 1462.63$ square inches $\div 144 = 10.1572$ superficial feet nearly

The base F G of the segment F D G = 34.5 inches, and perpendicular E D 13 inches, what is the solid content?

$34.5 \div 2 = 17.25^2 \times 3 = 892.6875 + 13^2 = 1061.6874 \times 13 = 13801.937 \times .5236 = 7226.6942 \div 1728 = 4.182$ cubic feet

Problem 109.

Required the number of ale gallons the segment A B C D contains; the base A B = 34.5 inches, and perpendicular height C D = 13 inches, Fig. 12, Plate 5.

$17.25^2 \times 3 = 892.689 \times 13^2 = 1061.689 \times 13 = 13801.967 \times .5236 = 7226.71 \div 282 = 25.63$ gallons

PART II.

ON LAND SURVEYING.

GENERAL OBSERVATIONS.

PREVIOUS to the commencement of a survey, it is requisite to have a knowledge of the country, either by observation, taken on some eminence, or from a well authenticated map; from which may be determined the most suitable position to fix the principal or base lines, so as to avoid obstructions that may prevent a continuation in measuring these lines to the extent desired or intended, also to select the most favourable and level part of the country for chaining. Sometimes it is advantageous to run these lines through or into part of the adjoining property, by which a better diagram is formed and many difficulties surmounted; as all these lines forming the diagram are the basis of succeeding operations, see Plate 14.

Having determined the position of the base lines, and observed particular objects that may be seen from different parts of the survey, proceed to pole or range out these lines very carefully in straight lines, which in many instances is a work of difficulty and labour, and of the greatest importance to the whole survey.

Provide a number of straight deal rods about ten or twelve feet long, spiked at the bottom, and white and red flags at the

top, by which they are easily distinguished at a distance and over the fences, which are frequently an obstruction, and require to be cut away to give a clear sight; this should always be done cautiously, so as not to injure the fence or make gaps.

It frequently happens that a prominent elevated object presents itself in the distance as a favourable point to direct a line, and poling become unnecessary. Flags or marks must be left at all the intended stations, and fixed with great care in the direct line. Provided the distant object can only be seen at intervals, then the head surveyor should remain at the commencement, or any determined point on the line, to direct the assistants, who have proceeded forward, in fixing the flags; when several of these flags are truly fixed, the assistants may then continue poling out the line without signals, and the process of chaining commenced, the surveyor at the same time observing the flags are correct with the distant object, and that they are truly perpendicular, as it is the foot of the poles that forms the lines.

During the operation of chaining the surveyor will have an opportunity of observing the most favourable points to construct other lines intersecting those in operation.

It is impossible to make a correct survey if any one of the principal lines are in the least crooked; the same applies also to the plotting, as the lines drawn on the plan represent the chain lines; if they are drawn the least curved, it has the same effect in disturbing the accuracy of the whole.

To impress on the mind how important it is that the lines in chaining, and the lines in plotting, should be perfectly straight, and a proper allowance made for the undulation of the ground, it is recommended to plot all the principal lines from a well-known accurate survey, and make only *one* of these lines a little curved, it will at once show the consequences that will occur to all the rest of the survey, therefore it requires as much attention in examining the straightness of the rule, and the correctness of the scale, as it does in poling the lines straight and the adjustment of the chain.

When the ground is hilly or thickly timbered, and no distant object visible, the best method of proceeding then is to plant the theodolite on rising ground about midway, so that, if possible, both ends of the line may be seen. Adjust the instrument perfectly level, and clamp the lower limb, loosen the clips that fix the telescope in the Y's, the upper limb being free, direct the telescope to the first station and clamp the upper limb; then reverse the telescope on the Y's; if the vertical wire in the telescope cuts the point required, let the instrument remain in that position until the whole line has been poled out, which is done by looking through the telescope and directing the assistants on both sides by signals as before.

The flag-poles, being few in number, should be fixed as far apart as may be clearly seen, and the intervening spaces filled up with straight sticks about a yard high, having a slit at top in which is fixed a piece of white paper; these will be found more generally useful for intervening stations, &c., the flag-poles being intended more for distant objects and particular stations.

Having shown the importance of straight lines, the next matter for consideration will be the form of the diagram or basis to the whole survey; this must be guided chiefly by the contour of the estate to be surveyed, and the nature of the ground as regards hills, woods, and water.

The two first principal lines should, if possible, intersect each other nearly in the middle at an angle as near 45 degrees as possible; these lines, also, should be the longest or bases of the two largest triangles; and when the connecting lines are added, the whole diagram will then consist of four large triangles. (See Plate 14.)

If the estate should be of a triangular form, the three principal lines may then be correctly proved, by having a line joining the two opposite sides, as Fig. 1, Plate 10.

When all the base lines of a survey are measured or chained they should be plotted before proceeding any further. First,

construct the largest triangle geometrically, marking off by the scale the points at which the other lines intersect; let those lines be drawn through the points and mark off their respective lengths, forming the whole diagram; after the first triangle is plotted, every intersection of lines entered into the book should read the same by the scale, if the survey is correct, and if so, every line, great and small, must prove itself.

Therefore under no consideration attempt to proceed with the survey until the whole diagram is proof against error in every line.

In very hilly districts the system of triangulating cannot always be carried out to the full extent, but may in parts; it then becomes necessary to use the theodolite to determine the position of the chain lines by taking angles, in doing which there are two methods; one is the number of degrees and minutes contained between one object and the magnetic needle; the other is the number of degrees and minutes contained between two objects. (See Instruments, Part V.)

This system requires great nicety, both in fixing the axis of the instrument directly over the apex of the angle, and reading the angle correctly by the vernier.

In order to have a check upon the angles so taken, frequent observations should be taken by the instrument to some fixed object that can be seen on different parts of the survey; if there is no object suitable, it is better to provide one or more by fixing up poles purposely, their position being accurately plotted on the plan. The measured length of these lines will also require particular attention, and the hypothenusal difference made to each line; this may easily be done at the same time, as the allowance to each chain is engraved on the arc of the theodolite according to the angle of acclivity or declivity. This subject is further explained hereafter.

Strict attention is required to both systems, and a combination of them will make a perfect surveyor.

There is an equal responsibility in plotting angles, probably

more, when the small radius of the protractor is frequently used to a long chain line; and however careful an angle may be pricked off, if that line is not drawn very delicately and truly through the two points, the radiation of the line will, at the extremity, be either considerably greater or less between the two extreme points: however, that may be examined by natural sines, or by trigonometrical calculation.

Where the ground is favourable, an angle can be better plotted by a measured line than by the protractor.

It is for this reason—the multiplicity of angles required by instrument surveying, and the time in taking them—that chain surveying is generally considered the most perfect.

The chain may, therefore, be considered the principal instrument in surveying. The chain should be thoroughly examined every day before commencing operations, for which purpose stakes should be driven in the ground the exact distance; or by comparing it with another chain kept purposely for that use, which is by far the best plan, as tricks have been played by moving the stakes.

In selecting a chain, let it be moderately stout with elliptic rings, having three rings between each long link, which prevents its coiling up.

To accompany the chain, there is always ten arrows or iron pins, about 13 inches long, formed with a ring at top, having slips of red cloth sewed to each, to make them easily seen in the ground.

THE CHAIN.

It is commonly called a four-pole chain, but more properly Gunter's chain, from the inventor, Dr. Gunter, an eminent mathematician.

It is equal in length to four poles, or 22 yards, or 66 feet. The whole length is divided into 100 parts, called links, each link being $\frac{22}{100}$ of a yard, or $\frac{66}{100}$ of a foot, or 7.92 inches. It is also divided or marked at every ten links by a piece of brass,

having points indicating the number of links from the ends, as 10, 20, 30, 40; the centre, or 50, is distinguished from the rest by a circular brass.

When reading the chain from one end, after passing the 50, the next brass will count 60, the next 70, and so on, these numbers 40, 30, &c., being subtracted from 100.

Ten of these chains in length and one in width are equal to one statute or imperial acre, as thus:

10.000 ×	1.00 =	100.000	square	links
.660 ×	.66 =	43.560	"	feet
.220 ×	.22 =	4.840	"	yards
.040 ×	4 =	.160	"	rods or poles*

By a statute made the 35th Edward I., an acre of land is to contain a quantity in any shape equal to 160 square poles or perches, of 16½ feet.

In many counties there is a customary acre, now gradually getting out of use.

It may be interesting to know the customary acre in different parts of England:

In Devon and some parts of Somerset . . .	15 feet to the perch
In Cornwall	18 "
In Lancashire	21 "
In Cheshire and Staffordshire	24 "
In the Isle of Purbeck and other parts of Dorsetshire	15 and 1 inch

The yard was settled by Henry I. from the length of his own arm.

In Wiltshire and its neighbouring counties the acre is different, as 120 statute perches are an acre, instead of 160 perches; equal to ¼ less.

The hide of land seems to have been the origin of land measure in the Confessor's time, as the caracote was by the Conqueror's new standard; either of which means the same as what we call a plough land; that is, as much as can be tilled and managed by one plough, having meadow and pasture sufficient to maintain the cattle thereto belonging. The hide of land consisted of from 100 to 120 acres, according to the nature of the land to be worked by the plough.

* To find the number of square links in any number of acres, roods, and perches, see Table, No. 13.

USING THE CHAIN.

The manner of using the chain should be carefully attended to, so that its length may be always correctly pointed off by the chain leader, as thus:

The line having been previously poled out, the surveyor standing at the station point holds one end of the chain, the assistant with the other end in his right hand, and the ten arrows in his left hand, which are transferred one by one, as required, into the right hand; on arriving at the extent of the chain, he turns partly round, holding the arrow perpendicular at the end of the chain, looking towards the surveyor, who springs the chain until it is in a straight line with the fore object, then, by motion of the head or hand, directs the leader to move the arrow accordingly, until it is in the proper point of the line, and the chain fairly stretched out; the arrow is then to be fixed in the ground, and the chain remain at rest until the surveyor has taken all the offset and remarks necessary, and at his signal proceed on to the next length, and so continuing, until the whole ten arrows are fixed or transferred into the surveyor's possession; the leader then proceeds without any pins, adjusting the chain in the line, which must remain at rest until the surveyor arrives at that end and puts one down, delivering the nine arrows remaining to the leader, each time carefully counting them at every change, and also at the end of every line, to prove that no mistake has occurred by dropping one or by false entry.

Some men are excellent chain leaders. It is necessary, in the first onset, to instruct them to keep the line, by a back object: that is, by placing himself in a line with the arrow last put down and the mark or pole at the station, or some distant object that may accidentally be in line; it saves much time and labour when the chain man is made to keep or pole out a line truly.

FIELD-BOOK.

The field-book is ruled into three columns: the centre contains all the dimensions of the chain from the station to the end of the line; the intermediate numbers are the points at which the offsets are taken at right angles to the chain, or at the crossing of fences, footpaths, &c.

The right and left columns contain the offsets that are taken on the respective sides of the chain; the margin on each side is for sketching the fences, buildings, and every necessary memorandum, such as the name of the field, the description whether arable or pasture (distinguished by the initial), the names of the adjoining proprietors or parish, &c., also the names of the occupiers. By carefully making these entries, it will be of great assistance in forming the reference.

Accurate and neat sketching will be of great assistance in plotting. Particular attention should be paid in showing the fences to which field they belong and where they change, at which point always take an offset; the ditch is always the boundary, and is noted in the field-book thus : the line denotes the ditch, the letter T shows the side on which the fence belongs; and of describing a fence in plotting when it changes, the same marks must be carefully attended to. Many expensive lawsuits have occurred through disputes in claiming the ditch boundary, particularly where much timber is growing.

OFFSETS.

When the ditch is next the chain, the offset is taken at right angles from the chain to the edge of the ditch, and when the ditch is outside, the offset is taken to the middle of the fence with the offset staff, and six or seven links added to it, as a general allowance for the ditch, about 4 feet 6 inches from the middle of the fence. In some districts the width of the ditch varies, therefore it is requisite at all times to inquire of the occupiers their local custom.

In measuring buildings, yards, &c., a tape is the most useful instrument for that purpose.

Sometimes there is what is termed a footset-hedge: that is, having no ditch either side; the offset must then be taken to the centre, and noted in the field-book thus .

A paling fence is known to belong where the nails are driven home, and described thus — — — —, excepting when it is the boundary next a road; it has then sometimes a ditch outside. The usual allowance for a ditch must in that case be made.

Walls are distinguished by two parallel lines =; footpaths and roads without fences by small dotted lines.

Buildings belonging to the estate or survey are usually tinted or hatched; and buildings not belonging have merely the outline.

The offset staff is also an important accompaniment to the chain; it is ten links long, painted white; each link is marked by a black painted ring, the bottom of the rod is shod with an iron spike, and the top has a stout open ring, as thus , to force or draw the chain through the bottom part of a fence. Offsets should, in general, be less than a chain. It is better to make a small triangle from the chain and have short offsets. See Line 1, Plate 11.

All stations or lines are numbered numerically, having a small circle round them, thus 1, as station or line one; and if two or more lines proceed from the same station, they are distinguished by a smaller figure over, thus 1^2 or 1^3, denoting that a second or third line commenced from that point or station.

Near to every fence a peg or mark should be fixed for an intended station or lines crossing it, and on other parts of a line where it is probable a line may cross it, and the number entered in the field-book; and against that number on the chain, in the offset column, a mark denoting that a line is intended to go from or through that line, and in the direction shown, as thus , denotes a line at right angles or there-

abouts on that side the line; or it may be two or three lines to the same point that will either commence or end at that point, as thus , showing the oblique or acute direction of the intended line or lines.

In measuring the principal lines of a survey, it is advisable to put down a peg at every ten chains, entering it in the field-book, as, in case of accident in dropping an arrow, it becomes doubtful which it was that had lost it; by going back to the last peg it is easily rectified. This precaution will be found useful in many other cases; particularly, when fields are very large, there would be no other marks left but these between the fences; and should a line be required to intersect or finish on it, the distance would be trifling to measure for the exact number on that line.

When a peg is put down for an intended station, that point may not be in the exact position by several links. The number originally entered in the field-book must not be erased; the pen must be drawn through it and the corrected number put underneath, and on the margin to show the working, that is, the number of links either added or subtracted from the original entry, as shown by Line 2, Plate 11. Reference to it hereafter removes all doubt of error, and explains the alteration; this will frequently occur in a large survey.

In all cases when a station is determined, cut a triangular hole round the station point, and fix the peg in the same hole in which the flag was fixed.

On any one of the lines nearest to the north and south, with a small mahogany box-compass lay alongside the line and take the magnetic bearing, so that a true compass bearing can be attached to the plan.

In taking angles for short lines, or for raising perpendiculars in cases of difficulty in crossing streams, or interruptions by buildings, &c., the box-sextant will be found to be of great assistance; for a description of which see Instruments, Part V.

In order to make an embellished plan of an estate, the surveyor cannot be too particular in noting every detail during his operations (particularly the course of hills, if any). He should also be well acquainted with ornamental writing, and have a knowledge of landscape drawing, to give the whole a picturesque effect.

SURVEYOR'S DUTIES.

There are a variety of duties belonging to a surveyor beyond that of measuring and mapping—viz.:

The assistance in the arbitration and settlement of disputed boundaries, manorial rights, the purchase and exchange of land for improvements, encroachments, diversion of roads, streams, &c.; a knowledge of timber falling, planting, and measuring; as well as a knowledge of mensuration of superfices and solids for measuring all kinds of materials and labour.

SURVEYING BY THE CHAIN ONLY.

Problem 1.

To ascertain the quantity of a piece of land without a plan.

It is sometimes required to ascertain the quantity of a small plot of ground without a plan, but more particularly in agricultural purposes, to know the quantity of reaping, sowing, mowing, &c., in which case it is only measured as far as it is tilled, and all deductions made, such as footpaths, ponds, &c.

When the sides of a field are tolerably straight, it is possible to approach tolerably near the truth, but in inexperienced hands the quantities are frequently very far from it, particularly when a piece is measured by ordinates; that is, by measuring several widths across, adding them together, and dividing the product by the number of times, and that quotient multiplied by the whole length, taken up the middle, the product will be square links; this method is erroneous, and should never be adopted.

When a field is measured by ordinates (see Fig. 1, Plate 8), the length should be taken as near the middle as possible, and at the end of every chain put down a mark; at right angles to the centre line measure from one side to the other, as shown on plan.

Rule. Add the first and last length together as one sum; then take the 2nd, 4th, 6th, and 8th lengths, add them together, and multiply that sum by 4; then take the 3rd, 5th, and 6th lengths, add them together, and multiply the sum of them by 2; collect all three sums together, and multiply by the common distance, or 100 links, and one-third of the product will be the area.

Another more simple method, without so much calculation: In measuring the line from A to B, put a mark at the centre or 50 of every chain; then measure from side to side through these points, add them together, multiply by the common distance, 100 links, or add two cyphers, the product will be the area in square links, which reduce to acres, roods, and perches, according to the rules before given.

This method is precisely on the same principle as the improved computation scale; as by that the fence at each end is equalised, and the length is taken in the middle of each chain, repeating that operation to the end without casting; so that it is similar to a piece of land perfectly straight, 3080 links long and 100 links wide, or 3.08000 = 3A. 0R. 14P.

Ex. 1.	A = 300	*a* 2 = 350	*b* 3 = 400	*Ex.* 2.	C = 325
	B = 300	*c* 4 = 450	*d* 5 = 430		D = 375
	——	*e* 6 = 400	*f* 7 = 400		E = 425
	600	*g* 8 = 350	——		F = 444
	6200	——	1230		G = 415
	2460	1550	2		H = 403
	——	4	——		I = 375
	9260	——	2460		K = 325
	100	6200			——
	——				3.08700
	3)926000				
	——				
	3.08666				

Problem 2.

Fig. 2, *Plate* 8. The line A B represents one side of a field, or part of the triangle A B C; the quantity of which is required without a plan.

In the former examples the ordinates were set out at equal distances, in this they are set out perpendicular to the line A B to the several angles of the fence, dividing it into triangles and trapezoids; each division is cast separately in the manner before shown; they are to be brought into one column and divided by 2, the quotient will be the area required.

Ex. 1.	A *a* = 106	*a* 1 = 60	*b* 2 = 85	*c* 3 = 50	*d* 4 = 100
	a 1 = 60	*b* 2 = 85	*c* 3 = 50	*d* 4 = 100	*e* 5 = 64
	6360	145	135	150	164
		a b = 146	*b c* = 95	*c d* = 150	*d e* = 128
Double Areas.					
360		870	675	7500	1312
21170		580	1215	150	1968
12825		145			
22500			.12825	.22500	.20992
20992		.21170			
5760				*e* 5 = 64	
				e B = 90	
2)83607				5760	
.41807 = 0A. 1R. 27P.	1A. 3R. 5P.				
And A B C = 1A. 1R. 18P.					

For the triangle A B C: Having fixed a mark at C, measure from A to C in a straight line, leaving another mark at D, as near as possible at right angles to A C; then measure from D to B, take half that distance and multiply it by the length before taken; this quantity, added to the former quantities, will give the whole area.

800 × 340 = 272000 ÷ 2 = 1.360 or 1A. 1R. 18P.

Or, measure the three sides and calculate the quantity by the rule. (See Problem 79.)

Problem 3.

To measure a field in the form of a trapezium without a plan, Fig. 64, Plate 4.

In measuring a four-sided field, whether the fences are straight or not, set marks up at A B C D, measure the line A C as a base, putting in marks for the two perpendiculars *a* and *b*, which is all that is required for the trapezium, if any, or all the fences are irregular; do as in the last examples, keeping each side separate; when all are measured, add them together.

A very near approximation may be obtained of any curvilineal figure.

Problem 4.

To measure a single field, Fig. 3, Plate 8.

Having provided the necessary poles, and the chain in perfect adjustment, proceed first to set out the lines to be measured by the chain, by fixing marks or poles at each corner, in such position so as to avoid long offsets. The surveyor should remain at each station to direct the assistant in fixing the poles at the determined point, provided there is no natural object. When these four lines are measured, it is imperative to measure a diagonal or tie lines, as A and B, otherwise the work could not be plotted. It must be remembered, that one diagonal, or one tie, is not sufficient to prove the accuracy of the work. When two tie lines are adopted they should always be taken to or from the same base, then the opposite line will prove the accuracy of the whole when plotted; also if a diagonal is adopted, a tie line must be measured either at the angle or from a point in the diagonal to one of the stations, then the tie line becomes the proof. These lines should always be taken on the most level or favourable ground.

Commencing at 1, take offsets at every bend of the fence and opposite an abutting fence, as at 390, and where it is intended to have a tie line, as at 268, put up a flag or mark, and notice it in the field-book as shown; also at the end of the line—in both cases describing by the character the direction the lines are intended to take. When arrived at the last chain of the line, be careful the flag or peg is left at the exact number that is entered into the field-book, and that all necessary re-

marks, such as names of adjoining proprietors, the fence marks, &c., are truly entered, and, before moving the chain to proceed on the next line, count the number of arrows held by each; if correct, deliver the whole to the chain man. Be particular, also, to notice stiles, gates, footpaths, ponds, &c.

In like manner proceed with all the other lines, then measure the diagonal or such tie lines as may have been determined on during the survey.

In every survey the magnetic north must be taken, and for such small surveys provide a small mahogany box-compass, place the edge of it in the direction of one of the lines, and read the angle the needle points to on the engraved circle, which is divided into degrees and half-degrees; sufficient for this purpose, enter it in the manner shown in the next example.

To plot the work, that is, to lay down the lines that have been measured, take the largest triangle first, in the manner before described, and if the proof lines answer correctly to the book, proceed then to set off the lengths on the chain lines, and from those points at right angles set off the offsets; through these points draw the fence lines.

When inked in, calculate the quantities by the methods before described.

Problem 5.

To survey the same field, inaccessible by wood or water, Fig. 1, Plate 9.

When the survey cannot be made within the fences, a different system in laying out the lines must be adopted, by extending one of the lines as a base at both ends, as line (1), leaving marks on the line as at 300, and 1325 for stations; the lines (2) and (4) are fixed by the tie lines A and B, and the line (3) is then a proof to the whole.

Commencing at (1), at which point leave a flag as the object for the tie line A, and another mark at 300 for line (4); at 536, notice the crossing of the ditch; at 1280, take an offset where the fence branches off; at 1325, put down a peg for the

next station (2); and at the end of the line leave a flag, remarking in the field-book the direction the tie line is intended to take.

Return to 1325 in the last line, and begin the second line. In crossing fences always take the ditch as the boundary; at 450, put down a peg for the tie line B (it may be well to remark here, that to avoid crossing and recrossing fences as much as possible, leave the eleventh arrow in the ground, and measure the tie line B, which enter in the manner shown, put down a peg, and bring away the flag, as that part of the survey is completed); then proceed on from the arrow fixed in the ground, noticing the building, footpath, and fence, finishing the line at 760; proceed on with the third line, and at the last chain ending at 1050, put down a peg. In chaining the fourth line at 300, leave a flag for the tie line A, marking the direction intended; at 453 it crosses the fence, at 596 it closes at 300 in (1), which completes the field work.

In plotting the chain lines, first draw line (1), mark off the length and stations thereon as at 300 and 1325; then describe the two triangles A and B, and point off the lengths of lines (2) and (4). In describing the triangle A, to obtain the length from *a* to *b*, subtract the number at *b* from the whole length, which write in the margin of the field-book to be left as reference (in large surveys this will often occur). Having described both triangles, apply the scale to the two points of line (3), which should agree with the length entered in the book, this being a proof line.

When short tie lines are introduced as a substitute for diagonals, or as proof lines in plotting the work, the angle lines may be further proved by multiplying the lengths of each line two, three, or four times; the angle may then be plotted by this enlarged triangle.

Problem 6.

The four following examples are introduced principally to show the method of laying out the chain lines, either for small

or large surveys. There is no end to the changes that can be made, which must be governed by circumstances.

Fig. 1, *Plate* 10. The boundary of this plot is triangular; although it is the simplest form, it is the foundation of all surveying by the chain only. Supposing this to be only one field, there would be just the same amount of labour. As before remarked, a triangle of itself is no proof of accuracy; it may be plotted as a triangle without detecting an error, consequently a fourth line is requisite, either from the apex to its opposite side, or by a line joining the two opposite sides as here adopted, which answers in this case two purposes, by taking up the fence and proving the triangle. It was not absolutely necessary to have a line purposely for the fence; that being straight, its position was determined by the two lines 1 and 3 crossing it. A line from 2 to 502 in 3 would also be a proof line.

Let it be remembered that when straight fences occur, and are crossed similar to this at the top and bottom, that unless it is crossed a third time in some part of the line, it should have a short tie line to fix its position, as an error may be made in the crossing and nothing to detect it by the length.

This is an excellent example for computing the quantities, first by reducing the fences to straight lines and casting by the scale, Fig. 1, Plate 42, second by the computation scale, Fig. 2, as described in Fig. 82; then compare the quantities; should the difference exceed a pole to an acre the operations must be repeated.

Problem 7.

Fig. 2, *Plate* 10. Most surveyors in commencing a survey, are apt to think that unless the base lines are laid out in taking up a fence, it is wasting time and chaining for nothing; which is a great mistake. In this example the main line is fixed nearly in the middle of the survey, and the succeeding lines so arranged they follow one another without running from one part to another; this is a very material point to be considered in all surveys.

Beginning at station 1 in chaining the line, every corner of the field can be observed; at 340 put up a flag for a tie line; at 708 put up another flag; this it will be observed is an important line, as it fixes the work on both sides; beginning again at the end of this line at 190, fix a flag in such position as will pass through the point at 708 and pass along the fence forming the side of the largest triangle, the other side of which will be a proof line, also the lines 2^2 and 6 with the short tie line complete that side of the survey.

Following up the survey by lines 2, 3, and 4, by a short tie line from 340 in 1 the whole is completed.

The plotting is extremely simple and effective. First construct the large triangle by the lines 1^2 and parts of lines 1 and 5, mark off the whole length of these lines, and draw line 2, which is a proof line. Then construct the triangle formed by the short tie line 4 = 397 links and part of 1, draw the line 3, which should be a proof, and correspond with the number entered in the book.

Let this be cast up, as before directed, by both the systems last stated.

Problem 8.

To survey a small estate divided by a road, Fig. 3, Plate 10.

The chain lines in this example are few and effective as a whole or as separate fields.

On the base line 1 are constructed two triangles, one by lines 1^2 and 9; the other by lines 2 and 4; having marked the different stations on them, line 5 should, by applying the scale, coincide with the dimensions entered into the field-book at every intersection, the small trapezium at the end of 2 is proved, also the trapezium formed by lines 6 and 7; line 8 also, taking up the road, is a proof, with exception to the small triangle 180 and 340 on line 1^2, which is entered on the margin of the field-book, every line in the survey proves itself, after plotting the two triangles before mentioned.

It is recommended to plot this survey to a larger scale, and compute the quantities by both methods.

Problem 9.

To survey three fields and detached cottage, Fig. 4, Plate 10.

The boundary of this plot is of more than ordinary occurrence. On the south side there is a prohibition not to trespass on the ground (not uncommon to surveyors), the narrow slip of meadow on the west, a wood on the east, the centre forming almost the sector of a circle.

Such a survey as this will necessarily command more lines to connect them under existing difficulties than otherwise would be if the adjoining land were free to chain over.

The first large triangle is formed by lines (1) and part of (1), part of (2), and (7), leaving marks or pegs on each for the necessary stations; line (3) is continued to take up the road, cottage, and garden, depending only on the short line of 161 links. Had there been no opposition, an excellent triangle could have been formed from the road into line (4), or from the end from (3) to (5), at the same time reducing the number of lines; line (4) commences at 161 in (3); at 852 have a flag, and another at 709, finishing it at 780 in (1); lines (5) and (6) require only the application of the scale to prove the greater part of the survey; the same also, to the short lines in the meadow, are all proved by the several intersections.

Should there be an error in any one of these lines, the whole would be affected, and every line must be re-measured until the error is discovered.

Problem 10.

To survey a small estate, Fig 1, Plate 11.

This example, if carefully studied, will point out the principle by which surveys of any extent should be conducted. It has before been remarked that the letter X (see Prob. 18) is considered the best foundation to work on, which is fully ex-

emplified in this case; for when the triangle formed by lines (7), (2), and part of (1^2), and the respective lengths marked off, the application of the scale to all those lines connected with them must prove themselves, or one of the three must be in error; therefore, as before stated, it is imperative on the surveyor to examine and correct them before proceeding further.

Having first obtained a tolerably accurate knowledge of the extent and position of the estate about to be surveyed, fix on a point where the longest and most important lines can be made through the centre of the estate, as at (1) and (1^2).

By referring to the plan, the survey commences at station (1), on which the several stations are fixed to take up the irregularities of the brook, leaving but little to enter into the field-book, excepting the crossings of fences; the small triangles between 556 and 870, and between 1195 and 1698, are filled up afterwards, or at the time when chaining the line.

The second line commences at the end of (1); this is also a principal line, although having only the intended stations and the crossings of fences to enter. It is worthy of notice, that in these lines there appears so little matter entered; in chaining them it gives the surveyor a better opportunity in fixing the minor lines to fill up the survey, also, from his first observations, he was prepared to intersect (1^2) near the middle of the line, leaving a peg for that purpose (see field-book) at 1050 in line (2). At the termination of this line the third line commences, crossing the first station at 404, which terminates at 596; now both these lines are proof lines to the greater triangles.

Commencing again at (1) or (1^2) is another important line of the survey, as it takes up a length of fence and fixes the position of a small field in a corner out of the general survey, which is completed by the lines (5) and (6); the seventh line ending at 1320 in (1^2) completes the other large triangle, also a proof line; the smaller lines following in succession finish the survey adjoining the brook.

Line (12), by the intersections of the great triangles, as also the small trapezium next the brook, is a further proof. In like manner the intersection of lines (13) and (15) fix the upper part of the survey, each line proving itself, and completes the survey.

In all cases endeavour to follow up the chaining successively, and avoid running from one part of the survey to another, which may be easily accomplished by bearing in mind the effect of the succeeding lines in conjunction with the principal lines of the diagram.

It must be observed, all the lines in this survey are confined within the boundary of the estate, and, from the peculiar and irregular form of it, the diagram may be changed by chaining over the adjoining lands, which must create more labour and loss of time in crossing fences (which should at all times be avoided as much as possible), and becomes very questionable whether in the end time and labour is saved by it; neither can the accuracy be called in question.

The plotting of this survey will be very simple and easy. Lay the largest triangles down first; next construct the triangle by lines (13) and (15), which being extended, and their lengths marked off, proves the connecting lines by the scale, as well as all minor lines.

The quantities have before been noticed. (See Problem 97.)

Estates are very frequently distributed, and small parcels lay at a great distance from the bulk; to connect them by the roads would be a waste of time, and make an ugly plan. Having the meridian line to each piece, they may be placed on the plan somewhat in the direction and in the true bearing; then one compass on the plan will answer for all.

Problem 11.

To survey a road, Fig. 1, Plate 13.

When no opposition is made to enter the adjoining lands, this example by the chain only is expeditious and accurate; if

the survey is confined within the boundary fences, the angles must then be taken by the instrument, which will be hereafter explained.

Although only a small portion of the road is shown, the construction of the chain lines fully exemplifies the previous observations on the truth of the intersection of two lines, and the proof they give to one another.

Commencing at (1), leave a peg or mark at 484 for station (2), and at 960 leave another mark, also one at the end of the line; return back to (2), and regulate the length of that line by the mark or flag set up at 960 in (1), sending a flag forward at the end of (3) to be fixed in the same line; then finish chaining the first three lines.

The end of the third line must be regulated by the flag fixed at the end of the first line, and a flag sent forward to fix in the line (4), and so on one line after another to the end of the survey.

To plot this: Draw line (1), and mark off the length and different stations on it; take in the compass the length of line (2), and describe an arc; then with the distance 510 from 960 in (1), intersect that arc, draw the lines, and mark off the length of (3). Draw the line (4) from the end of (3), through the point end of (1) apply the scale, and if the length from (4) to the end of (1) agrees with the entry made in the field-book, the foundation of the whole survey is correct; having marked off the length of line (4), apply the scale to line (5). This process must be repeated on every fresh base, and requires only one triangle by construction.

Note.—For small surveys a regular field-book is not so convenient as a sketch-book about the size of letter paper, in which, by correctly sketching the subject about to be surveyed, draw in the chain lines by dotted lines, as shown by this example.

Problem 12.

To survey a river or brook, Fig. 1, Plate 14.

This example is similar to the last, except that the land is

generally found to be more open and level, and therefore better adapted to sketching, as is here shown.

In either example, where wood or plantation interferes, then the use of the instrument is necessary, and even then it is difficult to obtain a clear sight for any distance without cutting a way purposely. For small surveys like these the box-sextant will be found the most convenient.

From the clearness of the field-book, there can be no difficulty in plotting both the examples to a larger scale.

Problem 13.

To continue a base line when obstructed by a piece of water, building, &c., Fig. 2, Plate 14.

This is one of the greatest calamities that can happen in chaining the principal line of a large survey. From the undulation of ground it is impossible for the surveyor in every instance to avoid such unexpected interruptions, although previously prepared for crossing a river, as shown in the next example.

It must be borne in mind that a base line is of such importance to a survey that it must not be abandoned; beyond this obstacle referred to, which is supposed to lay in a valley, the line has been accurately poled out to the extent on the opposite rising ground, therefore the intervening space has to be obtained to make the length complete.

On the base line, A B, raise two perpendiculars, either by the box-sextant or by the chain, as *a b* and *c d*; extend these lines to *b* and *d* equal to 400 links, then measure the distance from *b* to *d*, equal to 450 links, add this sum to the length of the main line up to *a*, say 1000, then will the line to *c* be equal to 1450 links; entering it into the field-book precisely as it occurred, then proceed with the chaining as before.

If the water should extend to the right, set out lines according to the nature of the figure, and sketch them in the field-book; this may be surveyed before proceeding on with the main line.

Problem 14.

To erect a perpendicular by the chain, Fig. 2, Plate 14.

Set off on the main line A B 40 links from B to *a*; fix one end of the chain firmly at B, and take the chain at 80 links, hold that firm at *a*, then take the chain at 50 links, pulling it tight both ways until they are quite straight, and put down a peg at *e*, which will be the point of the perpendicular; so that B *a* is 40, *a e* 30, and B *e* 50 links.

Another method:

At 40 links from *a* fix one end of the chain by the offset staff at B; let the assistant hold the chain at 80 links at *a*; then take the middle of the chain 50 links, stretch the chain firmly on the ground, and a mark or peg put in at *e* will be the perpendicular to B *a*.

THEODOLITE SURVEYING.

Problem 15.

To continue a base line over an inaccessible river, Fig. 3, Plate 14.

When the base line crosses nearly at right angles.

From *c* measure back 400 links to *a*, and fix flags at both points; raise a perpendicular at *c*, and set off 300 links at *d* and fix a flag; at *a* raise another perpendicular and set off 600 links at *b*, and fix a flag; now measure from *a* to *d*, which should be exactly 500 links, otherwise it must be corrected; then from *d* to *b* 500 links; this point also must be minutely correct, as the distance across the river depends entirely on the accuracy of the points *d b*. These points being truly settled, send the assistant over the river to fix a flag at *e* correctly in a line with the flags at *c a* and the flags at *d b*; then the distance across the river from *e* to *c* will be equal from *c* to *a*.

Another method, Fig. 4:

When the base line crosses the river obliquely.

First set out the line D B E as part of the survey to take up

the river; at any part of the line as E, with the box-sextant set at 90 degrees, having a flag previously fixed at C in a direct line with A B; keep moving on the line E B till you have brought the two flags together on the mirror, which will be the point perpendicular to E B; measure the distance from E to B equal to 400 links; measure the same distance from B to D; at D raise a perpendicular as before; directing the assistant to move till he is at the point of intersection at A, measure the distance from A to B equal to 475 links, which will be the distance across the river from B to C.

SURVEYING HILLY GROUND.

Chaining over hilly ground is not attended to by surveyors in general with that caution the subject deserves.

There are few countries where an extensive survey has to be made without a portion of hill and vale. A moderate gentle slope is by no means objectionable, as it enables the surveyor to have a better view of the country and to lay out the principal lines, in setting out which endeavour to command the sidelong ground for the chief lines, the hypothenusal angle will then be much reduced.

It must be remembered that when a survey is plotted it represents a perfect plane, and in chaining the lines they should be so conducted that every length should be a point exactly equal to the base of a right angled triangle.

For instance, by way of exemplifying the subject, suppose a flight of 20 steps, each step in width 1 foot, equal to 20 feet on the level, and the whole height 12 feet, the length of the ascent or hypothenusal line will be 23 feet 4 inches nearly in that short length (the difference is considerable, for as the angle of acclivity or declivity increases, so also will the difference increase); consequently, if a proper allowance in the length of the line measured up an incline is not made it is utterly impossible to plot a survey correctly.

It is the practice of many surveyors in chaining hilly ground

to hold up one-half the chain and drop a stone or one of the arrows to the ground, intending to make that a perpendicular point; it is perfectly ridiculous and imperfect.

The only true method is first to ascertain the angle, and make the allowance when plotting the survey, or by making the allowance on each chain at the time of measuring the line.

When surveying in a hilly country the surveyor should be provided with the table for reducing the hypothenuse (see Table, No. 12); take the angle by the box-sextant, or by a very simple and portable instrument for that purpose invented by the author. (See Quadrant, Fig. 1, Plate 38.)

The chief argument on hilly ground is the difference between the base and hypothenuse. (See Problem 19.)

In taking the angle of inclination or declination, be careful the forward object is equal exactly to the height of the eye when taking the sight, so that the imaginary line be parallel to the surface.

When an angle is measured by the theodolite, the allowance to each chain is engraved on the vertical arc according to the angle.

As a further elucidation on the subject of hilly ground, a pale fence or growing timber on the acclivity of a hill will not occupy more space than it will on level ground; but the rails to which they are fixed, having the same slope as the hill, must therefore be longer in proportion to its angle.

So also might be said in planting trees. This approaches very near to the first argument, that of the steps; the root of each tree must have a sufficient base for the nourishment of its root, having a certain width round it like so many steps; consequently there could not be more trees, and in some cases, according to the nature of the ground, there may be less than on the level ground.

It is rather questionable whether there be more corn or grass; at all events, it is well known the quality will be inferior.

The subject of quantity is already decided, as it is estimated from a perfect plane.

The only thing that can be reasonably admitted is the work of the labourer, consisting wholly of superficial and lineal measure, as mowing, reaping, hedging, ditching, &c.

When a hill approaches a semi-globular form it can only be an approximation, and the method of obtaining the difference is by fixing a number of poles round the hill, and observing the angles from the top; in practice the surveyor should avoid these difficulties as much as possible.

Those who are desirous of entering more minutely into the subject are referred to the calculations made by Dr. Hutton, and Dr. Maskelyne's observations in Scotland in the years 1775 and 1778. (See the Philosophical Transactions.)

TO SURVEY AN ESTATE OR PARISH BY THE CHAIN ONLY.

Problem 16.

Plates 15, 16. The chief point now is to draw the attention of the student to former observations on a more extensive survey; and he is strongly recommended to plot the whole of this survey from the field-book, to a scale of 3 chains to the inch, and compute the same by the several examples previously shown.

By this practice the knowledge gained will more than compensate for the trouble; merely reading over the field-book and referring to the plan will not impress the subject so firmly on the mind.

Agreeable to the instructions given in the General Observations, and referring to the plan, the lines (1), (2), and (3) constitute the largest triangle. In poling out line (1) a tree on an eminence in the distance assists greatly; in like manner, at the same station, the steeple in the distance was another excellent object; in poling out line (1^2) these two lines will at once show the importance of their position in relation to the

other portion of the survey; by extending these lines beyond the limits of the estate, proves at once the superiority of the diagram by the lines (1), (2), (3), (1^2), and (4).

When all these lines have been chained they should be immediately plotted.

First construct the largest triangle by the help of a beam-compass; point off the intersecting number on line (1) at 2255, and on line (2) at 2210; draw line (4) from that point through the intersecting point in (1), and set off the whole 2990 links; then apply the scale from the end of (4) to (1); if correctly measured, the distance by the scale of (1^2) must agree with the number entered in the field-book; if it does not agree, the error must be found before any further proceedings are made. The cause must arise either from bad chaining, or false entry in the field-book, or crooked lines in the poling, or an error in plotting.

When all the principal lines are plotted satisfactorily, every line afterwards must prove itself by the application of the scale, from one station to another, and at every point of intersection.

In the large triangle to the west the lines (6) and (7) govern the rest of the smaller lines in that triangle, completing that side of the survey.

The north part is governed by the lines (8) and (14), fixing all the inferior lines, and completing that part of the survey.

The triangles at the north-east, the lines (9) and (10) governing and completing that part.

The remaining portion is governed by lines (5), (19), and (16).

By adopting this system of confining the details of the survey within the district formed by the principal lines, it will prevent running about from one part of the survey to another; it is also more expeditious, and the plotting much clearer to be read.

As before observed, all chaining performed in the day, the lines should be plotted at night; in case of any accidental error

it is more easily rectified. Plotting the fences and other minutiæ remain for a wet day; never leave off chaining otherwise.

On several of the lines at (11) and (14) short lines are introduced instead of long offsets, which should always be avoided; it is better to measure these at the same time the line is chained.

Supposing this to be a survey of many thousand acres, the same system must be used; it would be only an extension of time and labour, and that may easily be reduced if required to be executed in a short time.

Two or four surveyors may be engaged at the same time; but all the principal lines forming the diagram should be first chained and plotted, and a division given to each surveyor, with a copy from the field-book of the base lines as far as may be required.

Observe not to roll the paper on which the plan is to be plotted; if possible, confine it to a board until the whole is completed.

At the commencement make a scale on the paper, and occasionally apply the plotting scales to it to ascertain if any difference has occurred by atmospheric influence.

SURVEYING WITH THE THEODOLITE.

In surveying with the theodolite its chief use is to measure the angle formed by two lines, in doing which there are two methods that may be applied to obtain the same result.

By the first the angle depends solely on the nicety of the magnetic needle, which is frequently very uncertain. The second method is more positive, though in both cases it requires equal attention in adjusting the instrument and reading off the angle.

There are many advantages to be obtained in surveying with the theodolite; in chaining hilly districts, the horizontal angle and the hypothenusal difference is given at the same time; it is valuable in taking angles of parts that are inaccessible, and

obviates many difficulties the chain by itself could not perform with equal accuracy and time.

In the survey of towns it is indispensable. It is also valuable in proving or determining the position of stations or distant objects, and in poling out long lines over hilly districts.

A full description of the Theodolite is given in Part V.

The whole of the plans that have been previously introduced to show the method of surveying by the chain only, are repeated in the following examples, with the view of drawing the attention of the student more directly to the difference of practice used in both systems, and by comparing one with the other he will more readily discern which of the two systems is best adapted for the survey.

The surveyor possessing a thorough knowledge of both systems is sure to overcome the greatest difficulties by a combination of the two.

There are no fixed laws to direct the surveyor at which part he is to begin, or what figure to make with the chain lines; the same surveyor could vary them and still be a good survey.

The best survey is that which has the least number of lines.

Problem 17.

To survey a single field with the chain and theodolite, Fig. 1, Plate 17.

In the former example (Fig. 3, Plate 8) the regular field-book is introduced, in this example the system of sketching is adopted; the chain lines and offsets are the same in both; instead of diagonals or tie lines, which were required in the former to plot the chain lines, the angles taken by the instrument produce the same effect.

As before remarked, a triangle by itself has no proof; so also in this case one angle is no proof. Therefore it is necessary that two angles should be taken on the same base; then the line opposite to them would become a proof line by the application of the scale.

Problem 18.

To survey a field outside the fences, Fig. 2, Plate 17.

The observations made on the last example will apply in every instance to this; the only difference is the chain lines, being outside the fences, cause more trouble in crossing the several fences, and making a clear sight through them for the instrument in taking the angles.

To plot these, draw the base line on which the angles were taken; place the centre of the protractor exactly to the station points, and zero, or 360°; with a fine needle prick off the angles as inserted in the field-book, draw lines through these points, mark off the respective lengths; the last line will be a proof to the whole.

Problem 19.

To continue a base line obstructed by buildings, water, &c., Fig. 3, Plate 17.

The former example (Fig. 2, Plate 14) is tedious when compared with the instrument.

In this case the box-sextant may be applied with considerable advantage in every respect; it answers the purpose of the theodolite.

The base line being stopped at *a*, put down a peg, set the vernier of the sextant to 90°, send a man forward with a flag-pole, having another fixed on the base line at A; then standing firmly on one heel directly on the point *a*, look through the hole in the box towards A, and direct the flag at *b* to move gently until it coincides with the mirrors in the box, which will be the point perpendicular to the base.

Then from *a* set off any convenient length sufficient to clear the pond, and fix the flag at that point (say 400 links); then on the other side of the water, at *c*, raise another perpendicular in like manner, and set off an equal distance and fix a flag at *d*. Then measure the distance from *b* to *d*, equal to 500 links (as the case may be), that being equal to the inaccessible dis-

tance from *a* to *c*; adding this to the number measured up to *a* will give the length of the base line to *c*; from thence the line may be continued.

Problem 20.

To continue a base line over an inaccessible river, Fig. 4, Plate 17.

First: When the main line A B crosses the river nearly at right angles, fix flags at *a* and *d* at a convenient distance from the edge of the river; at *a* erect a perpendicular, and measure off any convenient length, say 300 links, as at *b*; at this point place the theodolite and adjust it (see Instruments, Part V.); then measure the angle *a*, *b*, *c*, equal to 53 degrees; turn the telescope round, the lower plate being clamped, and set it to the same angle on the base line; fix the flag at *d*; measure the distance from *c* to *a* equal to 400 links, then *a d* will be equal to *a c*.

Problem 21.

To continue a base line when obstructed by a house, garden, and premises, Fig. 5, Plate 17.

This obstruction is of greater extent, and requires a different operation, to the former examples.

Fix the theodolite at *a*, set off an angle of 30 degrees on each side the base line, and mark off any convenient distance, say 400 links, as at *b d*; fix the theodolite over both these points, and set off 120 degrees each, intersecting each other at *e*; if correct should be equal to 400 links, the same as the other sides; then will *a b d* be equal to *b d e*, and both are equilateral triangles; the angle *e* will be equal to the angle *a*.

If possible to chain the line *d b*, fix a flag at 200 links from *d* or *b* at the point *c*; the line A B may be then continued.

The four lines, *a*, *b*, *e*, *d*, serve to take up the boundary fences to the premises, and probably the line *b d* may be used for the interior part; otherwise a sketch of the whole must be made within the lines *a*, *b*, *d*, *e*, and a separate survey altogether.

In a large survey such an obstacle as this happening in the middle of the base line, it should not be continued upon such a slender foundation as from *c* to *e*; better abandon the line or change the diagram. If it happened very near the end of the line, it may then be adopted if executed with peculiar care. But if the obstacle lies in a valley, and the ground on each side so elevated as to see over it and pole the line, then this process would be sufficiently accurate.

Problem 22.

In the four following examples the field-book is dispensed with, and the lines, with their lengths, and such angles as are requisite to plot and prove the accuracy of the survey, are inserted on the plan.

To survey a field with several sides, Fig. 1, Plate 18.

First fix flags or marks to all the stations; commencing at *a*, chain the lines regularly round the field, leaving marks at 173 in 2, and 176 in 4; then take the angles at *a*, *b*, *c*, which is all that is required.

It must be noticed that by extending line 6 to 173 in 2 it saves taking a very obtuse angle at station 2; as line 1 is fixed by the measured angle, so also will line 2 be fixed. The angle taken at *c* fixes line 4, consequently line 3 becomes a proof line, provided the lines and angles have been accurately taken; line 7 is similar to that of 6 by crossing part of the field from *c* to *b*; the angle taken at *b* determines line 6, therefore line 5 is a proof, and the survey is finished; by extending 6 and 7 two angles less are required.

Supposing the lines 1, 3, and 6 were extended to *d*, the intersection of 6 and 7, and line 2 fixes every line in the survey, line 5 being the only one not plotted and the last proof; comparing this and the example, Fig. 2, Plate 10, with Fig. 3, Plate 18, will clearly show which of the three examples are best; in the one instance there is the care and trouble with

the instrument and time in taking the angles and plotting them, the others require only the chain and offset staff.

Note.—When surveys are executed by the chain with the theodolite it is not advisable to take angles at every station; a multiplicity of angles would only create mistrust and confusion, nor is it possible to take angles with that great degree of nicety to be proved by the sum of the whole, as will be shown in the next example; and the method adopted in practice to prove angles taken on large surveys will be further explained.

Problem 23.

Fig. 2, *Plate* 18. Euclid, Book I., Prop. 32, Cor. 2: "All the exterior angles of any rectilineal figure made by producing the sides successively in the same direction, are together equal to four right angles."

Because the interior angle, A B C, and its adjacent exterior angle, A B D, are together equal to two right angles, therefore all the interior angles, together with all the exterior angles of the figure, are equal to twice as many right angles as the figure has sides.

It has been proved by the foregoing corollary that all the interior angles, together with the four right angles, are equal to twice as many right angles as the figure has sides. Therefore all the interior angles, together with all the exterior angles, are equal to all the interior angles and four right angles, and all the exterior angles are equal to four right angles.

The given figure has five sides; the sum of all the interior angles will be $= 5 \times 2 - 4 = 6$ right angles $= 90° \times 6 = 540$, viz.:

	Deg.	Min.
∠ A =	121	10
„ B =	103	45
„ C =	69	25
„ D =	142	50
„ E =	100	50
	540	00 Proof of angles only.

Problem 24.

To survey three fields divided by a road, Fig. 3, Plate 18.

Although the boundary of this plot is very irregular adjoining the stream, the number of chain lines are few and effective, requiring only three angles by the instrument, as A B C. The

necessary flags being fixed, commence chaining at A; on this line leave two station marks at 867 and 1662, length of line 2210; 2 commencing at the end of 1, leaving off at 550, there fix a flag; now return to 1662 in 1 and chain line 3; at 338 the end of 2 completes the triangle on base 1; at 400 put down a peg and continue the line to 586; this line by the triangle is fixed; return to 400 in 3 and chain line 4; at 192 put down a peg; leave the pin in the ground at 200, and proceed to measure the two short lines of the trapezium and enter them in the margin of the field-book, which finishes up that part of the survey; for these short lines take the angle at C 96° 10′ with the box-sextant; the trapezium is then proved; now commence again from the 200, at 740 have a peg, and at 1042 have a peg or mark, finishing the line at 1618, there fix a flag. Return to 1, stopping at the last flag 450, and finish the triangle A (which enter in the margin of the field-book): continue the line to 740. Although this line is fixed by line 4, it is possible an error may occur in chaining, therefore to guard against casualties take the angle at A; proceed with 5 and 6, take an angle at B; the line 5 is a proof to this trapezium; line 7 completes the survey; this is also a proof line.

The plotting of this survey is extremely simple. First plot the angle A, next the triangle D; set off the station and length of 1^2, and the station and length of 3; the lines 1^2 and 3 are fixed; apply the scale to line 4; if it agrees with the length entered in the field-book the survey is correct. Now plot the angle B and set off the length; this line is fixed; apply the scale to 5 and 7; the chain lines being all proved, plot the offsets, and all is complete.

This example shows the great advantage of combining the two systems, and not to depend wholly on the use of instrumental angles.

Problem 25.

To survey three fields peculiarly situated, Fig. 4, Plate 18.

This example possesses more difficulties in projecting the

chain lines than any preceding it. By comparing it with Fig. 4, Plate 10 (the same plot), it is at once very evident that the theodolite reduces labour and time by the reduction of chain lines.

Commencing at line (1), at 239 measure up the small triangle and enter it in the margin of the field-book; at 762 leave a flag, to be seen from the road, for line (6), which, by taking an angle, will be a fixed line; finish chaining line (1) and the small triangle at the left; next proceed with line (2), which passes through the hedge near 575. It will be observed this line divides a very crooked fence, making the offset short on both sides, and avoiding the wood; at the end of this line take an angle to fix line (3), supposing permission was not given to cross the field next the road, otherwise the angle would be useless, as lines (3), (4), and (5) would be proof lines. The cottage-garden in the one case must be taken up by a small trapezium; in the latter, it can be taken up by offsets from the two lines (3) and (6). An angle must be taken at the end of (5), which determines the position of that line; in fact, by measuring the whole of (6), and the two angles on that line, determines the whole survey.

To plot this survey first draw line (1), mark off the different lengths, and the angle at 762; draw line (6), mark off the length 507 links, and prick off the angle 108° 0′; draw line (5), mark off the station 383, and the length 766 links, proved by the scale, as also line (4).

In every kind of survey the diagram or chain lines should always be kept in the mind, to determine on certain angles by which the work is to be plotted, as it is only then that the correctness or defects of the survey can be discovered, which shows the necessity of plotting the chain lines immediately.

It is always better to have a line or angle too much, than to want one, as frequently a very trifling line or angle would prevent many difficulties.

Problem 26.

To survey a small estate, Plate 19.

The great irregularity in the boundary of this estate shows that the survey by the chain only increases the labour, though that would not be so in every case; much depends on the fences within the boundary.

The base line (1) is similar to that in the former example, Fig. 1, Plate 11, and has the same effect in fixing the position of the small field at the extent.

The triangle formed by (4) and (4^2) is confirmed by the small tie line; all the other lines are fixed by the angles taken with the instrument, and are chiefly on the base line; when plotted the remaining lines are all proof lines.

Problem 27.

To survey a road within the fences, Plate 20.

This example is well adapted for the Prismatic Compass, a full description of which is given in Part V.

The centre of the instrument is fixed directly over the station point in the same manner as the theodolite; the engraved card is divided into degrees and half degrees, and attached to the magnetic needle. It must be observed that the card reads reversed—that is, north for south.

In taking an angle by this instrument, place the box containing the needle as level as possible, look through the prism to the forward object, and bring the thread in the slide to coincide with it and the flag in front; when the card is perfectly steady take the angle.

In a survey of this kind, where extreme accuracy is not required, the prismatic compass, from its portability and readiness in fixing, makes it preferable to any other instrument.

To plot this, draw a meridian line through every station, as at A B, &c.; lay the straight edge of the protractor along the line with 360° at the top and the centre at the station, then prick

off the number of degrees required; repeat this at every station, then proceed to plot the offsets.

Problem 28.

To survey a river or brook, Plate 21.

This example, when compared with the former (Plate 14), shows that the survey by the chain only is in this instance the best, provided the ground is free from obstructions, such as wood; the lines would be chained in much less time than that occupied in taking the angles, as also in plotting.

The land adjoining the stream may be supposed to be nearly level, therefore well adapted for the box-sextant; the angles taken by it are the bearing of two lines.

The first and second lines may be plotted by the tie lines, and proved by the measure of the angles; lines 4 and 5 must be plotted from the measured angles; short tie lines should at all times be taken, however trifling the check; it is sufficient for such a small survey.

The smaller triangles to fill up the survey may be sketched, as shown by the field-book.

Problem 29.

On surveying an estate or parish, Plate 22.

Referring to the previous remarks on extensive surveys, the first main object is to select the best position for two lines intersecting each other near the middle of the survey, and by other connecting lines to form a diagram, so that the system depended entirely on correct chaining the lines that constantly intersect each other, forming a series of triangles.

The system of surveying with the theodolite differs from the former system, in not commencing the survey with the two base lines before pointed out.

In this system, the accuracy of the survey depends on the truth and adjustment of the theodolite in the first instance; second, in the care and attention in reading the angles and entering them in the field-book correctly.

When angles are taken from the magnetic meridian, the greatest attention must be given in watching the needle, to notice that it is properly sensitive and has no metallic attraction near while taking an angle, such as the chain and arrows, or knives in the pocket, &c.; if this is not particularly attended to the angles will always be uncertain.

In large towns the attractions are numerous, such as gas-pipes, railings, &c.

When angles are taken by the bearing of two lines, then the needle is not required, excepting once, in taking the north point to fix on the plan.

The field-book is kept precisely in the manner before described, and when an angle is taken it is entered on the margin; but when two or more lines begin at the same station, or end at that station, before moving the instrument take all the angles, and make a small diagram on the margin of the book similar to those on the plan, numbering each line and the angle against it.

It is not requisite an angle should be taken at every station; such practice not only consumes much time, but creates confusion and difficulty in plotting.

As in the former example, when there is no natural object to direct the chaining, the line must be poled out, the offsets and all other minutiæ entered in the book as before. The angles that are chiefly required are those connecting the boundary lines and roads, as shown by the plan; the smaller lines seldom require an angle.

Commencing with line *1*, the steeple in the distance is a good fore object for the chaining. Plant the instrument directly over the station at 1, and put it in adjustment (see Theodolite, Part V.); bring the zero of the vernier and 360° to the opposite side of the eye-glass, and clamp the upper part; then bring the needle to 360° in the box, and clamp the lower part, relieving the upper; moving the telescope very steadily, observe the first chain line directed to the steeple, reading 30 degrees.

Looking to the right hand along the fence an object is there fixed for a station, turn the telescope to it, the angle reads 130° 30′; and, in order to have a check on all the angles at different parts of the survey, to prove both the chaining and the angles, take extra angles frequently from different stations to one particular object, either natural or one fixed purposely. In this instance a tree is the object, therefore an angle is taken at station (1) 38° 30′, and at the end of line (1) another angle is taken 70° 30′. Now the intersection of these two lines will fix that object on the plan, consequently when an angle is taken at station (3) or at station (26), or any other part of the survey, whatever the angle may be when it is plotted, the lines when drawn should intersect at the point or tree. Another way of proving the work in succession is, having the angles taken at each end of the line, the sum of those two gives the third, so that when those angles are plotted, measure the other angle by the protractor; it must read the correct angle if the previous work is correct. (See Problem 22.)

This process may be practised at different parts of the survey, and where there is no object put up a flag, and proceed as before.

Returning to (2), take the angles by the brook and road; also to the tree; continue in the same manner round a certain portion of the survey, taking the roads as a boundary as far as the case will admit, until finishing at the same point at which the survey commenced, as from (1) to (11), leaving pegs or marks on all the lines as stations for the interior fences.

Referring to the plan, observe that when lines from (1) to (11) and (12) to (13) are plotted, the only line that requires an angle measured by the instrument is (20); such lines may be taken with the box-sextant, after which every other line in that division is a proof line, and requires only the application of the scale.

The remaining portion of the survey is closed in the same

manner, and the numerous angles required on this side are properly checked by angles to the tree or other object in the centre.

This system of surveying may fairly be termed a combination of the two, which cannot fail in producing an accurate survey.

One thing is to be observed, that when an error occurs either in taking an angle or in chaining, it is more readily discovered when the angles are taken from the meridian line than when taken by the other method.

In plotting, describe two or three circles by the outer edge of the protractor on different parts of the plan adjacent to the work, as shown by plan, and draw several meridian lines; it is then easier to draw the chain lines; and mark off as many stations on the circles as are in that locality, noting the angle and station.

Before attempting to plot the fences, let all the chain lines be first plotted, the number, length, and angle put against them; it will save much time, instead of having to refer frequently to the field-book.

The larger a protractor is, the better; the angles are more minutely pricked off. The wheel protractor is the most perfect instrument, having two verniers, and may read to seconds if required; but, unless strict attention be paid to the adjustment, by keeping the points truly in a line with the centre, the common protractor is far better. Incorrect plotting is just as bad as incorrect surveying.

For practice, plot this survey on a larger scale—the angles and chain lines only; then, to prove the survey, calculate by the figure A B C D E F all the triangles. Angles should also be taken from one station to another across the survey without chaining, as from C to D and from E to F, and, by joining all these lines, the whole survey is reduced to a figure of six sides.

Note.—The detailed portion of this survey is filled up as shown by the field-book, Plate 15.

Problem 30.

To survey the boundary and roads of an estate, Plate 23.

In this example is shown a portion of the field-book, applying to this and the last plan.

This survey is intended to show only the boundaries and the roads, coinciding exactly with the observations made in the last example, proving at once how easily the whole estate may be filled up without further use of the instrument.

Distant objects should always be preferred in fixing the lines for chaining, particularly when using the theodolite, as angles may be taken from various parts of the survey to the same object, and be an excellent check to the whole of the survey.

Plot this survey to the same scale as the last for practice; compute the details of the former and the gross quantities of this, and compare the total quantities, which should agree within a few poles.

MINING SURVEYING.

Problem 31.

Latching or dialling a pit, Plate 24.

Latching or dialling is a term used by miners, when the survey of a mine is required of the course that has been excavated from one shaft of the mine to another.

This underground survey is then transferred to the estate plan by means of the shafts, which are accurately shown on the surface; otherwise it is working like a mole in the dark, and the course might run into another property.

Referring to the plan (which is an actual survey), it will be seen the lengths are taken by the chain or tape in links, and the angles taken by the needle, or magnetic meridian, with a circumferentor called by the miners a dial.

The legs of this instrument are made with a screw-joint in

the middle, and has a set of extra points to screw on when the mine is low, sometimes not more than a yard high.

The survey commences from the shaft No. 16 to No. 17. The chain lines and angles are all plotted from that point, in the same manner as described in Plate 20.

The circle shows the protractor with the meridian line through the centre, and all the angles marked thereon for plotting.

The field-book (if it may be so termed) is on the margin, containing only the numbers, lengths, and angles.

Great nicety is required in taking the angles and the lengths; the last point finishes at the centre of the shaft No. 17.

TOWN SURVEYING.

Problem 32.

Plate 25. The plan here represented is part of the town of Cheltenham, surveyed and published by the author.

In surveying parishes, it frequently occurs that large towns form a considerable portion; the streets are generally very irregular, preventing the possibility of continuing the course of triangulation.

When an opportunity presents itself to run a base line through the town from one side to the other, connecting itself with the general survey, it should be embraced, and fix on it stations for every street branching from it. Such a line as this will be a basis for the angles required to be taken by the theodolite. It is not required to take angles for every street, because, where two or more angles are taken on the same base, their position is fixed, and the lines running through the ends of them become fixed also, and many of them will close into the lines of the general survey outside the town.

As before described, there are two methods of taking angles by the theodolite: one, the angle measured from the magnetic

meridian; the other, the angle measured between two lines. There is a very great objection to the first, the uncertainty of the angle being correct because of the numerous attractions to the needle from iron railings, gas, water, and rain pipes, lamp-posts, &c., therefore the latter is more certain.

The four-pole chain is generally used, but when the survey is plotted to a large scale, and great accuracy required, the 100-foot chain should be used; the offsets would then be taken with the tape in feet and inches, instead of links.

When flag poles cannot be fixed, take any object at a distance, as a lamp-post, corner of a building, &c.

Many station points are frequently referred to, either for taking angles or for starting fresh lines, and require to be found very accurately.

Therefore measure from each angle of the buildings nearest to it in feet and inches, as shown at C and B, Fig. 2; their intersections will give the true point.

The angles of buildings also require to be very minutely fixed; measuring an offset at right angles from the chain is not sufficiently accurate, therefore with the tape measure two distances from the chain, intersecting each other at the point required, as at 25 feet on the chain it is 37 feet to *a*, and at 50 feet on the chain it is 38 feet 3 inches to *a*, forming a small triangle; the same is done on the other side to *b*; the next angle is at *c*, and on the opposite side at *d*; the fronts of the buildings are straight from *b* to *c*, and from *a* to *d*; and so proceed on throughout, taking offsets in this manner only at the angles and for the stations. The subdivisions between each angle or corner of streets, &c., are all measured afterwards; the chief thing at first is to get all the lines and angles measured and plotted.

The line A H is directed to a lamp-post, or it might be extended to a line on the general survey, as at H; a station is left at C for lines D and E, to take up the opposite streets; also another station at B, for the two roads G and F. This line

will be sufficient to show the method of keeping the book, and the system to be adopted throughout in taking the dimensions.

When there are a sufficient number of lines measured, proceed then to take the angles. It will be found in course of the survey that angles will not be required to every line or street, for when one or two of the principal lines are plotted, many others will fall into the work as proof lines; for example, Fig. 1.

Much information will be gained by a careful study of the plan. The dotted lines represent the chain lines; the north part called the block plan, marked A, having only the fronts of the buildings or streets; the south part, marked B, represents the plan when finished with all its details.

On the line *a b*, there are ten stations determined on in the first line. Commencing again at *c*, take the angle *a c d*, on which there are two stations, *g h*; commence again at *e*, take the angle *a e f*. Now, it is evident, if the two former angles and the lengths are measured correctly, that the line *g f* will be a proof line, and also the line *i h k*; the station at *k* will be a fixed point.

We now commence at *f* and proceed to *l*, at which point it will be necessary to take the angle *m l f*, and that angle will prove the length of *c d* and *n k*.

Now measure by the chain the line *l m*, passing through *d n*, that being the end of the second line; therefore the distance *l* to *d* must prove itself, as that point was before determined by the angle taken at *c*.

At *n*, on the line *l m*, leave a station, then will *l m* be a fixed line for a combination of the survey on the north part.

Now measure the line from *n* to *k*, on which leave stations at *o* and *p*; the lines *n k* and *o g* will be both proof lines.

So that in the great portion of this part of the survey three angles only are required, and every measured line afterwards proves itself.

The same principle is adopted on the south side, taking the longest and straightest streets to form a basis, and those lines

that are nearest to a right angle, as *q* to *r*, upon which numerous stations will be fixed.

Observe always to keep the work together as much as possible, so that it may be closed in, as shown by the lines *e* to *s* and *s* to *t*.

When the whole of the survey is completed, and the block plan plotted, proceed then with the details.

Prepare a book about the size of letter paper; copy off each block separately on an enlarged scale. With the tape, first measure the fronts of each house, passage, &c., separately, as thus (see Field-book): 24.3, 5.6, 20.0, and 29.3; add all these together, which should be equal to the measure of the whole length from *b* to *c*.

Then proceed with the depths of the houses and the back premises, taking diagonals from one angle to the other. A sketch of the premises having been previously made.

In many instances there are passages at the back, as at *u v*, which afford considerable assistance in measuring the details, particularly when straight, as at *u*; when they are crooked as at *v*, the prismatic compass will be found useful, as the angles must be taken by the needle.

In all cases take the supplement as well as the angle. (See Field-book.)

RAILROAD SURVEYING.

Problem 33.

Plate 26. To survey for a railroad.

When a survey has to be made for a railroad, it takes a long irregular tract of country from one important town to another, generally from 10 to 20 chains in width, in some cases more.

The course the intended line is proposed to be made is marked out by the engineer on the Ordnance or county maps.

The first and most important part in surveying for a line of railway is to determine and accurately pole out the base lines,

which should be as near as possible to the line described on the Ordnance map.

In chaining a base line the greatest attention and accuracy is required, as the only check to its length is when the levels are taken. Many bills in Parliament have been lost entirely through the inaccuracy of the plans and sections, involving not only the loss of a year, but the enormous amount of money required for the Parliamentary preparations.

Unfortunately these surveys are often got up in haste, and frequently under great opposition, all contributing to the many difficulties the surveyor has to contend against.

When a long line of country has to be surveyed a portion is allotted to each surveyor, as shown by the plans; each portion has to be connected at each end, requiring the greatest accuracy in fixing the base lines.

The most perfect method of connecting these lines, is by continuing them into each other's work, and fixing them by intersecting triangles.

Base lines that are connected by an angle taken with the theodolite are very ambiguous, from many causes; for instance, where the country is hilly there may be only a short distance from the instrument to the flag, and the line probably of considerable length, depending wholly on the angle taken by a six-inch radius; a very trifling error in reading the angle would throw that line greatly out of its real position. By the former method, if the lines are accurately poled out, the intersecting lines not only prove the correct position of each base, but they also serve to take up the details, and a great saving of time. Sometimes a base has been hung on by a fence only; such practice is highly censurable.

In a long line of railway it is impossible always to keep in the valley; a portion of the line will be rough and hilly; in such cases the theodolite is indispensable.

At the end of each base line the surveyor for that portion

should leave a distinguishing mark by cutting a triangular hole, and one or two other marks on his line, to enable the next surveyor to continue that line into his work, and adopting such tie lines to the two bases as in his judgment will be most desirable.

Again, supposing the surveyor of the middle portion should begin his base line before the first base line is finished; in that case leave a distinguishing mark at the beginning of his base, and other marks, as before, to point out the direction, as well as marks on the last fence line; then the surveyor of the first portion will pole back the second base and tie lines into his work, or such other lines as the case may require.

If every surveyor adopted this simple method, base lines could never be plotted wrong.

Problem 34.

Plate 26, *Fig.* 1, represents the portion of a line, the ending and beginning of two other portions.

Commence by poling out the base line from B to *p*; at B is a distinguishing mark denoting the end of the first base, and at E another mark showing the direction of the line; from B continue that line to B^1, where it intersects the second base line; at D B^1 measure back from B^1 to E, and the tie line E D.

The second base may be said to commence at D, and ends at *p*; in chaining the line set up all the requisite stations or flags for the details; the line D F takes up the fence to the left, leaving flags for the cross fence from F to H, intersecting the base line at G, forming a large triangle and proof to the tie line E D; all the other fence lines within that triangle are proof lines as A C.

We now pass on from I to K, taking up the fence; and from K to O, intersecting the base line at S; the intermediate fence line proves the triangle I K S; from O proceed to L on the base line, which is a proof to I K besides the line T P Q;

finish the line O L to M; from M a short tie line, M Q, determines the position of the third base, which is proved by the line N R *p*.

Where fence lines are not determinately fixed, as *c b*, and without a box-sextant, take a short tie line with the chain, as at *a b*.

By adopting this system, the base lines cannot fail being correct, and every fence line becomes a proof, excepting in a very few instances.

Problem 35.

Fig. 2, *Plate* 26. To survey the same ground with the theodolite.

In this example the same base and fence lines are adopted as in the last example.

It is of no importance whether the angles are taken before or after the chaining.

When base lines are obtuse, it is therefore better to pole the line back and fix a flag, as at *a*, and measure the acute angle A B *a*.

Beside the angles taken at B and C, there would require at least six other angles for the fence lines, as at *c d e f g*, after which all other fence lines will prove themselves.

Now by comparing the two plans it will be seen that there is but very little more chaining in the first example than in the second.

Considering the time in taking angles by the theodolite, the care required to preserve the instrument from accident, it will be found that the first example is preferable to the second in the saving of time; the angles taken by the chain are more to be relied on, supposing the country tolerably fair for chaining.

Where a ravine occurs, adopt any of the examples shown, Plate 14.

MAPPING.

Mapping is the art of delineating the surface of the ground, and embellishing it to represent the natural appearances by colours, making the whole descriptive.

To excel in this art as a perfect draughtsman, a knowledge of landscape drawing will assist greatly in colouring the various parts to give it a natural effect.

The different styles of printing should be well studied. Great neatness is also required in drawing with the compasses, scale border, &c. Sometimes an abstract reference of the quantities is introduced, representing a scroll of paper and a vignette of some particular object on the estate.

Plans drawn on vellum or parchment are more difficult in executing, and never to be depended upon for accuracy after, both being effected by atmospheric influence more seriously than paper.

For practical purposes, surveys should never be plotted to a less scale than three or four chains to the inch; for convenience they may be reduced to any scale, by the following different methods:

The most perfect instruments for reducing plans are the eidograph and pentagraph. The proportional compass is a most valuable instrument, particularly for enlarging or reducing buildings, &c. (See Instruments, Part V.)

The system of reducing by squares or triangles, laid down to their respective scales, although tedious, is very accurate.

Enlargement of plans should never be done by the pentagraph; either re-plot from the field-book, or by triangles and squares.

In copying plans there are various methods; the two following are the best: The paper being confined to the drawing board, lay the original plan over it; then with a fine needle prick through all the lines and buildings; draw pencil lines through all these points and ink them in.

The next method is, to lay a piece of tracing paper over the plan and carefully copy all the lines, &c.; then place this tracing copy smoothly on the paper prepared for the new plan; underneath the tracing place a sheet of very thin tissue paper prepared either black or red, and with a very fine-pointed ivory tracer go over all the lines, and prick off all the buildings with a fine needle; the impression of coloured paper will be left. This is the most expeditious mode.

Another method is sometimes used, applicable only to small drawings. Provide a large sheet of plate-glass (which should be mounted in a frame), upon which the original plan and the paper is fixed in such a position to receive the strongest light; the lines are then visible through the paper, and traced off with a hard pencil.

A north light is best for an office, in which should be firmly fixed up a strong drawing board; provide also several other drawing boards made to the different sizes of drawing paper; also several T squares, and angle or set squares, straight edges, centrolineads for perspective, &c.

The beam-compass is a most necessary instrument in plotting large surveys. The most approved is the French invention. The sliding boxes are packed in a small case; the beam may be made by any carpenter—simply a straight piece of wood made to fit the boxes, and of the length required. They are far more convenient than those that are divided with verniers and tangent screws.

A box of curves or arcs of circles is useful, not only for drawing curves of railways, but for architectural drawings of arches and other purposes. They are made of hard wood, from half an inch to any radius, which is always marked on them; therefore, whatever the radius is on the ground according to the scale of the plan, so is the number of the curve, as thus:

Multiply the number of chains radius to the curve by the number of chains to the inch the plan is drawn.

Suppose the plan to be 3 chains to the inch, and the curve

20 inches, then $20 \times 3 = 60$ chains, or $\frac{3}{4}$ of a mile. Again, if the curve is 40 inches, and the plan 4 chains to the inch, then $40 \times 4 = 160$ chains, or 2 miles radius; and so on.

In highly finished plans, the hills are shaded or stippled with light Indian ink, or short curved lines, repeated so as to vary the depth of shade.

In like manner the ploughed lands are represented by drawing narrow parallel lines representing the furrows. To represent gravel, mix light Indian ink, with a toothbrush by the finger slightly touch the hairs, and spirt it over the part required, having first cut out a piece of paper the form of the piece, to protect other parts of the drawing from being injured.

MERIDIAN LINE.

The magnetic meridian does not show the true north, which is always moving so many degrees east or west, called the variation (at present it is about 23° 15′ west).

When a compass is drawn on a finished plan, the true north is generally drawn from the meridian line.

There are several methods of finding the variation by astronomical problems, seldom resorted to or required by surveyors in their ordinary course of business.

The most simple and ready method (see Fig. 4, Plate 35) is, by drawing on a perfect smooth and level plane, open to the morning and afternoon sun, three or four concentric circles; in the centre of the circles fix a straight piece of wire truly perpendicular, of such height that its whole shadow may fall upon all the circles at equal hours before and after twelve o'clock.

From about eight o'clock in the morning until four o'clock in the afternoon, about which hour the extremity of the pin's shadow will fall without the circles, particularly note the time in the forenoon when the extremity of the shortening shadow's point touches the several circles, and then make marks.

In the afternoon of the same day, and the same distance of time from twelve o'clock, watch the lengthening shadows as before, making marks on the circles where the shadow falls.

Lastly, find the middle point exactly between two marks on the same circle, and draw a line from the centre through that point, which will be the true meridian line or north point.

Remove the pin, and fix a short-pointed pin in its place, on which place the magnetic needle; when it is at rest, mark the point it cuts on one of the circles at the north end; from that point draw another line through the centre, measure that angle accurately, which will be the number of degrees' variation.

To measure the angle, see Problem 14.

DIVISION OF LAND.

The division of land is applied to many cases; as the general inclosure of a parish in which there are sundry claimants, each receiving land in proportion to their claims, the quantity being guided by the value per acre.*

In some cases an exchange of land is made between two adjoining proprietors for the mutual improvement to their estates, taking value for value, or quantity for quantity.

In either case it requires a correct system to arrive at a true balance, therefore a division has to be made according to circumstances, to accomplish which the following examples are given.

In small plots of level ground having straight fences, whether rectangular or triangular, there is no difficulty in laying out a division without a plan, as the dimensions required for the calculation can be made at the same time the division is made.

In all other cases an accurate plan, and quantity of the land

* Whatever the value is per acre, in calculating it must be reduced to shillings and decimals, to obtain an accurate result, in the same manner as the quantities are entered. The value per acre is usually marked in private characters, such as letters of any particular town or object, containing the exact number of letters in lieu of figures, as "Altringham," "Mayflower," &c.

to be divided, must be made before any calculation can be made, or the allotments staked out.

To facilitate the calculations required in the divisions, see Tables, Nos. 11 and 13.

Problem 36.

Fig. 1, Plate 27.

Example 1. It is required to cut off 1 acre 3 roods from a parallelogram containing 3 acres, parallel to the side A B, equal to 600 links. See Table 13.

Rule. Divide the square links in the quantity to be cut off, by the number of links in the side; the product will be the length of the other side; thus:

$$1.75000 \div 600 = 292 \text{ links}$$

Note.—Explanation to Table 13. The first column shows the number of acres, the second column the number of square links contained in the required number of acres; the roods and perches the same; each part is taken out separately and added together.

Problem 37.

Fig. 2, Plate 27.

Example 2. It is required to cut off 2 acres from a rectangular piece of ground, containing 3 acres 2 roods 16 perches, from a fixed point at *a*.

Rule. Draw an assumed line, as *a b*; then find the quantity of *a b* C D equal to 2.550; that being an excess of .55000 square links, therefore divide that sum by half the length of *a b*, equal to 620 links; the product will be 88 links (nearly), provided it was to be set out parallel; the figure of the excess, *a b c*, being a triangle, the perpendicular *c d* will then be 176 links; then will *a c* C D be the quantity required.

Problem 38.

Fig. 3, Plate 27.

Example 3. To divide a piece of building land of a rectangular figure, containing 6 acres 3 roods, between A B C, being of equal value per acre; the proportionate share of A equal to 7 parts, B = 5 parts, and C = 3 parts.

Rule. As the sum total of the proportionate shares is to the whole quantity of the estate to be divided, so is each proportionate share to its respective claim; thus:

	Acres.	Acres.	
$7 + 5 + 3 = 15$ ::	6.750 : 7 =	3.150	= A's share
$7 + 5 + 3 = 15$ ::	6.750 : 5 =	2.250	= B's „
$7 + 5 + 3 = 15$ ::	6.750 : 3 =	1.350	= C's „
Total . .		6.750	

Problem 39.

Example 4. The same estate is to be divided between A B C, the value of the whole being 750*l.*; A is to be allotted land equal in value to 350*l.*, B equal to 250*l.*, and C equal to 150*l.*; thus:

£	Acres.	£	Acres.	
750 :	6.750 ::	350 :	3.150	= A's share
750 :	6.750 ::	250 :	2.250	= B's „
750 :	6.750 ::	150 :	1.350	= C's „
Total . .			6.750	

Problem 40.

To set out the above quantities on the ground.

Rule. Divide each separate quantity by the length of the side D E equal to 675 links; thus:

Quantity.		Links.	Links.		Feet.
3.150	÷	675 =	467	or	308.220
2.250	÷	675 =	333	or	219.780
1.350	÷	675 =	200	or	132.000
10 chains =			10.00	or	660

And so on in like manner for any number of shares.

Problem 41.

Fig. 4, Plate 27.

Example 5. To divide a triangular piece of ground into a given number of parts by right lines drawn from an angle to its opposite side.

Rule. Divide the base of the triangle into the required number of parts, then draw a line from the vertex to the division point; it will be the proportion required.

A triangular piece of ground, containing 26 acres 2 roods, is to be divided into three parts, bearing the proportion to the numbers 4, 2, 1, equal to 7; the length A B equal to 28 chains.

By construction:

Divide the base A B, equal 28 chains, into 7 equal parts; from the angle C draw a line to the fourth division as C *a*, and from C to the sixth division as C *b*, dividing the triangle as required.

Arithmetically:

Rule. As the sum of the ratios is to the length of the base A B, so is each respective part to the length required. As:

7 : 28 :: 4 = 16 chains = A *a*
7 : 28 :: 2 = 8 " = *a b*
7 : 28 :: 1 = 4 " = *b* B

To find the quantity contained in each. As:

Ratios.	Acres.			Acres.	
7 :	26.500	::	4 :	15.14286	= A C *a*
7 :	26.500	::	2 :	7.57143	= *a* C *b*
7 :	26.500	::	1 :	3.78571	= *b* C B
				26.50000	or 26A. 2R.

Problem 42.

Fig. 5, Plate 27.

Example 6. The triangular field A B C is to be divided between two persons in the proportion of 2 to 5, having an equal right to the pond, or an occupation road to that point.

Geometrically:

Rule. Divide the side of the triangle A B into 7 equal parts; from C draw the line C E; parallel to which from *a*, the fifth point, draw the line *a* D; then draw the line D E, the division fence required.

Problem 43.

To lay this out on the ground:

From the plan, scale the length from C to D equal to 300 links, and from D to B 312 links; if correct, fix a stake at the point D.

Problem 44.

Fig. 6, Plate 27.

Example 7. The triangular field A B C is to be divided into three equal parts, reserving to each the right of water at the pond, or as an entrance from the occupation road.

Rule. Divide the line A B into three equal parts, as A *a*, *a b*, *b* B; draw the line C D; then from *a*, draw the line *a c* parallel to C D; and from *b*, draw the line *b d* parallel to C D; draw the lines *c* D and *d* D, which will be the division fences required.

To lay out the same on the ground, measure the lengths from the plan as before, and drive stakes at the points *c* and *d*.

Problem 45.

Fig. 7, Plate 27.

Example 8. To divide a triangular field A B C, containing 4A. 1R. 8P., or 4.300, into three equal parts, parallel to the side B C, the length of each side being 10 chains.

Rule. Divide the side A B into three equal parts as *a b*; then find the mean proportion between A B and A *a*; by multiplying the two together, the square root of their product will be the mean proportion required, as A *c*; then draw a line from *c* parallel to B C, and B *c* C *d* will be equal to $\frac{1}{3}$ of the triangle.

Thus: A B $= 1000 \times$ A *a* $= 666' = \sqrt{666000} = 816 =$ A *c* the mean

In the same manner divide the remaining part, A *c*, into two equal parts as at *b*; find the mean proportion between A *c* and A *b* as before; mark off the length from A to *e*; draw the line from *e* parallel to B *c*, then *c d e f* will be the second part, and the triangle A *e f* will be the third part.

Thus: A *c* $= 816 \times$ A *b* $= 408 = \sqrt{332928} = 576 =$ A *e* the mean

Problem 46.

Another method:

Example 9. *Rule.* Find the content of the whole triangle and subtract from it the quantity to be cut off. As similar triangles are in proportion to the squares of their like sides, the triangle

A c d will be to the triangle A B C, as the square of A c is to square of A B.

Then, as the whole quantity is to the square of the side A B, so is the remaining quantity to the square of A c; extract the square root, and the product will give the division required.

$$1st.\ 10.00^2 \times \overset{\frac{1}{3}ABC}{1.433} = 143.300 \div 4.300 = 33.325$$
$$\text{And } \sqrt{33.325} = 577 = A\,e$$

$$2nd.\ 10.00^2 \times \overset{\frac{2}{3}ABC}{2.866} = 286.66000 \div 4.300 = 666514$$
$$\text{And } \sqrt{666514} = 816 = A\,c$$

Note.—The same method must be repeated for any number of parts the triangle is required to be divided.

Problem 47.

Fig. 8, Plate 27.

Example 10. It is required to cut off a portion of land equal to two acres, in the direction shown by the dotted line A B.

Note.—When a piece of land has to be cut off, and the boundary very crooked, and no fixed point to measure from, to fix the stakes of the true line of division, that portion must be accurately measured, and a chain line fixed as near to the division as can be guessed, as at A B, and at each end put in temporary stakes. The survey must then be plotted, and scaled to this assumed line, from which the correct line C D can be calculated, returning then to the field to make the amendment.

When calculated, it was found to be minus 1 rood 3 poles, or .268 decimals; therefore divide the decimals .26800 by the length of A B equal to 660 links, the quotient will be 40 links nearly.

Then set off 40 links at each end of the line A B, which will be the exact division required.

Problem 48.

Fig. 9, Plate 27.

Example 11. It is required to cut off a portion of a field from a given point, A, to its opposite side, equal to 1 acre 1 rood.

Note.—Let this also be surveyed and the quantity calculated, as in the last example.

From the plan draw an assumed line as A B equal to 640 links; find the contents of the piece to the line A B equal to 1.887, or 1A. 3R. 22P., being minus the quantity required by .362 decimals, or 0A. 1R. 18P.

Then divide .36200 by 640, the length of A B, the quotient will be 56 links, the width to be added if the amended line was parallel. As the increased quantity would form a triangle, this quotient must be doubled, or 112 links for the perpendicular to a, then will the line A a be the exact division required.

To set this out on the ground:

From the plan, scale the distance B a equal to 130 links, and at the point a put in a stake. Scale, 4 chains to the inch.

Problem 49.

Fig. 10, Plate 27.

Example 12. A field containing 3A. 0R. 6P. is to be divided into four equal portions, in such manner that access to the pond of water be given to each. By the Table, No. 13, 3 acres 6 poles = 3.03525 square links.

Thus: 3.03525 ÷ 4 = .75881 = to each share

No. 1. Set off the line A F as a fence; draw B F an assumed line; calculate the quantity A B F = .59040, which is less than the quantity required by .16841; divide this quantity by the length of B F = 380 equal to 44 links, being half the perpendicular; therefore set off 88 links, which will complete the first trapezium A B *a* F.

No. 2. Draw the guess line C F, and calculate the quantity of *a* C F equal to .49099, less than the quantity required by .26782; divide this by the length C F equal to 378 = 71, the half; set off the perpendicular, 142 links, which completes the trapezium *a* C *b* F.

No. 3. Draw the guess line E F; calculate the quantity A E F equal to .56121 less than the quantity required by .19760; which divide by E F = 350 links; the quotient 56 the half, or 112 the perpendicular to *c*, which completes the trapezium A E C F.

No. 4. The remaining trapezium is to be cast up according to the previous examples, and is found to be .75881, the quantity required.

Problem 50.

Example 13. To divide a common field amongst sundry claimants according to the value per acre of the various parts of the common, and in proportion to the sum of each proprietor's share.

Rule. Divide the yearly value of each person's estate by the value per acre; then, as the sum of all the quotients is to the quantity of the whole common, so is each particular quotient to the quantity of each particular share.

It is required to divide a common containing 500 acres amongst A B C D E, whose respective claims per annum are as follows:

	£		sh.				£	sh. dec.
A's claim,	105	at	4	per acre	A . .	$105 \div 4 =$	26.25
B's „	250	at	5	„	B . .	$250 \div 5 =$	50.00
C's „	480	at	6	„	C . .	$480 \div 6 =$	80.00
D's „	750	at	7	„	D . .	$750 \div 7 =$	107.143
E's „	880	at	8	„	E . .	$880 \div 8 =$	110.00
					Sum of the quotients .		373.393

	sh. dec.	acres.	sh. dec.	a. dec.
Then, for A . . .	373.393 :	500 ::	26.25 :	35.150
B . . .	373.393 :	500 ::	50.00 :	66.934
C . . .	373.393 :	500 ::	80.00 :	107.130
D . . .	373.393 :	500 ::	107.143 :	143.465
E . . .	373.393 :	500 ::	110.00 :	147.291
Sum of the whole common .				499.990 or 500

Another method:

Divide the yearly value of each person's estate by the value per acre, and find a factor, or common multiplier, by dividing the sum of the whole common by the sum of the quotient; then multiply each particular quotient by the common factor, will give the quantity to each share.

$$\text{Thus: } \frac{500}{373.393} = 1.339 \text{ the factor}$$

	£ sh.	sh. dec.		a. dec.	
Then A	$105 \div 4 =$	26.25	$\times 1.339 =$	35.150	A's share
B	$250 \div 5 =$	50.00	$\times 1.339 =$	66.934	B's „
C	$480 \div 6 =$	80.00	$\times 1.339 =$	107.130	C's „
D	$750 \div 7 =$	107.143	$\times 1.339 =$	143.465	D's „
E	$880 \div 8 =$	110.393	$\times 1.339 =$	147.291	E's „
Sum of the quotients		373.393		499.990	sum of the whole

Problem 51.

In dividing or allotting an extensive parish or common, the land will have various qualities or value, consequently it is impossible in all cases to put out allotments of one value; it becomes necessary on the part of the surveyor to calculate the quantities of each value; a line is therefore drawn on the plan to distinguish the changes.

An allotment will, in some cases, be made up in value by land of different qualities; the following system is then adopted:

Rule. Every portion of the common must be surveyed,

plotted, and computed, according to the several annual values that are set on them; they are then collected together to find the value of the whole common or parish.

Ascertain the annual values of each estate having rights of common.

Then, as the annual value of all the estates is to the annual value of the whole common, so is each respective estate to its annual share of such common.

Or, as the price per acre of each portion of the common is to one acre, so is each annual share to the quantity due thereon.

The quantity of each claimant being truly found, must be set out in such situations as may be considered beneficial to the property to which they belong.

Problem 52.

Fig. 11, *Plate* 27, represents the plan of a common containing 69 acres 25 poles, to be allotted in proportion to the value of the five claimants' estates.

The dotted lines on the plan divides the common into the several portions of value, which are numbered respectively, with their quantities and value per acre, in private characters,* viz.:

	£			Quantity. a. dec.	Value per acre.	Annual value. sh. dec.
A's estate,	300	per annum	No. 1	14.041	28/	393.148
B's "	200	"	2	14.253	29/	413.337
C's "	280	"	3	9.996	33/	329.868
D's "	125	"	4	9.928	32/	317.696
E's "	95	"	5	9.058	32/	289.856
			6	10.390	30/	311.700
Total	1000 = 20000 sh.		7	1.491	18/	26.838
				69.157		2082.443

	Total estate. sh.	Value of common. sh. dec.	Annual value of each.	Value of each share. sh. dec.	
As	20000 :	2082.443 ::	6000 :	624.733	A
	20000 :	2082.443 ::	4000 :	416.489	B
	20000 :	2082.443 ::	5600 :	583.084	C
	20000 :	2082.443 ::	2500 :	260.305	D
	20000 :	2082.443 ::	1900 :	197.832	E
			20000	2082.443	

* When a parish or common has to be inclosed, commissioners are appointed under the act, whose duty it is to value the land. Letters of the alphabet to denote the value, to prevent confusion with other numbers, and also for secresy, are used; in this instance the word MAYFLOWERS, consisting of ten different letters, is adopted.

A's share . sh. dec. 624.733

	Quantity.	Value.
No. 1	14.041	= 393.148
Pt. 6	7.299	= 218.785
Pt. 5	0.400	= 12.800
	21.740	= 624.733

B's share . sh. dec. 416.489

	Quantity.	Value.
Pt. 6	3.091	= 92.915
Pt. 5	8.658	= 277.056
Pt. 7	0.810	= 14.580
Pt. 4	0.998	= 31.938
	13.557	= 416.489

C's share . sh. dec. 583.084

Pt. 2	14.253	= 413.337
Pt. 4	4.581	= 146.593
Pt. 3	.330	= 10.894
Pt. 7	.681	= 12.260
	19.845	= 583.084

D's share . sh. dec. 260.305

Pt. 3	7.888	= 260.305

E's share . sh dec. 197.832

Pt. 4	4.349	= 139.158
Pt. 3	1.778	= 58.674
	6.127	= 197.832

ABSTRACT.

	a. dec.	sh. dec.
A	21.740 =	624.733
B	13.557 =	416.489
C	19.845 =	583.084
D	7.888 =	260.305
E	6.127 =	197.832
	69.157 =	2082.443

Note.—The several quantities to make up the value of each share is entered on the plan, each of which must be accurately calculated and scaled, together making the sum required. A guess line must then be drawn as near to the division as can be, and adjusted in the manner shown by the first examples.

The rate to be raised to defray the expenses amounting to 150*l.*

Rule. First find the factor by reducing the amount into shillings; divide that product by the number of acres in the whole common; the quotient will be the factor; then multiply each person's allotment by the factor.

3000 ÷ 69.157 = 43.37 (sh. dec.) the factor

	a. dec.		sh. dec.		sh. dec.
A =	21.740	×	43.37	=	942.864
B =	13.557	×	43.37	=	587.967
C =	19.845	×	43.37	=	860.778
D =	7.888	×	43.37	=	341.903
E =	6.127	×	43.37	=	265.730
	69.157				2999.242 or 3000

FORM OF COLLECTED REFERENCE FOR APPORTIONMENT.

[*Name of Parish.*]

Apportionment of Rent Charges in lieu of Tithes.

Landowners.	Occupiers.	Arable.	Grass.	One-fifth Arable.	One-eighth Grass.	Total of last Two Columns.	Money Value of Rent Charge Apportionment. C.M. 1.100.				Corn Rent Charges on Imperial Bushels, and decimal parts of an Imperial Bushel, of Wheat, Barley, and Oats.		
											Wheat, 7s. 0½d. p. bushel.	Barley, 9s. 11½d. p. bushel.	Oats, 2s. 9d. p. bushel
		s. dec.	s. dec	s. dec.	s. dec.	s. dec.	s. dec.	£	s.	d.			
Ablett, John	Ablett, John	57.375	48.100	11.475	6.013	17.488	19.237	0	19	2	.90999	1.61404	2.3232
Cottenham, James..	Linton, William ...	279.073	115.038	55.814	14.379	70.193	77.212	3	17	2	3.66369	6.94825	9.3535
French, Rev. Henry	Moor, John............	116.188	...	23.237	...	23.237	25.560	1	5	7	1.21464	2.15428	3.10101
Linton, William	Self.........................	4402.132	3286.140	880.428	410.767	1291.195	1420.908	7	1	2	67.47378	119.67720	172.2626
Ditto, as Lord of the Manor	Linton, William	...	302.152	...	37.769	37.769	41.546	2	1	6	1.95055	3.45965	4.9797
Ditto	Chapman, George, and others	...	57.960	...	7.245	7.245	7.969	0	8	0	.37982	.67368	.9697
Ditto	Doggett, James	...	12.800	...	1.600	1.600	1.760	0	1	9	.08309	.14737	.2121
	Total.........	5658.605	4757.886	1131.721	594.734	1726.455	1900.000	95	0	0	99.15439	159.88743	230.14141

1726.455) 1900.000 (1.100 the common multiplier

	£	s.	d.
Total rent charge	95	0	0
Tithe	90	0	0
Parochial charges...	5	0	0
Total...........	95	0	0

The apportionment to be delivered. See the form as given by the commissioners.

LAYING OUT LAND.

Problem 53.

To lay out a square containing a given number of acres.

Determine the side of the square that is to contain the number of acres required.

Rule. Reduce the acres, roods, and perches to square links, the square root of which will be the side of the square.

To set out a square equal to 5 acres:

$\sqrt{}$ of 5.00000 = 7.071, the side of the square nearly

Problem 54.

To lay out a parallelogram to a given quantity, having one side given.

Rule. Divide the given quantity in square links by the length of the side, the quotient will be the length of the other side. This rule applies also to a rectangle.

To set out a parallelogram whose side is 9 chains; thus:

$5.00000 \div 900 = 556$, the perpendicular nearly

Problem 55.

To lay out a parallelogram whose base shall be greater than the perpendicular in the proportion of 2, 3, 4, &c.

Note.—When the dimensions are required in feet, the given quantity must also be given in square feet.

Rule. Divide the quantity given in square links by the number of times the base is greater than the perpendicular, and extract the square root of the quotient, which will be the length of the perpendicular; multiply the product by the proportionate number of times the base is greater, will give the parallelogram required.

To set out a parallelogram equal to 5 acres, whose base is 3 times the perpendicular; thus:

$5.00000 \div 3 = 1.66666$
$\sqrt{1.66666} = 408$ links, the length of the perpendicular
$408 \times 3 = 1224$ links, the length of the base

Problem 56.

To lay out a given quantity in the form of a triangle having either the base or perpendicular given.

Rule. Divide the given area by half the base (if the base be given), or half the perpendicular be given; the quotient will be the base or perpendicular required.

To set out a triangle equal to 5 acres on a base equal to 4 chains; thus:

$5.00000 \div 200 = 2.500000$, the length of the perpendicular

Problem 57.

To lay out any given quantity of land in a circle.

Rule. Divide the area by .7854, then extract the square root of the quotient for the diameter; take half the diameter and describe a circle, which will be the quantity required.

It is required to lay out five acres in a circle:

$$5.000 \div .7854 = \sqrt{6.366183} = 2.523, \text{ the diameter}$$

Problem 58.

To lay out any given quantity in a regular polygon.

Rule. Find in the Table No. 14 the area of the polygon required to be set out, the side of which is 1.

Divide the proposed area by the number in the table; extract the square root of the quotient, and the root is the side of the polygon required.

Multiply the side thus found by the corresponding radius in the column marked (Radii of Circum.), and the product will be the radius of the circle circumscribing the polygon.

Set out a pentagon containing 5A. 3R. 16P.

$$5.85150 \div \overset{\text{tab. num.}}{1.72048} = \sqrt{3.40295} = 1.8445 = \text{the side required}$$
$$1.8445 \times .8507 = 1.56911, \text{ the radius of circle}$$

With a strong cord equal to the length of the radius describe a circle; then take the length of the side and point off the number of sides on the circle. (See Fig. 69, Plate 4.)

Problem 59.

To lay out any given quantity of land as an ellipse.

When one of the diameters are given.

Rule. Divide the given area successively by .7854 and the given diameter; the quotient will be the other diameter.

When both diameters are required to be in a given proportion or ratio.

Divide the given area successively by .7854 and the product of the terms of the ratio and the square root of the quotient, multiplied separately by the terms of the ratio will give the two diameters.

Example 1. Lay out an ellipse equal to 5 acres, the transverse diameter 8 chains.

$$\frac{5.000000}{.7854 \times 800} = 796 \text{ links, the conjugate diameter}$$

Example 2. Lay out an ellipse, equal to 5 acres, the proportion of the diameters as 9 to 5.

$$\sqrt{\frac{5.00000}{5 \times 9 \times .7854}} = 118$$

Then $118 \times 9 = 1062$, the transverse diameter
And $118 \times 5 = 590$, the conjugate diameter

To construct an ellipse, see Problem 69, Part I.

OBSERVATIONS ON INCLOSURES.

In the year 1801, and reign of George III., an act was passed called the General Inclosure Act; since which there are five acts of amendments for the inclosure of commonable and waste lands, and allotting land, in lieu of tithes, common, and cottage rights, the claims for which are various in different parishes. In all cases, every parish about to be inclosed applies to Parliament for its own act, subject, notwithstanding, to certain conditions under the general act.

The many advantages and improvements that presented themselves induced a great portion of the kingdom to embrace the great powers to be exercised by it. Farms in small quantities that were scattered about the parish in every possible direction were allotted in large quantities near to the farm homestead.

The tithes belonging to the rector and vicar, called great and small tithes, were usually taken in kind, that is, by a tithing-man collecting every tenth, or by composition. Both systems were bad, and caused constant litigation. Under this act, land was allotted equal to the value of the tithes, after deducting all roads, drains, &c.

The lord of the manor and the cottager were also allotted land equal in value to their respective claims.

The improvements that were made by the formation of new roads and drains afforded every facility for great improvements in cultivation at a less cost.

To carry this into effect, full powers were given to commissioners duly appointed, whose duty was to value the whole, and appoint a surveyor and other officers required.

The surveyor should have a sound geometrical knowledge, well acquainted with decimals, as every matter of calculation is worked by decimals to the fraction of the smallest coin, and the decimal of a pole; consequently his plan, when finished, would have a very large number of small pieces of land, of different occupiers and owners, and when the total quantities are made out, and the value of each piece brought to a total also, under their several heads, and balance with the gross amount of quality and quantity in the parish.

In the first place, a very accurate plan of the parish must be prepared, showing minutely all waste, baulks, commons, open field lands, and old inclosures. The several quantities are carefully computed and carried to separate columns in the reference, viz. the old inclosures are carried to one column, distinguishing the arable, pasture, and wood; the homestalls are carried to the grass column, &c. &c.

The open field lands, commons, baulks, and such old inclosures as are given up as allottable land, are carried to another column, having a separate set of numbers on the plan.

The old roads and water-courses passing over commons and allottable lands are carried out in quantity to allot. New roads and water-courses are deducted out of the total quantity, and entered as such in allotting.

The values are put on the plan in private characters to each respective piece, transferred to the reference, and cast out in shillings and decimals, producing the total value of the allottable part of the parish.

An abstract or allotting book is then made showing each proprietor's numbers and values, the total value of his share in shillings and decimals, for which he is to receive equal value in his allotments according to the original values.

In addition to the open field lands, the proprietors have other claims to add and receive value in land, horse, sheep, and cow commons, and common rights, which is generally noticed in their written claims at the commencement of an inclosure,

and submitted to the commissioners and their clerk; if admitted, it is entered into a book for that purpose made by the clerk, and a copy is delivered to the surveyor, who has to make out a calculation sheet, or general reference, ruled in columns under every head of the different claims, and every particular of value and quantity.

A scheme is then submitted to the proprietors, showing as near as possible the situation they are to be placed to receive their allotment or allotments, in lieu of their original holdings and rights, after proportionate deductions have been made for manorial rights, tithe, gravel-pits, drains, roads, public fencing, recreation-ground, &c.

The most difficult part of an inclosure is in giving general satisfaction to the proprietors in the situation their allotments are placed, according to the judgment of the commissioners, that being at variance with the proprietors as to value and location.

When the scheme is pretty well matured, a division book is made, according to the lines drawn on the plan, the pieces and parts of pieces of land are numbered and scaled, and the proportionate value adjusted, until each proprietor has his exact share. This is a very delicate operation to the surveyor, as the whole of the allotments must balance with the original quantity and value.

All roads and drains are set out prior to any of the allotments.

After the allotments have been staked out, and the plan to correspond, the clerk then divides them in their just proportion of copyhold and freehold, &c., which is accurately laid down on the plan by the surveyor.

The surveyor has to make out a rate according to the expense of inclosing the parish, submit it to the commissioners, on the approval of which it is then given to the clerk to collect.

Example. Open field properties and old inclosures charged with a rate to defray expenses.

	Open Fields.	Old Inclosures.	Open Field. Rate.	Old Inclosure. Rate.	Total Rate.
	shill. dec.	shill. dec.	shill. dec.	shill. dec.	shill. dec.
A .	4761.291	2341.576	6720.611	1796.282	8516.893
B .	2763.423	1381.726	3900.600	1059.958	4960.558
C .	1765.912	832.526	2492.604	638.653	3131.257
D .	17646.173	8376.947	24907.751	6426.166	31333.917
	26936.799	12932.775	38021.566	9921.059	47942.625

The rate to be raised on the above property amounts to 2397*l.* 2s. 7½d., or 47942.625 (shill. dec.); and it is supposed 23s. in the pound on the open field, and 12s. 6d. in the pound on the old inclosures, will raise the above sum. But, in case it should not, the said sums are to be increased or diminished in exact proportion, so as to raise the exact sum—viz. 2397*l.* 2s. 7½d.

First reduce the required sum into shillings and decimals. Then find factors for the open field land and the old inclosures, thus:

```
2397  2  7½          7½       As 20 : 23 :: 1 = 1.15 factor for open field
  20                  4            s.   s.
————————           ————
47942.625         48)300.625
                      sh.
                      23       As 20 : 12.5 :: 1 = .625 factor for old inclosure
                      12.5
                     ————
                     35.5 sum of the two
```

Then multiply the annual value of the open fields and the old inclosures by their relative factors, the amount of which will give the total sum according.

```
        sh.                  sh.    dec.
26936.799 × 1.15  = 30977.31885   open field
12932.755 ×  .625 =  8082.984375  old inclosure
                    ——————————————
                    39060.303225  rate raised being deficient 8882.322
47942.625
39060.303
—————————
 8882.322 less than the sum required
                                              sh.
As 39060.303225 : 35.5 :: 47942.625 = 43.5727 the required sum to be raised
As 35.5 : 43.5727 :: 23.   = 28.2302 in the pound, open field
   35.5 : 43.5727 :: 12.5 = 15.3425      do.      old inclosure
                            ———————
                            43.5727 as above
```

Then 28.2302 ÷ 20 = 1.41151 improved factor for open fields
15.3425 ÷ 20 = .767125 do. old inclosure

			Open fields.		Old inclosure.
A	4761.391	× 1.41151 =	6720.60986	2341.576 × .767125 =	1796.28149
B	2763.423		3900.59990	1381.726	1059.95656
C	1765.912		2492.60245	832.526	638.65151
D	17646.173		24907.49651	8376.947	6426.16547
			38021.30802		9921.05503
			9921.05503		
			47942.36305 the amount of rate required		

Therefore upon the same principle can any rate for a parish or county be calculated.

ON PLANE TRIGONOMETRY.

Plane trigonometry is the art of measuring and computing the sides of plane triangles, or of such whose sides are right lines.

As this work is not intended to teach the elements of mathematics, it will be sufficient to point out a few of the principles, and give the rules of plane trigonometry for those cases that occur in surveying. In most of these cases it is required to find lines or angles, whose actual admeasurement is difficult or impracticable; they are discovered by the relation they bear to other given lines or angles, a calculation being instituted for that purpose; and as the comparison of one right line with another right line is more convenient and easy than the comparison of a right line to a curve, it has been found advantageous to measure the quantities of angles, not by the arc itself, which is described on the angular point, but by certain lines described about that arc.

The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 minutes; each minute into 60 seconds; and so on.

The sine of three angles of every triangle, or two right angles, are equal to 180 degrees.

The sum of two angles in any triangle, taken from 180 degrees, leaves the third angle.

In a right angled plane triangle, if either acute angle be

taken from 90 degrees, the remainder will give the other acute angle.

When the sine of an obtuse angle is required, subtract such obtuse angle from 180 degrees, and take the sine of the remainder, or supplement.

If two sides of a triangle are equal, a line bisecting the contained angle will be perpendicular to the remaining side, and divide it equally.

Before the required side of a triangle can be found by calculation, its opposite angle must first be given, or found.

The required part of a triangle must be the last term of four proportionals, written in order under one another, whereof the three first are given or known.

In four proportional quantities, either of them may be made the last term; thus, let A B C D be proportional quantities:

As first to second, so is third to fourth, A : B :: C : D

As second to first, so is fourth to third, B : A :: D : C

As third to fourth, so is first to second, C : D :: A : B

As fourth is to third, so is second to first, D : C :: B : A

Against the three first terms of every proposition or stating must be written their respective values taken from the proper tables.

If the value of the first term be taken from the sum of the second and third, the remainder will be the value of the fourth term or thing required, because the addition and subtraction of logarithms correspond with the multiplication and division of natural numbers.

If to the complement of the first value be added the second and third values, the sum rejecting the borrowed index will be the tabular number expressing the thing required. This method is generally used when radius is not one of the proportionals.

The complement of any logarithm, sine, or tangent, in the common table, is its difference from the radius 10.000.000, or its double, 20.000.000.

The complement of an arc is what it wants of 90 degrees.

The supplement of an arc is what it wants of 180 degrees.

A sine or right sine of an arc is a line drawn from one extremity of the arc perpendicular to the diameter, as B F, or the supplement to the arc B D E. (Fig. 1, Plate 28.)

The versed sine of an arc is that part of the diameter intercepted between the arc and its sine; as A F is the versed sine of the arc A B.

The tangent of an arc is a line perpendicular to the diameter touching the circle, as A H.

A secant is a line drawn from the centre C through any point of the circumference until it intersects the tangent as at H; then C H is the secant of the arc A B; also E I is the tangent and C I the secant of the supplemental arc B D E; and this latter tangent and secant are equal to the former, but are accounted negative, as being drawn in an opposite direction to the former.

The co-sine, co-tangent, co-secant of an arc, are the sine, tangent, and secant of the complement of that arc, the co. being only a contraction of the word complement. Thus, the arcs A B and B D being the complements of each other, the sine, tangent, or secant of one of these is the co-sine, co-tangent, or co-secant of the others; so B F the sine of A B, is the co-sine of B D; and B K the sine of B D, is the co-sine of A B. In like manner, A H the tangent of A B, is the co-tangent of B D; and D L the tangent of D B, is the co-tangent of A B. Also C H the secant of A B, is the co-secant of B D; and C L the secant of B D, is the co-secant of A B.

Corollary. Hence several remarkable properties easily follow from their definitions; as—

1st. That an arc and its supplement have the same sine, tangent, and secant; but the two latter, the tangent and secant, are counted negative when the arc is greater than a quadrant, or 90 degrees.

2nd. When the arc is zero, or nothing, the sine and tangent are nothing, but the secant is the radius C A.

3rd. Of any arc A B, the versed sine A F and co-sine B K

are equal to the radius C A—the radius, tangent, and secant forming a right angled triangle C A H; so also do the radius, co-tangent, and co-secant, another right angled triangle C D L. All these right angled triangles are similar to each other.

The sine, tangent, or secant of an angle, is the sine, tangent, or secant of the arc by which the angle is measured, or of the degrees, &c., in the same arc or angle.

The method of constructing the scale of chords, sines, tangents, and secants, usually engraved on instruments for practice, is shown by Fig. 1, Plate 28, called a Trigonometrical Canon.

A trigonometrical canon is a table showing the length of the sine, tangent, and secant to every degree and minute of the quadrant, with respect to the radius which is expressed by unity, or 1, with any number of cyphers. The logarithms of these sines, tangents, and secants are all ranged in the tables; and these are most commonly used, as they perform the calculations by only addition and subtraction, instead of the multiplication and division, by the natural sines, &c., according to the nature of the logarithms.

Having given an idea of the calculations and use of sines, tangents, and secants, we now proceed to resolve the several cases of trigonometry; previous to which it will be proper to add a few preparatory notes and observations.

Note 1. There are three methods of resolving triangles—namely, geometrical construction, instrumental operation, and arithmetical computation.

In the first method, the triangle is constructed by laying down the sides by a scale of equal parts, and the angles from the scale of chords or protractor. Then measuring the unknown parts by the same scale for the lengths of the sides, and the angles by the scale of chords or protractor.

The second method, by logarithmic lines, commonly called Gunter's scales. These scales are to be perpendicularly over each other, as thus: the 10 on the line of numbers, 90 on the sines, and 45 on the tangents. (See Sector, Plate 41, Part V.)

In working proportions with these lines, attention must be paid to the terms, whether arithmetical or trigonometrical, that the first and third term may be of the same name, and the second and fourth of the same name. To work a proportion, take the extent on its proper line from the first term to the third with the compasses, and applying one point of the compasses to the second, the other applied to the right or the left, according as the fourth term is to be more or less than the second, will reach to the fourth.

In the third method, the terms must be stated according to rule; which terms consist of the given lengths of the sides, and of the sines or tangents of the given angles taken from the logarithmic tables; in which case the second and third terms are added together, and from this sum the first must be subtracted, excepting when radius is not concerned in the analogy, by taking the arithmetical complement of the first term, and adding to it the logarithms of the second and third terms, the natural number of which aggregate logarithm is the fourth term of the proposition.

Note 2. A triangle consists of six parts—viz. three sides and three angles; and in every case in trigonometry there must be given three parts to find the other three. Also of the three parts that are given, one of them must be a side, because with the same angles the sides may be greater or less in proportion.

Note 3. All cases in trigonometrical surveying are comprised in three varieties—viz.:

1st. When two angles and a side are given.

2nd. When two sides and the included angle are given.

3rd. When three sides are given.

Problem 60.

Case 1. The following proportion is to be used when two angles of a triangle and a side opposite to one of them are given to find the other side:

Rule. As the sine of the angle opposite the given side is to

the given side, so is the sine of the angle opposite the required side to the required side.

When two sides and an angle opposite to one of them are given to find another angle, use the following rule:

Rule. As the side opposite the given angle is to the sine of the given angle, so is the other given side to the sine of the required angle. (Fig. 2, Plate 28.)

Demonstration. Let A B C be the proposed triangle, having A B the greater side and B C the least; take A D equal to B C, consider it as a radius, and let fall the perpendiculars D E and C F, which will evidently be the sines of the angles A and B to the radius A D or B C. Now the triangles A D E, A C F, are equiangular, they therefore have their like sides proportional—namely, A C : C F : : A D or B C : D E; that is, the side A C is the sine of its opposite angle B, as the side B C is to the sine of its opposite angle A.

Note.—In practice: To find an angle, begin the proportion with a side opposite to a given angle; and to find a side, begin with an angle opposite to a given side.

An angle found by this rule is ambiguous or uncertain, whether it be acute or obtuse, unless it be a right angle, or unless its magnitude be such as to prevent the ambiguity, because the sine answers to two angles, which are supplements to each other, and, accordingly, the geometrical construction forms two triangles with the same parts that are given (see Prob. 63); and when there is no restriction or limitation included in the question, either of them may be taken. The number of degrees in the table answering to the sine is the acute angle; but if the angle be obtuse, subtract those degrees from 180°, and the remainder will be the obtuse angle. When a given angle is obtuse, or a right angle, there can be no ambiguity, for then neither of the other angles can be obtuse, and the geometrical construction will form one triangle.

Note.—All the three angles of a triangle are equal to 180°; therefore, if the sum of any two of them be subtracted from 180, the remainder will be the third angle.

Problem 61.

Fig. 3, *Plate* 28. In the triangle A B C; given the sides A B = 4969, B C 4580, and ∠ A 42° 40′, to find the side A C and ∠ s. B and C.

By Geometrical Construction.

Draw the line A B = 4969 from a scale of equal parts; then from the scale of chords with the radius of 60 describe the arc *a b*; from the same scale of chords take in the compasses 42° 40′; from *b* intersect the arc at *a*; draw the line A C; take in the compasses 4580 = B C; with one foot at B intersect the line at C; draw the line B C; apply the scale from A to C equal to 6756.

The angles B and C are found by the protractor.

By Gunter's Scales.

To find the ∠ C.

As B C : sin. ∠ A :: A B : sin. ∠ C

Rule. Extend the compasses from the first to the third term on the line of numbers—viz. from 4580 to 4969; with this distance place one foot of the compasses in 42° 40′ on the line of sines, the other will reach to 47° 20′, equal to the angle C.

To find the angle B.

Subtract the sum of the two angles A and B from 180° =, leaves 90° ∠ B, which is a right angled triangle.

To find the side A C.
As sin. ∠ A : B C :: sin. ∠ B : A C

Extend the compasses from 42° 40′ to 90° 0′, the angle B on the line of sines; with that extent set from 4580 will point to 6756, the side required.

Problem 62.

The Same. In all right angled triangles, when the base and lesser angle are given, to find the hypothenuse and perpendicular.

Given the side A B = 4969 and ∠ A = 42° 40′, to find the side A C.

By Logarithms.

As radius	10.000000
: the side A B 4969	3.696269
:: secant of ∠ A 42° 40′	0.133530
: the hypo. A C = 6757	= 3.829799

To find the secant, subtract the co-sine ∠ A 42° 40′ from radius = 0.133530.

To find the perpendicular B C.

As radius	10.000000
: the side A B 4969	3.696269
:: tangent ∠ A 42° 40′	9.964588
: the per. B C 4580	= 3.660857

Arithmetical complement is commonly used when radius is not the first term in the analogy.

Problem 63.

Fig 4, *Plate* 28. In the obtuse triangle A B C; given the side A B 6880, the angle A 108° 0′, the angle C 42° 00′, to find the sides B C, A C, and the ∠ B.

By Geometrical Construction.

Draw A B 6880 by the scale; at A with the chord of 60 describe an arc *a c*, and set off 108°;* draw A C and B C; with the scale measure A C equal to 5141, also B C equal to 9779.

The angle B is found as before.

As sin. ∠ C : AB :: sin. ∠ A : B C

By Logarithms.

To find the side B C.

As the sin. ∠ C 42° 00′ ar. co.	0.174489
: the side A B 6880 . . .	3.837588
:: sup. of sin. ∠ A 108° 00′ .	9.978206
: the side B C 9779 . .	= 3.990283

To find the side A C.

As sin. ∠ C 42° 00′ a. c. .	0.174489
: the side A D 6888 . . .	3.837588
:: sin. ∠ B 30° 00′ . . .	9.698970
: the side A C 5141 . .	= 3.711047

* When the angle is greater than 90°, take half the angle and set it off at twice, as from *a* to *b*, and *b* to *c*.

The Same. In the obtuse triangle A B C; given the side A B = 6880, the angle B = 30° 00′, the side A C = 5141, to find the two values of the angles A C″ B and A C B.

As the side A C = 5141 a. c. .	= 6.288953		180° 00′
: sin. ∠ B = 30° 00′ . . .	= 9.698970		41.59
:: the side A B = 6880 . . .	= 3.837588	sin. ∠ A C A =	138.01
: sin. ∠ A C″ B 41° 59′ . .	= 9.825411		

Problem. 64.

Examples for Practice.—By Logarithms.

Fig. 5, *Plate* 28. In the acute angled triangle A B C; given the base A B 675, the angle A 47° 05′, the angle B 75° 0′, to find the ∠ C and the sides B C and A C.

Ans. The ∠ C 57° 55′; the side B C 586; the side A C 773

Problem 65.

Fig. 6, *Plate* 28. This example may be considered as the commencement of an extensive survey, or one in which a lighthouse and battery, a distance at sea, are required to be accurately shown; the base line A B being the only available spot for accurate measurement, and taking the necessary observations.

The whole to be calculated by logarithms and Gunter's scale.

Given the side A B 1200, the ∠ C A B 96° 00′, ∠ D A B 41° 29′, ∠ A B D 103° 30′, and ∠ C B A 44° 4′.

Required the angles	A C D = 96° 00′	Required the sides	A C = 1300
"	A C B = 39.56	"	C D = 1663
"	B D C = 74.40	"	B D = 1385
"	B D A = 35.01	"	A D = 2034
		"	B C = 1860

Each of these examples may be repeated by changing the sides and angles.

Case 2. When two sides and the included angle are given, to find the other angles and side.

Rule. As the sum of the two given sides is to the difference of those sides, so is the tangent of half the sum of the two unknown angles to the tangent of half their difference.

Half the difference thus found, added to half their sum, gives

the greater of the two angles, which is the angle opposite the greatest side, and subtracted from the half sum, will give the less angle.

If the third side is wanted, it may be found by Case 1.

Problem 66.

Fig. 7, *Plate* 28. In the triangle A B C; given the sides A B 1200, A C 1300, and the angle A 96° 00′, to find the side B C and angles B and C.

By Logarithms.

As the sum of the sides A B + A C = 2500 ar. co.		6.602060
: their difference 100		2.000000
:: the tan. of ½ the opp. angles = 42° 00′ . . .		9.954000
: the tan. of ½ diff. . . .	2 4 . . .	= 8.556497
Their sum .	44 4 ∠ B	
Difference .	39 56 ∠ C	

The two sides and angles being known, the third side is found by Case 1.

To find the side A B.

As sin. ∠ C 39° 56′ ar. co.	0.192535
: the side A B 1200	3.079181
:: sin. ∠ A 96° 00′ supplement	9.997614
: the side B C 1860	= 3.269330

Problem 67.

Fig. 8, *Plate* 28. The exact distance between two churches is required, being intercepted by wood; given the length A B 6880, A C 9779, and the ∠ A 30° 0′; required the ∠ s. C and B, and side B C.

By Logarithms.

As the sides A B 6880 + A C 9779 = 16659 ar. co.		5.778351
: their difference 2899		3.462248
:: the tan. of ½ the opposite ∠ s.	75° 00′	0.571948
: tan. of ½ difference	33 00	= 9.812547
Their sum	108 00 ∠ B	
Difference	42 00 ∠ C	

Then, as sin. $\angle$ C 42° 00′ ar. co.	0.174489
: the side A B 6880	3.837588
:: sin. $\angle$ A 30° 00′	9.698970
: the side B C 5141	= 3.711047

Problem 68.

Case 3. When the three sides of a triangle are given, and the angles required.

Rule. Let fall a perpendicular from the greatest angle to the longest side or base, which divides it into two segments, and the whole triangle into two right angled triangles.

Then, as the base or sum of the two segments is to the sum of the other two sides, so is the difference of those sides to the difference of the segments of the base.

Half the difference of the segments thus found, added to the half base, gives the greater segment, or subtracted from the half base gives the less segment, and the whole triangle will be divided into two right angled triangles, with two sides and one angle given to each; the remaining sides and angles of which may be found by the rule to the first case.

Problem 69.

Fig. 9, *Plate* 28. In the triangle A B C; given A B 6880, B C 6756, C A 4960, to find the angles A B and C.

By Geometrical Construction.

Set off the base line A B from a scale of equal parts; take the length of each side from the scale, and strike arcs intersecting in C; then from C let fall a perpendicular.

By Gunter's Scale.

To find the difference of segments.

As A B : B C + A C :: B C − C D : diff. of segments

Extend the compasses on the line of lines from 6880 A B to

11716, the sum of the sides B C and A C; that extent will reach from 1796, the difference of the two sides, to 3058, the difference of the segments, or fourth number.

To find the angle B.

As B C : radius :: B D : the co-s. ∠ B

Extend the compasses from 6756 B C to 4960 B D, the greater segment; that extent from the radius on the sines will reach to 47° 14′ the ∠ B C D, the complement of which is 42° 46′ the ∠ B.

To find the angle B.

Extend the compasses from 4960 A C to 1911, the less segment; that extent from radius on the sines will reach to 22° 39′ ∠ A C D, the complement of which is 67° 21′ the ∠ A: the supplement of the sum of the angles A and B equal to 69° 53′, the angle A C B.

By Logarithms.

As 6880 : 11716 :: 1796 :: 3058 diff. of segments

Half diff. of segments 1529

Half the base 3440 + ½ the diff. 1529 = 4969 greater segment
Half the base 3440 − ½ the diff. 1529 = 1911 lesser segment

To find the ∠ A.

As the side A C	4960	3.695482
: radius		10.000000
:: the side A D	1911	3.281261
		13.281261
: the co-sine ∠ A 67° 21′		= 9.585779

To find the ∠ B.

As the side B C 6756	3.829690
: radius	10.000000
:: the side B D 4960	3.695482
	13.695482
: the co-sine ∠ B 42° 46′	= 9.865792

The supplement of the angles A and B equal to 69° 53′ = ∠ C

Problem 70.

Example for Practice.

In the triangle A B C; given the side A B 930, the side A C 600, and the side C D 650; required the angles.

First, by geometrical construction.

Second, by Gunter's scale.

Third, by logarithmic calculation.

Problem 71.

Miscellaneous Questions for Practice.

Fig. 10, *Plate* 28. It is proposed to throw a bridge across the river; what is the exact distance from C to D?

Given the line A B 1017, the ∠ A 41° 29′, the ∠ B 35° 01′, to find the line C D.

To find the ∠ C.

∠ B 35° 01′ + ∠ A 41° 29′ = 76° 30′ − 180° 00′ = 103° 30′ ∠ C

To find the side A C.		To find the side B C.	
As sup. sin. ∠ C 76° 30′ .	9.987832	As sup. sin. ∠ C 76° 30′ .	9.987832
: side A B 1017	3.007321	: side A B 1017. . . .	3.007321
:: sin. ∠ B 35° 01′ . . .	9.758772	:: sin. ∠ A 41° 29′ . . .	9.821122
	12.766093		12.828433
: side A C 600 .	= 2.778261	: side B C 692 .	= 2.840601

Then by Case 3:

As 1017 : 1292 :: 92 : 116.877 diff. of segments

58.438 half diff.

Half the base 508.5 + 58.438 = 566.938 the greater segment

" " 508.5 − 58.438 = 450.062 the lesser segment

Then by Case 1:

As radius	10.000000
: the side B D 566.938 . .	2.753582
:: tangent ∠ B 35.01 . .	9.845496
: the side C D 399. . .	= 2.599078 the perpendicular required

Problem 72.

Fig. 11, *Plate* 28. The distance is required between two points, A B, on the summit of two hills; given the angle of

declivity at A 41° 29′, and length from A to C 600; the angle of acclivity from C to B 35° 01′, and length 692.

By Case 1:

As radius	10.000000	As radius	10.000000
: the side A C 600 .	2.778151	: side C B 692 . . .	2.840106
:: the co-s. ∠ A 41° 29′	9.874568	:: co-s. ∠ C 35° 01′ .	9.913276
: *a* C 450	= 2.652719	: C *b* 567	= 2.753382

Then *a* C 450 + C *b* 567 = A B 1017

Problem 73.

Fig. 12, *Plate* 28. This and the preceding figure will show the principle of obtaining not only a correct distance, but also a tolerable section of the ground from one eminence to another, and the valley between them, and if carefully performed, the heights from the base or datum line A E are ascertained with tolerable effect. Care should always be taken that the object fixed to take the angles should be of the same height as the telescope of the theodolite.

Commencing at A, the angle of acclivity is 24° 10′, and length to B 1350; from B the angle of declivity is 28° 0′, the length 1140; from C the angle of acclivity is 25° 20′, and length 1560; and so on for any distance.

Plot this to a large scale, and set off the angles by a good protractor.

Problem 74.

Fig. 13, *Plate* 28. When a station or other object falls within one of the large triangles formed by the three objects.

Let A B C represent three towers, whose distance from each other is known; to find the distance from the tower D, measure the angles A D C, B D C, A D B; plot the angles by the protractor, the point of intersection will be the point D.

Problem 75.

Fig. 14, *Plate* 28. To find the height of a building as A C.

At B measure the angle F B C; set off any distance B D as

a base; measure the angle B D C; C B D is the supplement of F B C, and B C D is the supplement of C B D + B D C.

Then as sine ∠ B C D : B D :: C D B : B C; B C being found, we have in the right angled triangle F B C, the side B C, and the angle F B C, to find F C, which is found by this proportion:

As radius is to sine of the angle F B C, so is B C : F C; F C added to A F, the height of the instrument, gives the height of the tower.

Problem 76.

Fig. 15, *Plate* 28. To take the map of a country.

First, choose two places so remote from each other that their distance may serve as a common base for the triangle to be observed, in order to form the map.

Let A B C D E F G H I K be several remarkable objects, whose situations are to be laid down in the map.

Make a rough sketch of these objects, according to their positions in regard to each other; on this sketch the different measures taken in the course of the observations are to be set down.

Measure the base A B, whose length should be proportionate to the distance of the extreme objects from A to B; from A, the extremity of the base, measure the angles E A B, F A B, G A B, C A B, D A B, formed at A with the base A B.

From B, the other extremity of the base, observe the angles E B A, F B A, G B A, C B A, D B A.

If any object cannot be seen from the points A and B another point must be found, or the base changed, so that it may be seen, it being necessary for the same object to be seen at both stations, because its positions can only be ascertained by the intersections of the lines from the ends of the base with which they form the triangles.

It is evident from what has been already said, that having the base A B given, and the angles observed, it will be easy to find the sides, and from them lay down, with a scale of equal

parts, the several triangles on the map, and thus fix with accuracy the position of the different places.

In forming maps or plans, where the chief points are at a great distance from each other, trigonometrical calculations are absolutely necessary.

But where the distance is moderate, after having measured a base and observed the angles, instead of calculating the sides, the situation of the points may be found by laying down the angles with a protractor; this method, though not so exact as the preceding, answers sufficiently for ordinary operations.

Problem 77.

Fig. 16, *Plate* 28. To ascertain the height of a building.

Measure the line F E from the foot of the building, so that the angle C D A may be neither too acute nor too obtuse; thus suppose E F = 130 feet, place the theodolite at D and measure the angle A D C = 34° 56′.

Then as radius is to the tangent 34° 56′, so is E F = 130 feet to A C; which, by working the proportion, you will find to be 89 feet 7.92 inches, to which adding 4 feet for D E, or its equal to C F, you obtain the whole height, 93 feet 7.22 inches.

The box-sextant may be applied to this with great approximation to the truth, on which should be engraved Table 18.

PART III.

LEVELLING.

PREVIOUS to the introduction of railroads, the art of levelling was confined to civil engineers, whose practice was chiefly in the formation of canals, docks, harbours, &c.

Levelling may now be considered a branch of surveying, as the surveyor is required by the agriculturist in draining marshy lands, or in improving roads, and in making sections of ground for various other purposes.

There are a variety of methods in obtaining the difference of level between two or more places to answer the purpose of drainage, &c. &c.

The bricklayers' level, the use of which is so generally known, requires no comment.

The difference of level is also performed by three laths of equal length, and is used in the following manner: First, with the assistance of the bricklayers' level, drive two pegs into the ground, and adjust them until they are level; then, at a convenient distance, drive a third peg gradually down, till the observer at the first staff can see the top of each head in a perfect line with each other; then continue this operation by removing the first staff on forward. Care must be taken to hold the laths upright.

Any inclination of ground may be found in like manner by

setting the first two pegs the number of inches required in a certain length.

The mountain barometer, a highly interesting instrument to the philosopher and traveller for determining heights.

The aneroid, another kind of barometer lately invented by M. Vedi, of Paris, much more portable than the former.

The only true method of forming an accurate section of the irregularities of the surface of ground from one distant place to another, for the construction of railroads, canals, &c., is by the spirit level.

Levelling is also performed by the theodolite, but seldom practised on engineering works.

The description of the spirit levels generally used in engineering surveying, with the method of adjustment, as also a description of the most approved level staff, is fully explained in Part V. (See Instruments.)

Levelling is the art which instructs us in finding how much higher or lower any given point on the surface of the earth is, than another given point on the same surface; or, in other words, the difference in their distance from the centre of the earth.

These points are said to be level which are equidistant from the centre of the earth. The art of levelling consists, therefore, first, in finding and marking two or more level points that shall be in the circumference of a circle whose centre is that of the earth; second, in comparing the points thus found with other points, in order to ascertain the difference in their distances from the earth's centre.

Problem 1.

Fig. 5, *Plate* 29. Let the circle be supposed to represent the earth; A the centre; and the points G D F, touching or upon the circumference, are level, because they are equally distant from the centre. Such are the surface of still water, seas, lakes, &c., tending to a natural level, or curve, similar to the earth's convexity.

Problem 2.

Fig. 6, *Plate* 29. To find how much higher the point B is than C, and C lower than D, we must find and mark the level points E F G upon the radii A B, A C, A D, thereby comparing B with E, C with F, and D with G; we shall discover how much B is nearer the circumference of the circle than C, and consequently how much farther from the centre of the earth, and so on of all the other points.

Of the different methods of marking out the level points.

The first, which is the most simple and independent, is by the tangent of a circle; the two extremes of the tangent give the true level points when the point of contact is in the middle of the line.

But if the point of contact with the circumference be at one of the extremities of the line, or in any other point except the middle, it will then only show the apparent level, as one of its extremities is farther from the circumference than the other.

Thus, the tangent B C, Fig. 5, marks out two true level points at B and C, because the point of contact D is exactly in the middle of the line B C, and its two extremities are equally distant from the circumference and the centre at A.

The tangent D C marks two points of apparent level, because D, where it touches the circumference, is not the middle of the line; and, therefore, one of its extremities D is nearer to the centre than the other at C.

C is farther from the centre in proportion as it is more distant from the point of contact D, which constitutes the difference between the true and apparent level.

Every point of the apparent level, except the point of contact, is higher than the true level.

As the tangent of a circle is perpendicular to the radius, we make use of the radius to determine the tangent, and thus mark the level points.

Let A represent the centre of the earth, A D the radius, and B D C the tangent, the two extremities B C are equally distant

from the point of contact at D, consequently the angles D B A, D C A, will be equal; the angles at the tangent point are right angles, and the radius common to both triangles; the sides A B and A C are equal, and the points B C are the two level points, because equally distant from the centre.

It is evident from this, that if from any point of the radius two lines be drawn, one on each side, making equal angles with it, and being of an equal length, the extreme points of these lines will be level points.

Thus, if from D of the radius D A, two equal lines be drawn D K, D L, making equal angles K D A and L D A, then will K and L be equally distant from the centre; though the level may be obtained by these oblique lines, yet it is far easier to obtain it by a line perpendicular to the radius.

When the level line is perpendicular to the radius, and touches it at one of its extremes, the other extremity will mark the apparent level, and the true level is found by knowing how much the apparent one exceeds it in height.

It follows clearly that the heights of the apparent level at different distances are as the squares of those distances (see Rule), and, consequently, that the difference is greater or smaller in proportion to the extent of the line, as the extremity of this line separates more from the circumference of the circle in proportion as it recedes from the point of contact.

Thus A being the centre of the earth, D F the arc that marks the true level, and D E C the tangent that marks the apparent level, or level taken by any instrument for that purpose, it is evident that the secant A C exceeds the radius A D, by C F, which is the difference between the apparent and the true level; and it is equally evident that, if the tangent extended no farther than E, this difference would not be so great as when it extended to C, and that it increases as the tangent is lengthened.

When the distance does not exceed 75 feet the difference between the two level points may be neglected, but if it exceed

300 feet or thereabouts, then the error resulting from the difference will become sensible, and require to be noticed.

In the general practice of levelling the curvature and refraction is seldom noticed, as the corrections are avoided by placing the instrument always equally distant from the two levelling staves.

Problem 3.

To find the height of the apparent above the true level for a certain distance.

Rule. Square the distance, and divide the product by the diameter of the earth, equal to 42,018,240 feet, or 7958 miles; the quotient will be the required difference.

Example 1. Required the difference between the apparent and true level, the distance B C = 5280 feet, or 1 mile.

$$5282^2 = 27878400 \div 42018240 = 0.66348 \text{ feet, or } 7.96176 \text{ inches}$$

Example 2. Required the difference when the distance is 10 chains, or 660 feet.

$$660^2 = 435600.00 \div 42018240 = 01036 \text{ feet, or } .12432 \text{ inches}$$

Another method:

$$\text{When the distance is} \left\{\begin{array}{l}\text{feet}\\ \text{yards}\\ \text{chains}\end{array}\right\} \begin{array}{l}\text{the square of}\\ \text{the distance}\\ \text{multiply by}\end{array} \left\{\begin{array}{l}.000000287\\ .000002583\\ .00125\end{array}\right\} \begin{array}{l}\text{equals the differences in inches,}\\ \text{when refraction is not taken into}\\ \text{account}\end{array}$$

Example 3. Required the difference, the distance 80 chains, or 1 mile.

$$80^2 = 6400 \times .00125 = 8.000 \text{ inches}$$

Example 4. Required the difference, the distance 1760 yards.

$$1760^2 = 309760 \times .000002583 = 8.000 \text{ inches}$$

If the distance is considerable, and refraction must be attended to, diminish the distance in respect to calculation by $\frac{1}{12}$.

Example 5. Required the correction for curvature and refraction, the distance being 20 chains.

$$\frac{20}{12} = 1.7 \text{ and } 20 - 1.7 = 18.3^2 \times .00825 = .4186 \text{ inches}$$

See Table, No. 9.

SIMPLE LEVELLING.

Problem 4.

The term simple levelling is when the level points are determined from one station, whether the level be fixed at one of the points or between them.

Required the distance between two points, A and B, Fig. 1, Plate 29, distance 5 chains.

Plant the instrument firmly in the ground midway between the two points, and adjust the level carefully according to the directions given. (See Level, Part V.) The line of sight is shown from C to D. The figure-staff is to be held up perpendicular on the points A and B, and directing the instrument to the staff at A, the height is 9.25; turn the instrument steadily round and direct it to the staff at B, the height is 6.14; then subtract the less from the greater will be equal to 3.11, or 3 feet 11 tenths.

Therefore, supposing the ground from A to B had to be made perfectly level, the ground at B E would have to be excavated 3.11, as shown by the dotted line, which may be called the datum line.

COMPOUND LEVELLING.

Problem 5.

When a line of country has to be levelled to show the true surface of the ground, called a section, it is by a repetition of the former example, and the difference between the first and last height, distinguished as the back sight and fore sight, and that difference is carried on every time the instrument is removed; the staff that was before called the back sight advances until it becomes the fore sight; the former staff remains stationary to the point, only turning it to face the instrument, which is now placed equidistant between the two stakes as before; these operations are continued from end to end.

It must be strongly impressed on the mind that these two heights regulate the whole section; the FORE sight must never

move from the point until motioned to advance, or until the instrument has been set and the sight taken. Between the BACK and FORE sights other heights may be taken, particularly in very uneven districts, called INTERMEDIATES; these heights do not in any way affect the casting or reducing the levels, which will be hereafter explained.

In taking a course of levels through a country for a long distance many changes take place in choice of the best ground, sometimes from opposition, and various other causes; it is therefore requisite to fix certain marks, as a coping, top of a particular post, or any fixed object that can by reference to the book be easily found by any other person, either to commence other levels from that point, or to take up those points when checking the former levels; these marks are called bench marks, which should be fixed at all roads for the purpose principally of taking cross sections, and many other parts where cross sections are required.

At the same time the heights are entered into the book the lengths between each staff must be entered opposite them; the distances are generally taken by a separate set of assistants having nothing to do with the levels, their duty being strictly confined to chaining, giving the distances of the crossings of fences, footways, roads, streams, &c., and are accordingly entered into the book opposite their respective heights. All remarks should be noted also before the removal of the instrument.

When a course of try levels are taken, the lengths are sometimes taken from a plan.

However careful the levels may have been taken, there is no proof against error. It becomes therefore necessary in all cases to go over the ground again; this is called check levelling, in performing which it is not requisite to take up all the former details in chaining, only the roads, streams, and bench marks are necessary. These levels must be very carefully taken, and when completed the book is cast up or reduced, and these particular points are then compared with the former levels. Where any

difference presents itself, it may be further examined by taking fresh levels from the nearest bench mark; sometimes an error may be confined, but very rarely, as if once an error occurs it is generally carried through and affects the whole section; this shows the importance of check levelling.

If the same person checks his former levels, it is better to commence where he finished. Any number of persons may also be engaged at the same time, by giving to each a certain portion between two bench marks.

In passing over streams both banks should be taken, and, where possible, the middle of the stream; the same observation also applies to roads, as frequently they are considerably above or below the adjoining ground.

In levelling through a town it is necessary to mark out distinctly the point where the line crosses from a fixed point; the lengths also will be obtained in like manner. This part is tedious, as the levels would have to be continued round from one street to another, making only an addition of heights in the book which are not required in the plotting for the section.

In the same manner, where the levels are obstructed by bogs or any other impediment, the levels must be continued until an opportunity arrives to get into the line marked out.

It is to be remembered that it is not absolutely necessary to plant the instrument on the direct line; it should be placed in the best position that can be chosen to take advantage of the lowest point of one staff and the highest of the other, that is where the ground is suddenly undulating.

When the instrument in the first instance is carefully adjusted it requires but little alteration during the day, particularly the eye-piece, but the object-glass may occasionally require to be moved by the milled screw as the distances vary; there should be only the parallel plate-screws to adjust on each move of the instrument.

A great deal of labour is saved, and time gained, in having a good staff-holder, who may be very soon drilled to hold the staff perpendicular, and to understand the motions when to move.

When commencing a course of levels, if there be no datum height given, always assume one, say 100 feet, more if the country is very hilly, so as to avoid having any of the levels below datum; when that is the case it is subject to errors in casting or reducing the levels; if any particular datum line is afterwards required it can be easily adjusted.

The datum line is the horizontal line upon which all the lengths are pointed off, and from which the heights are marked off perpendicular to it.

When the line is staked out for constructing, stakes are driven into the ground nearly close to the surface at every chain's length, and the heights taken off each stake; where a change takes place for the curves, two or three stumps are then driven to note the tangent point corresponding with the working plan and section.

The horizontal scale, or the scale to the datum line, is always the same as the plan, 3, 4, or 5 chains to the inch, according to circumstances; and the vertical scale, that is the scale used for marking the heights from the datum line to the ground surface, also varies from 20 to 100 feet to the inch. A section is never plotted with the heights to the same scale as the horizontal line; sometimes the same scale is used by taking the horizontal at 3 chains to the inch, and the vertical scale at 30 feet to the inch.

Therefore when a section is drawn it does not present the natural surface of the ground, but a deformed figure, otherwise it would not be possible to calculate the cuttings and embankments, and other engineering matters.

The leveller should have nothing to do beyond attending to his levels and the staves; a tracing of the ground to be levelled, having the line carefully drawn on it, is given to the assistants deputed to measure the lengths, and report to him from time to time as required, and to mark out the line by putting up marks at the fences as the work proceeds.

Having endeavoured to explain the several matters requiring attention in taking the longitudinal levels, it will be necessary to make a few remarks on cross or transverse levels.

These levels are chiefly, as before noticed, principally roads, both public and private, as well as where the ground is very side-laying; they are generally taken about 5 chains each side the line; on tolerably level ground they can be taken at the same time the other levels are being taken, and entered into the book the same way as the intermediates are, noting in the remarks the distance from the line on each side they are taken; but when it is hilly it will be necessary to take them distinctly, beginning from the bench mark as before noted, and taking a height in the middle of the road where the line crosses. When there are many cross sections required, it is better to take them separately. The scales used for plotting cross sections are usually larger than those for the general section. (See Plate 30.)

LEVEL BOOK.

The greatest attention is required in keeping the level book; the more simple it is constructed, the less liability will there be in making errors. There are a variety of forms, some complicated with numerous unnecessary columns.

The two forms here introduced, No. 1 and No. 2, are selected for conciseness and legibility, and most generally used in practice.

Problem 6.

Fig. 2, *Plate* 29. This example is purposely introduced to show the difficulties and liability to errors in casting out the levels arising from the datum line A C, crossing the section at D and E, thereby changing the regular system of adding and subtracting the back and fore sights, and in reducing those heights, whereby it becomes necessary to mark the changes by the algebraic symbols of plus and minus.

The surface line is shown by the line A D E B.

The strong black vertical lines denote the points where the level staves are held, which are figured with feet, tenths, and hundredths.

The spirit level is shown between them, and the dotted line represents the line of vision as taken by the telescope when the

instrument has been perfectly adjusted, and is parallel to the datum line.

This example should be carefully studied, and compared with the following example, Fig. 3, Plate 29.

When taking levels in the field, all that is entered into the book are the back, intermediate, and fore sights, with the distances at each point, where a height is taken and the remarks required.

The levels that have been taken during the day should always be cast up and proved every night before proceeding further with the levels.

The level may be taken as an intermediate, by measuring the height to the centre of the eye-glass, as is shown in this example, and noted in the book.

In all cases, when the height in the column of back sights exceeds the heights in the fore sights' column, the latter is subtracted from the former, and the difference entered in the column marked RISE, as at 7.00 marked A *a* on the section and 1.55 at *b*, leaves 5.45 rise.

And when the back sight is less than the fore sight, it is subtracted and carried to the fall column, as 0.80 at *b* and 9.20 at *d*, leaves 8.40 fall.

This is the general rule on every principle of keeping the book.

When all the heights of the back and fore sights are reduced and carried to their respective columns, then proceed to reduce them to the column called reduced levels.

The first reduced level (see Level Book, No. 1) is 5.45 plus, or above datum; the next number is 8.40 difference in the fall column, which is greater than the last reduced number 5.45, and shows at once it is below datum; consequently the order is changed, and instead of adding to 5.45, that number must be taken from 8.40, carrying the difference 2.95 to the reduced column as minus or below datum.

Now proceed to the next move of the instrument. The back sight is 1.65 as at *d*, and the fore sight 1.00 as at *e*, both of which are below datum, the difference being 0.65 rise; this

number is subtracted from the last reduced 2.95, bringing the reduced level to 2.30.

At this last point it must be remarked that, when the datum crosses the section, the heights in the rise column are subtracted and the fore column added, being quite the reverse when the datum line continues below. It is at these points of crossing where the great liability of error exists.

The instrument being again removed, the back sight is 12.00 at *e*, and the fore sight at *f* 1.85; now the line crosses again at E, between the back sight and the intermediates, the last reduced level is subtracted from 10.15 and carried to the reduced column 7.85, plus or above datum, and the preceding intermediates in the same manner, all being plus or above datum.

The next change of the instrument alters the calculation; the height in the rise column 3.40 is now added to the last reduced number 7.85; the total height B C = 11.25 above the point of commencement at A.

When all the back and fore sights are reduced, then, before reducing the intermediates, prove the whole by casting up the back and fore sights, and subtract the one from the other, as shown by the book; if the remainder is the same as at the last reduced number 11.25, the levels are correct.

The same proof can be made by casting up the columns of rise and fall.

Then proceed in like manner with the intermediates, observing the changes in crossing the datum line.

This example shows distinctly the trouble and difficulties that arise through not providing a number to prevent the datum line running through the section.

Problem 7.

Fig. 3, *Plate* 29. In the last example is shown how to reduce the levels when the datum crosses the section.

In this example the same dimensions are adopted, and an assumed number given to avoid the difficulties before described, producing the same section.

The line A B shows the surface line, and C D the datum line, on which the lengths are marked off; and from each point a perpendicular line is drawn, on which the several heights are pricked off.

All that has to be remembered is, that the heights in the rise column are to be added to the last reduced level; and the heights in the fall column are to be subtracted from the last reduced level. (See Example 2.)

Then, to prove that the two calculations have been carefully reduced, add all the back sights together, and all the fore sights together, and subtract one from the other; the difference should be the same as the last reduced level, that is 31.25; also the same proof will be obtained by adding up in like manner the rise and fall columns, and their difference will be the same, observing to take in the amount of the assumed number 20.00.

Problem 8.

Example 3 represents the portion of a line of levels for a railway, and the general method of keeping the level book, which was fully explained in the last example.

It has just been observed that, on commencing a course of levels, it is recommended to assume a number according to the nature of the country, say 100 feet.

This example is supposed to commence on a certain part of the line, at a bench mark that is above datum 112.73, that being the last reduced level, and from which all succeeding heights are to be calculated.

It will be observed that in some instances heights are taken right and left of the line, as at 12.00, &c. This is merely to show the particular position of the ground at a certain point; or a cross section may be taken in a similar manner, without interfering with the section of the main line.

Supposing these levels were to be continued over to another page, the last reduced level 71.32 will be carried over, and continued on in the same manner.

CHECK LEVELS.

Problem 9.

Example 4 is a reduction of the former method of levels. When a course of levels has been taken and the section plotted, there is no proof that it is truly correct; it is therefore requisite either to level back over the same ground and take up the several bench marks, or by running a course of check levels—that is to say, to prove chiefly the bench marks that were made in the first course of levels and the terminating point, and comparing the reduced levels of both books, which will immediately show on which point of the line a difference exists, if any, and at what part will require levelling.

Check levelling requires only the back and fore sights, and occasionally an intermediate, with all bench marks. It is not necessary to follow the line; all that is required is to start from the same bench mark and end at the same as the first levels; if a road is near the line, it will be the quickest and best levelling. Check levelling requires no plotting nor chaining.

LEVEL BOOK NO. I.

Example 1. In plus and minus. See Fig. 2, Plate 29.

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distance.	REMARKS.
0.00	...	...	...	0.00	0	
7.00	1.35	5.45	...	5.45	136	+
0.80	0.65	0.15	...	5.60	236	
	4.25	...	3.45	2.00	324	Instrument
	7.60	...	6.80	1.35	396	–
	9.20*	...	8.40	2.95	523	–
1.65	4.05	...	2.40	5.35	651	
	1.00*	0 65	...	2.30	790	
12.00	7.50	4.50	...	2.20	877	+
	4.60	7.40	...	5.10	943	
	4.60*	7.40	...	5.10	1016	Instrument
	1.85*	10.15	...	7.85	1096	
5.75	2.35	3.40	...	11.25	1204	
27.20	15.95	19.65	8.40			
15.95		8.40				
11.25		11.25				

N.B. The stars are put to distinguish the back and fore sights at the time of casting them out.

Example 2. In plus. See Fig. 3, Plate 29.

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distance.	REMARKS.
20.00	...	...	...	20.00	0	
7.00	1.55	5.45	...	25.45	136	
0.80	0.65	0.15	...	25.60	236	
	4.25	...	3.45	22.15	324	Level
	7.60	...	6.80	15.35	396	
	9.20*	...	8.40*	17.05	523	
1.65	4.05	...	2.40	14.65	651	
	1.00*	0.65	...	17.70	790	
12.00	7.50	4.50	...	22.20	877	
	4.60	7.40	...	25.10	943	
	4.60	7.40	...	25.10	1016	Level
	1.85*	10.15	...	27.85	1096	
5.75	2.35*	3.40	...	31.25	1204	
47.20	15.95	39.65	8.40			
15.95		8.40				
31.25		31.25				

Example 3. Levels from to .

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distance.	REMARKS.
				112.73	...	Mr. A.'s B. M.
8.23	8.23	...	0.00	112.73	230	Edge of ditch 2 ft. deep
	9.40	...	1.17	111.56	250	Over fence
	3.54	4.69	...	117.42	...	About 2 chains up fence
	7.86	0.37	...	113.10	700	Fence
	9.00*	...	0.77	119.96	800	Footpath
6.74	9.40	...	2.66	117.30	9.50	
	8.46	...	1.72	118.24	12.00	Up fence to brook, 2 ft. 6 in. deep
	4.26	2.48	...	122.44	12.35	Middle of lane × section
	8.92	...	2.18	117.78	12.60	Over fence
	10.43	...	3.69	116.27	...	1 chain to right
	10.27*	...	6.53	105.43	16.20	
1.93	3.65	...	1.72	103.71	20.00	Fence
	9.62	...	7.69	97.74	20.20	Over fence
	12.35*	...	10.42	95.01	23.90	On bank
3.40	6.57	...	3.17	91.89	24.10	In ditch
	4.33	...	0.93	94.08	24.20	Top of bank
	0.64	2.76	...	97.77	...	Corner of plantation—left × section
	7.38*	...	3.98	91.03	26.50	
1.24	5 96	...	4.72	86.31	32.50	
	5.89	...	4.65	85.38	...	2 chains to left
	6.52	...	5.28	85.75	34.00	Edge of ha-ha, 3 ft. deep
	10.21	...	8.97	82.06	37.00	
	12.84*	...	11.60	79.43	39.30	Fence
3.67	5.56	...	1.89	77.54	44.60	Middle of road, × section
	11.81	...	8.14	71.29	...	Edge of pond
	11.78*	...	8.11	71.32	57.00	To Mr. B.'s B. M. on lower hook of gate
25.21	66.62	112 73				
	25.21	41.41				
	41.41	71.32				

Example 4. Check levels from to .

Back. Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distance.	REMARKS.
						Line along the Bolton Road from canal bridge
				318.53	...	At A from Mr. B.'s levels
14.71	7.57	7.14	...	325.67	...	
	0.42*	14.29	...	332.82	...	B. M. on lower hook of gate
8.74	2.12*	6.62	...	339.44	...	
11.45	4.90	6.55	...	345.99	...	Opposite gate to house
	2.23*	9.22	...	348.66	...	
1.21	9.67*	...	8.46	340.20	...	On line, Duxbury branch
2.08	6.05*	...	3.97	336.23	...	
0.78	9.08*	...	8.30	327.93	...	
6.18	2.27*	3.91	...	331.84	...	Lane to left
0.35	12.75*	...	12.40	319.44	...	Opposite cottage
1.58	9.41*	...	7.83	311.61	...	B. M. on milestone
0.19	11.26*	...	11.07	300.54	...	Lane to farm-house
0.16	7.45*	...	7.29	293.25	...	
1.76	8.13*	...	6.37	286.88	...	
6.18	11.62	...	5.44	281.44	...	
	18.41*	...	12.23	274.65	...	B. M. at Mr. B.'s station
55.37	99.25					318.53
	55.37					43.88
	43.88					274.65 proof

Example 5. Cross sections.

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distance.	REMARKS.
				263.65		
0.75	3.45	...	2.70	260.95	100	
	11.20*	...	10.45	250.50	300	
0.10	10.50	...	10.40	240.10	500	Centre of line, 240.10
	15.90*	...	15.80	94.30	700	
2.20	5.60	...	3.40	90.90	600	
	8.00	...	5.80	85.10	800	
	10.00	...	7.80	77.30	900	
	12.75*	...	10.55	66.75	1000	

LEVEL BOOK NO. II.

Example 6. Levels from   to   .

Distance.	Staff.	Reduced Levels.	Plane of Collimation.	REMARKS.
chains.			105.16	B. M. from Mr. A.'s levels
0.00	0.56*	...	105.72	
5.00	7.05	98.67		
6.50	7.31	198.41	...	B. M. top of coping
7.00	15.70	90.02		
9.00	2.23*	103.49		
	15.30*	...	118.79	
9.50	11.60	107.19		
9.80	10.75	108.04	...	Fence
10.20	2.84	115.95		
11.00	2.80*	115.99		
	2.72*	...	118.71	
11.50	9.50*	109.21		
	3.55*	...	112.76	
12.00	0.61	112.15		
12.50	0.00	112.76		
13.00	7.22*	105.54	...	B. M. on stump of tree
0.00	5.70*	...	111.24	
1.50	4.92	106.32		F. S. = 27.95
2.00	1.85*	109.39		B. S. = 9.87
	0.67*	...	110.06	Cross section 18.08
2.50	10.10	99.96		Last reduced level 105.54
3.00	14.70*	95.36		
	3.50*	...	98.86	Proof 87.46
3.80	11.40*	87.46		
	15.80*	105.54	121.34	
13.50	3.38*	07.96		
	13.60*	...	131.56	
14.00	0.16	131.40		
14.50	14.67*	116.89		F. S. = 84.32
				B. S. = 71.12
	1.17*	...	118.06	Cross section
15.00	10.40	107.66		13.20
16.00	14.90*	103.16		Mr. A.'s B. M. = 105.16
	5.72*	...	108.88	Last red. level = 91.16
16.50	4.50	104.38	...	Instrument
17.00	14.60*	94.28		
	12.70*	...	106.98	
18.00	15.20	122.18	...	Off line
19.00	15.02*	91.16		

Example 7. Levels taken at Seacombe, prior to my design for the town of New Brighton, opposite Liverpool.

Distance.	Staff.	Reduced Levels.	Plane of Collimation.	REMARKS.
			138.00	B. M. corner of wall at gate
129	3.32*			
224	7.04			
326	11.43			
416	13.08*			
	0.89*			
480	4.04			
545	14.59*			
	0.17*			
790	10.55*	...	...	B. M. for cross section
	0.52*			
1050	10.86*	...	...	Fence
	1.52*			
1227	6.21			
1420	6.52*			
	1.59*			
1720	7.79	...	...	B. M. for cross section
1740	6.89	...	...	Fence
1800	13.90*			
	0.96*			
2100	10.33*			
	1.31*			
2300	6.44	...	...	B. M. for cross section
2472	6.38*			
	0.75*			
2666	9.09*			
	0.75*			
2840	13.01*	...	...	Opposite corner of wall
	0.79*			
2900	3.01			
2980	13.02*			
	0.04*			
3048	13.84*			
	5.35*			
3070	17.25*	...	...	Foot of well above beach by quarry
	3.63*			
3150	6.17*	...	...	High-water line

Example 8. First cross section from 2300′ on first line.

Distance.	Staff.	Reduced Levels.	Plane of Collimation.	REMARKS.
			62.01	
	12.21*			
240	5.74			
270	0.30*			
	16.96*			
300	0.00	...	...	Top of cop
350	2.00			
410	12.39*			
	7.61*			
500	2.00			
670	16.00	...	...	Hollow
750	7.58			
800	17.43*			
	0.00*			
857	4.46			
930	8.46			
1080	17.98*			
	0.66*			
	6.46	...	...	B. M. end of 3rd line 44.93
1120	9.55			
1230	5.42	...	...	Top of ridge
1300	16.10*			
	8.91*			
1400	12.32	...	...	Foot of brow
1515	4.50			
	11.96	...	...	B. M. corner of road
1760	13.31			
1990	10.60*			
	12.49*			
2180	4.50			
2550	15.23*	...	...	Deep hollow
	2.57*			
2845	4.71*			
	1.17*			
2950	5.78	...	...	On top of brow
	17.42*			
	6.75*			
2990	16.12*	...	...	B. M. beach on slate opposite battery

Problem 10.

See Example 5. In taking cross sections there requires no alteration to the previous advice on levelling, it is merely a transverse section of a short length that is required; and the

only matter of importance is to notice the crossing of the longitudinal section and the reduced level at that point, from which must be calculated the heights of the cross section, as shown in the margin.

Problem 11.

See Example 6. Introduces another method of keeping the level book No. 2.

As before noticed, simplicity and conciseness in the book is most required for levelling; there is but little to enter beyond the heights and distances. In this example there is one-third reduction of calculation as well as columns.

The manner of proving the calculations is the same as the former examples by the back and fore sights; it requires but little study to prove its superiority.

The first column contains the distances; that is of small importance, whether it be first or last.

The second column contains *all* heights that are taken from off the staff at one reading; that is, back, fore, and intermediate sights.

The third column contains the reduced levels from which the section is plotted.

The fourth column is the plane of collimation, commencing, as in former examples, with the fixed bench mark, and continuing as the same, or a series of parallel proved lines, to carry on the further calculations.

The fifth column is for remarks, sketching, &c.

All the staff heights are entered into one column: the first or back sight, 0.56, is added to the given reduced number, 105.16, and the amount is carried to the next column, called the plane of collimation, 105.72; then ALL the other heights are subtracted from that number; the last height, 2.23, will be the fore sight. Then draw a line at every removal of the instrument, and proceed in like manner as shown by the example.

The next removal: the back sight, 15.30, is added to the last

reduced number, 103.49, equal to 118.79, which is carried to the column of collimation; so proceed throughout.

A cross section has been purposely introduced to show that a short line may be taken at the same time.

As 7.22 was the last fore sight, the cross section proceeding from that point, the instrument is removed to a consistent distance; the assistant holding a staff on the same point, a back sight was taken at 5.70, which is added to the last reduced level 105.54, and carried to the next column, 111.24, as before; and so proceed.

We now return to the main line: 105.54, the last reduced level (that number in practice would be omitted), adding the back sight 15.80 to that number, 121.34 is carried to the plane of collimation; and so proceed.

To balance the heights and prove the last reduced number, collect all the back sights together, as 0.56, 15.80, &c.; cast them up separately.

Also the fore sights in the like manner, subtract the one from the other (as the case may be); if the product agrees with the last reduced number the book is correct.

Then proceed to cast out the intermediates taken on each remove of the instrument as in the former examples, which are all subtracted from the last number in the plane of collimation.

In all cases show the working of the balance as in the former examples.

Problem 12.

See Example 7. Is a part of a course of levels taken over an exceeding rough country; they are here introduced for practice.

PLOTTING SECTIONS.

Problem 13.

By referring to the section, Fig. 3, Plate 29, it will at once show the manner a section has to be prepared for Parliamentary purposes.

The first thing is to draw a perfect straight line, as a datum line, which is a matter of the greatest importance and sometimes of difficulty, notwithstanding its apparent simplicity. (See Plotting Scales and Straight Edge, Part V.)

The whole accuracy of a section is entirely dependent on the datum line being straight. First set off all the lengths from the level book, taken from the column of distances; then place a straight edge against the datum line, and with a set square draw in all the perpendicular lines from each length, on which prick off all the heights from the reduced levels; then draw a pencil line through each point, which will form the surface or ground line, noting particularly all roads, streams, &c. &c.; then carefully draw a fine ink line, and insert from the book all necessary remarks, as roads, streams, &c.

In like manner draw the cross sections, which are usually drawn to a larger scale. The manner of keeping the book for cross sections is shown by Examples 5 and 6.

The gradients are then drawn to equalise the cuttings and embankments, as nearly as possible forming a series of ascending and descending lines, also horizontal or level lines, all of which represent the line of rails; they are distinguished by a strong black line, the inclination being boldly written underneath each, and the heights at each end.

Tunnels, viaducts, and bridges must be very minutely described, as to the heights, the span, and mode of construction, with every other particular; also the deviation of roads, streams, &c.

Every cutting or embankment exceeding five feet must have the height inserted.

PARLIAMENTARY PLAN AND SECTION.

Problem 14.

Fig. 1, *Plate* 30, represents the portion of a plan for a proposed railway, which must always be constructed strictly to the standing orders of the House of Lords and the House of Com-

mons; a non-compliance to any of the rules therein stated prevents any further proceedings to obtain a bill for its construction during that session.

There is very little difference in the plan than what has been already explained.

The strong black line denotes the course of the proposed line of railway, on which the miles and furlongs are correctly marked to agree with the section; and the length of tunnels, which are distinguished by a dotted line.

Curves less than one mile radius must have their radius distinctly written. The two parallel dotted lines, called limits of deviation, usually about five chains each side the line of railway, are to show the full extent the powers of the bill extends to construct the railway, and any deviation in the course of the line may be made within those limits; beyond that a fresh application to Parliament is required, provided the same could not be obtained by private contract.

Fig. 2 represents a portion of the plan upon an enlarged scale, showing a proposed diversion of road, so that it might be more distinctly shown. This plan also applies to clusters of buildings belonging to, or occupied by, sundry persons; a number is required to be inserted to each, corresponding with the reference book prepared by the solicitor to the bill, as also the parish and county boundaries, the distinction of the public, turnpike, and occupation roads, the description of property, numbering fields, names of fields, occupiers, owners, leaseholders, &c. All this latter part is the solicitor's duty.

The surveyor's duty is to prepare an accurate outline of the properties, with all their details.

Particular attention must be paid to the scales, and that they correspond with the sections.

Problem 15.

Fig. 3, *Plate* 30, represents the longitudinal section to the last described plan. The datum line must always be determined from some well-defined point, and all heights are mea-

sured from this line; the mileage must also be shown to correspond with the plan.

The heights of the gradients at each end must be legibly written, and the rate of inclination; the lengths and heights of all tunnels and viaducts, the last describing the number of arches and the span of each.

All bridges over or under turnpike, public, or occupation roads, must be minutely described by their widths, heights, and approaches; as also bridges over rivers, canals, streams, &c.; the diameter of all culverts must be shown.

Strict attention generally must be given to the standing orders to secure the passing of a bill through both Houses of Parliament.

Fig. 4 are the cross sections of roads, chiefly showing how much above or below the line, and the alterations proposed to be made on the surface.

Roads that are crossed on the level must be particularly described.

The scales must also be inserted, describing them distinctly to the part they belong.

Plans and sections must not be plotted to less than four inches to the mile.

Enlarged plans not less than one quarter of an inch to every hundred feet.

The vertical scale to sections not less than one hundred feet to the inch. The scales to cross sections and the horizontal scales not less than one inch to every three hundred and thirty feet; and the vertical scale not less than one inch to every forty feet, and extend not less than two hundred yards on each side of the centre line of the railway.

And wherever the extreme height of any embankment, or the extreme depth of any cutting, shall exceed five feet, the extreme height over, or depth under, the surface of the ground shall be marked in figures upon the section.

The construction of public roads, &c., see Gradients.

WORKING SECTION.

Problem 16.

Fig. 1, *Plate* 35. A working section is a definite profile of the ground taken at the centre of the line of railway; it is usually plotted to two chains to the inch horizontal, and twenty or thirty feet to the inch vertical.

The levels that are taken to form this section are termed permanent levels; the heights are taken at every chain from the top of stumps that have been previously driven in as before described; and every minutiæ is particularly defined, such as the top of banks, depths of ditches, water-courses, roads, pits, the middle and sides of roads, &c. The method of keeping the book is precisely the same as before shown by Examples 3 and 6.

Perpendicular lines are drawn from the datum line to the surface at each chain.

Parallel to the datum line two columns are drawn: one containing the heights from the datum to the surface; the other, the heights and depths of the cuttings and embankments from the surface to the formation level—that is, the line below the surface of rails.

In each of the cuttings or embankments are written the number of cubic yards, and numbered successively, and their slopes. (See Transverse Sections, Plate 23.)

The heights of the cuttings and embankments should all be calculated from the column containing the same.

The inclination of the gradients must be distinctly written, and the heights at each end.

The datum line must be figured at each chain, furlong, and mileage, describing particularly the point or place the datum is fixed.

Tunnels, viaducts, and bridges, are all to be minutely described, with their heights, lengths, &c., as well as all culverts, whose lengths are obtained from the plan and by the cross sections.

Where the ground is undulating, numerous cross sections

must be prepared for the purpose of calculating the quantities by the rules hereafter described. (See Plates 23 and 40.)

LEVELLING WITH A THEODOLITE.

Problem 17.

Fig. 4, *Plate* 29. Levelling with a theodolite is more properly taking the angles of elevation and depression from one known point to another, or ascertaining the perpendicular, B F, of a right angled triangle, by measuring the angle, B A F, and the hypothenusal length, which has to be reduced to the base A F, and the intermediate heights are obtained, similar to the former examples, by holding a staff at every change of height at C D E, and observing by the intersection of the cross wires in the telescope the number on the staff in a direct line to B.

At every change of line, as B G, the operation is the same, only reversing the triangle, making B H the base and H B G the triangle, the line B H will be parallel to A F, and the line H G parallel to B F.

The height of the instrument from A to I, or the centre of the telescope when levelled, must be added to B F, forming the parallelogram A F I K, then I K would be the datum line from which all the heights must be calculated.

This method requires great accuracy, and the instrument very exact in all its adjustments.

Having the distance and the angle accurately measured, the difference of level is calculated, thus:

Add to the logarithm of the measured distance, the logarithmic tangent of the vertical angle, and their sum, rejecting 10 from the index, will be the logarithm of the difference of level, in feet and decimals (provided the lengths are measured in feet); if the lengths are measured by the chain and heights required in feet, add to the above logarithm the logarithm of 9.819544, which will give the number of feet and decimals.

Figs. 1 *and* 2, *Plate* 40, represent two cross sections taken by the quadrant; and in the same manner will the theodolite produce a section.

THE MOUNTAIN BAROMETER.

This instrument is used in measuring heights by the weight of the atmosphere; it was invented in 1643, by Torricelli.

There are various forms of the barometer; the one best suited for meteorological observations consists of a tube about 33 inches in length, which is inverted into a small reservoir or cistern, and, in order to maintain the mercury in the cistern always level, the cistern is constructed partly of leather, that, by means of a screw at the bottom, the surface of the mercury in it may be so adjusted as to have it always at the place from which the scale commences.

Some barometers are furnished with a gauge or float, that, in great elevations or depression, the observer may perceive when the mercury in the cistern sinks too low or rises too high.

The wheel barometer cannot be trusted to for correct heights; it merely shows if the mercury be in a rising or falling state. It may rather be considered as an ornamental piece of furniture than as having the slightest pretensions to a scientific instrument.

For a full description and use of this instrument and the aneroid, see "Manual of the Mercurial and Aneroid Barometers," by John Henry Belville, of the Royal Observatory, Greenwich. Price 1s. Published by J. and R. Taylor, Red Lion-court, Fleet-street.

THE ANEROID BAROMETER.

This instrument has lately been invented by M. Vidi, of Paris, for ascertaining the variations of the atmosphere. Its action depends on the effect produced by the pressure of the atmosphere on a metallic box, from which the air has been exhausted and then hermetically sealed.

Some later improvements have been made by Mr. Dent, watchmaker, Strand.

GRADIENTS.

Gradients are the ascending and descending lines of a railway, dividing the section, showing the natural surface of the ground nearly into equal parts; the portion on the upper side of the line is called cutting, and the lower portion embankment. The earth is removed from one to the other, forming a level surface from one fixed point to another; at this point either another gradient is formed, or it is carried on the side level to the datum line until it changes again at another fixed point.

There is great judgment required in fixing gradients, such as economy in the earthwork and bridges, the limit of ascent and descent, and the restriction in the proper height for bridges over roads, rivers, &c.

In the laying out of gradients public and turnpike roads command particúlar attention; level crossings are to be avoided as much as possible, or they must be crossed 20 feet above or below them; if then it is impracticable, the road has to be either raised or lowered to meet the level of the rail, as the case may be, and sunk to gain the depth required; if it passes under the railway, the inclination of the approaches in the turnpike raised will be 1 in 30, and other roads 1 in 20.

Problem 18.

To find the rate of inclination of a gradient between two fixed points.

Rule. Divide the distance contained between the two points by the difference of the two heights, the quotient will be the rate of inclination.

Note.—The lengths on the horizontal or datum line are usually divided into chains, therefore they must be reduced to feet, the same as the vertical height.

Fig. 1, *Plate* 35. Given the length A B = 34 chains 45 links, or 2273.70 feet, and the height from datum to A = 109.90 feet, and to B = 122.15 feet.

Thus: $122.15 - 109.90 = 12.25 \div 2273.70 = 185$, or 1 in 185

Problem 19.

To find the height at every chain from the datum to the gradient.

Rule. Divide the number of feet in a chain by the rate of inclination; the quotient must then be added to the lower given height if the gradient ascends, and subtracted from the greatest height if the gradient descends, as shown in the middle column, which, subtracted from the heights in the lower column, will give the heights and depths of the cuttings and embankments shown in the upper column.

Thus: 185 − 66000 = .356 the decimal to be added or subtracted

Problem 20.

Given the height at one end of the gradient, the rate of inclination, and the length, to find the height at the other end of the gradient.

Rule. Divide the given length by the given gradient, the product will be the difference between the two heights, which add or subtract to the given height.

Thus: 2273.70 ÷ 185 = 12.29 + 109.90 = 122.19 the height at B
Or: 2273.70 ÷ 185 = 12.29 − 122.19 = 109.90 „ at A

Problem 21.

To find the rate of inclination in 1 mile.

Rule. Divide the number of feet in a mile by the given gradient, the product will be the rate required.

Thus: 5280 ÷ 185 = 28.540 or 28½ in a mile nearly

Let the next gradient, B to C in Fig. 1, be cast through in the same manner.

LAYING OUT A LINE RAILWAY ON THE GROUND.

If the plan provided in the first instance is not truly accurate, it is indispensable that a fresh survey should be made before any other proceedings are entered into, as by it in particular the lines and curves are projected, the quantity of land and the earthwork calculated, and the longitudinal section governed.

The line being carefully drawn on the plan, the curves clearly described by their radius, and tangent points drawn, a tracing is given to the surveyor to stake out the same.

In the general observations of Part II. is described the most perfect method of ranging out a straight line, and the care required in chaining, which equally applies to this case. A stump has to be driven in the ground very truly in the line level with the surface at every chain, extending beyond the tangent points where required; at the tangent points it is usual to put a stake on each side the regular one, to distinguish it from the ordinary stakes.

The curves adopted to a line of railway are arcs of circles of different radii; some have the same arc inverted, others there are having two or more arcs combined; all of which should have their tangent points accurately defined, that being the correct point of junction. Some curves are united by a short length of straight line between them.

In laying out curves there are numerous obstacles to contend against, and they require different methods to lay them out. Under all circumstances they should be very carefully plotted to a larger scale; it will afford great assistance to the surveyor in his operations, and enable him to determine on the best method to adopt.

WOODEN CURVES.

The wooden curves are usually made to inches and half and quarter inches up to a very large radius; amongst the many, there may be none to draw the curve required, which seldom happens; when that is the case, it must be remedied by geometric construction. The following rule may be applied generally.

Problem 22.

Fig. 7, *Plate* 29. It is required to find the radius to a curve, the tangent points B C being fixed, A B and C D the straight portion of the line.

Erect perpendiculars to the lines A B and C D from the

points B and C, their intersection at O will be the centre, and O B and O C the radius.

Should the curve be required to go through a fixed point as at E, the tangent points must be extended to B′ and C′, and by the same rule, as Fig. 34, Plate 2, the centre of the arc will be found.

TO LAY OUT A RAILWAY CURVE.

Problem 23.

Fig. 1, *Plate* 31. The following method has been most extensively practised, requiring only the chain to form the curve; it requires to be executed with peculiar care, as an error committed in setting out one of the points will extend itself to every succeeding operation, and cause considerable deviation from the true curve.

The only calculation required is the first and second offset, therefore a table is unnecessary. This rule applies to all curves that are set out by this method.

Given A B and C D, portions of the main line; B and C the tangent points to be connected by the curve whose radius is 40 chains.

Rule. First reduce the radius and the distance into feet, then square the distance set off on the tangent line, and divide the quotient by twice the given radius, the product will be the first offset in feet and decimals, which multiplied by 12 will reduce it to inches and decimals. (See Table, No. 6.)

Example 1.

$$\overset{\text{ch.}}{40} \times \overset{\text{ft.}}{66} = 2640 \text{ feet, and } 2640 \times 2 = 5280$$
$$\text{Then } 66^2 = 43560 \div 5280 = .825 \times 12 = 9.900 \text{ the first offset}$$

Then twice the first offset will give the second offset, and all succeeding offsets to the curve required.

$$\text{As } .825 \times 2 = 1.650 \times 12 = 19.80 \text{ inches, the second offset}$$

Having ascertained the first and second offsets, set out one chain from B to a, in line with A B; then set off at right angles to A B the first offset a 1 equal to 9.90 inches, and fix a mark accurately on the point.

Then range out the line from B through 1 to *b* equal to one chain, and from *b* at right angles to B 1 *b* set off the second offset equal to 19.80 inches.

With the last dimension 19.80, set off from every chain produced in like manner every offset to the end of the curve.

The objection to the above method, by the numerous points hanging on to so delicate an operation, may be easily remedied, and be a perfect check on the whole, by adopting the same system upon a larger scale, taking at least four times the distance at each operation, and filling up the intermediate spaces afterwards.

In order, therefore, to arrive at perfect accuracy, it will be necessary in the first instance to calculate the lengths of the lines B F and B 3 F by the rule before given, which upon a larger scale would be considerable, whereas on the smaller scale it is but trifling, but will make considerable difference when repeated frequently.

Problem 24.

Fig. 2, *Plate* 31. To lay out a curve (with the chain only) by offsets from the tangent.

Range out the main lines A B and C D until the tangents intersect each other at E, and there fix a flag; let B and C be the two tangent points, and the radius 40 chains.

The first offset in this example is obtained by the same rule as in the former example, and all the offsets are set off one chain apart from the lines B E and C E at right angles as before; the succeeding offsets, 2, 3, 4, &c., are to be respectively the square, or 4, 9, 16 times the first offset; as, for example, the radius given is 40 chains:

						ft.	in. dec.
a =	.825 ×	1 =	0.825 ×	12 =		0	9.900
b =	.825 ×	4 =	3.300 ×	12 =		3	3.600
c =	.825 ×	9 =	7.425 ×	12 =		7	5.100
d =	.825 ×	16 =	13.200 ×	12 =		13	2.400
e =	.825 ×	25 =	20.625 ×	12 =		20	7.500
f =	.825 ×	36 =	29.700 ×	12 =		29	8.400

The offsets on the tangent from E to C are precisely the same, only reversed.

The main line should be set off from the tangent point C at every chain.

Although the last two offsets *f f* may not meet on the curve, it is of no consequence, as the points obtained by the offsets do not give the regular distance of one chain on the curve, which is subsequently done, as will hereafter be shown.

When the curve is longer than ¼ the radius the offsets become very long, and subject to great inaccuracy; when that is the case, the following example will be found more practicable.

Note.—A common cross staff—merely a piece of board with two grooves cut at right angles, fixed on a short staff—may be applied with considerable advantage in setting out long offsets.

Problem 25.

Fig. 3, *Plate* 31. Another method to lay out a curve by offsets from its tangents, the extension of the main line being obstructed by buildings, &c.

The tangents B E and C E were found to be obstructed in their intersections at E by buildings, or otherwise, too great a distance from the curve to set out the offsets within the limits, it was therefore necessary to obviate these difficulties by introducing other tangents, as F G and G H.

The radius of the curve is 40 chains, or 2640 feet, and the angle at the centre 60 degrees.

Having carefully laid down the curve on the plan and drawn the several tangents, the sides or lengths of each were proved by the following trigonometrical calculations. (Case 1.)

As radius	=	10.000000
: is to the side O B* = 2640 . . .	=	3.421604
:: the tan. of the ∠ at centre 15° 0′ .	=	9.428052
: the length of B F 707	=	2.849656

Then measure off the tangents B F and C H, and leave flags; range out the line F G H and measure the same, leaving a flag at the centre G; if the whole length measures 1414 feet, or 24

* For convenience, the radius lines are not extended to the centre.

chains 43 links, the position and measure of the tangents are correct.

The radius being the same as the former example, the offsets also will be the same.

In all cases it is most advisable to set out and prove the position of the tangents before setting out the offsets.

Problem 26.

Fig. 4, *Plate* 31. To set out a curve by offsets from a chord line. Radius 20 chains.

The obstructions on the convex side of the curve prevent the preceding examples being adopted; therefore, to obviate the difficulty, the two following examples are substituted:

Let B C, Fig. 4, be the curve to be connected to the lines A B and C D. (The tangent lines and offsets shown by dotted lines are merely introduced to show the method of obtaining the offsets from the chord line.)

In this instance, if possible, chain the tangent line out to E equal to 10 chains, and there fix a flag; at right angles to E B A set off 165 feet as at G, or 2.50 chains, and put up a flag.

The method of obtaining the tangent offsets is shown in the first example; the chord offsets are obtained by subtracting each height respectively from the greatest, or 165 feet, thus:

	Tangent offsets. ft.	in. dec.		Chord offsets. ft.	in. dec.
No. 1 =	1	7.80	–	163	4.20
2 =	6	7.20	–	158	4.80
3 =	14	10.20	–	150	1.80
4 =	26	4.80	–	138	7.20
5 =	41	3.50	–	123	8.50
No. 6 =	59	4.80	–	105	7.20
7 =	80	10.20	–	84	1.80
8 =	105	7.20	–	59	4.80
9 =	133	7.80	–	31	4.20
10 =	165	0.00	–	0	0.00

See Table, No. 7.

Then at the tangent point B, at right angles to A B, set out the first half chord offset 165 feet to the point F, and proceed on regularly up to G; when the ground will admit of it, the flag at the centre would be desirable; at G set out the angle

F G E, equal to 60 degrees; and again set off 165 feet to E, from which point measure off the half chord to C; if the flag at C cannot be seen from E, set out the angle 90 degrees (see Diagram); if required, the curve may be continued; sometimes half chord lines may be set out with less trouble than the whole chord.

Problem 27.

Fig. 5, *Plate* 31. The method adopted in this example differs only in setting out the whole chord by an angle either from the tangent point B, equal to 30 degrees, or by the angle C B A, equal to 150 degrees; the offsets are the same as before.

The calculations are made from the radius of 20 chains.

The angle between the tangent and chord is equal to half the angle at the centre in all cases.

If the curve has to be continued, plant the instrument at C, and set off the double angle, equal to 120 degrees.

Problem 28.

Fig. 6, *Plate* 32. To lay out a curve intercepted by a river, and other obstacles preventing the use of the chain, and substituting two theodolites.

This method has many advantages that is not contained in other examples, particularly on hilly or sloping ground, and where a wide stream of water has to be crossed, as in this instance.

The angles E B C and E C B, as before stated, are regulated by the two radii, and the angle at the centre, that being double the angles E B C or E C B. These angles may be divided into any number of equal parts, as B *a*, B *b*, C *d*, C *c*, &c. See the 21st proposition of the third book of Euclid, that all angles contained in the same segment of a circle are equal to one another.

The radius is equal to 20 chains, or 1320 feet.

The angles E B C and E C B are each 30 degrees, which,

for example, is divided into five equal parts, or 6 degrees each.

Plant the theodolites correctly over their respective points B and C; adjust the instruments and set the verniers to zero; then direct both telescopes to E, the point of intersection of the two tangent lines A B and C D; at this point clamp the lower plate, and bring the vernier to the first angle, equal to 6 degrees, whilst the other theodolite will fix the opposite angle at 24 degrees, and so on to the two last points. When the verniers are brought to the respective angles, as at *a*, *b*, *c*, &c., an assistant must be at those points with a pole, waiting the signals from each of the telescopes, shifting the pole until both sights have brought it to the correct point; this operation must be done to each of the points, and a stake driven down.

If it be required to set out each point one chain, or 66 feet apart, the following rule will give the first angle, then the succeeding angles are obtained by multiplying that angle by the number of times, as 1° 26′ by 2 is equal to 2° 52′, and so on.

Therefore to find the angle so that the points in the curve shall be 66 feet apart (or any other distance), as by the following rule:

Divide the distance by twice the radius, the natural sine of the quotient will be the required angle, or by logarithms, thus:

As 66 feet ÷ 2640 twice the radius equal to .025 = 1° 26′	Or, the log. of 66 feet	=	1.819544
	Radius	=	10.000000
			11.819544
	Twice radius 2640 . .	=	3.421404
	Sin. 1° 25′—56″ . .	=	8.397940

Problem 29.

Fig. 7, *Plate* 32, is similar in its operation, having only one theodolite, and the angles are calculated in the same manner.

The theodolite is planted at B, and set to the first angle; one end of the chain is held fast at B, the other end at 1 is strained out, and moved about, until by signals from the instrument;

the line of vision intersects the pole held at the end of the chain, which will be the first point; the chain is then moved on and held firm at this point, the other end at 2 waiting the signals from the instrument as before. At all these points stakes must be fixed; so proceed to C, at which point place the theodolite if the curve has to be continued.

This method has similar objections to the first example, except that by this the instrument overcomes difficulties which the chain could not.

Problem 30.

Fig. 8, *Plate* 32. To lay out a compound curve.

This example is introduced to show the method of adopting two curves of different radii, and connect them with the proper tangent points B and C.

The curve B E has a radius of 40 chains, and the angle at the centre 50 degrees; the curve E C has a radius of 80 chains, and the angle at the centre 18 degrees 20 minutes.

As on former examples, the plan should be consulted, and the angles carefully measured, so that the tangent points B E and E C may be minutely defined.

The lengths B a and a C may be correctly obtained by calculation, as shown by Fig. 3; the angle B a O is equal to 64° 30′, therefore the angle B a E is equal to 129 degrees, which may be set out by the theodolite, leaving a flag at the point E equal the length of B a.

The offsets are calculated from the tangents as in Fig. 2, and if too long an intermediate tangent should be introduced, as shown in Fig. 3.

The curve E C having a greater radius will require similar calculations, for the tangents E b and b C, as well as the offsets from them, all of which will come within the limits.

To insure accuracy and prevent extra labour, it is better to fix all the tangents first.

It frequently happens that several curves of different radii are combined, as in the following example.

The centre of every curve must always be fixed on the last radius, as at E O, Fig. 8; and supposing another curve was to join on at C, the centre of it would be on the line C F, and that portion of the line would form the first radius of that curve.

Problem 31.

Fig. 9, *Plate* 32. To lay out an inverted curve.

Curves similar to this are frequently adopted, under many peculiar circumstances, to avoid some particular property, or, in an engineering point, to avoid a tunnel, &c. &c., and are sometimes of different radii.

In this example the radii are the same, consequently one calculation answers for the whole, adopting the same rules as before given.

A B and C D show the termination of the main lines, and the respective tangents carried out at *a*, *b*, *c*, *d*. The centre of the curve E C is fixed in direct position with the line A B, the points B O and O *c* forming a perfect parallelogram, and the line O E O dividing it into two equal parts, so that the invert curve B E is the same in every respect as the curve E C; the tangents and offsets will be the same also.

Provided the offsets to the tangents B *a*, *a* E *b*, and *b c* are too long, the tangents *e f* and *g h* must be introduced, then the whole of the tangents will be the same, and require but one calculation.

By referring to the diagram, it will be seen the curve *c* C has the same radius; but its connexion with the tangent line C D having a different angle, changes the tangents at *c d* and *d* C, and would require a fresh calculation.

The preceding examples are carefully selected to meet the many difficulties or obstructions that are likely to occur in setting out railway curves. It must rest with the judgment of the surveyor as to the particular method most suitable to adopt.

Too much care cannot be exercised in the operations. It must be borne in mind, that however careful the calculations

and observations are made, the plan being a perfect level surface, some of the most easy diagrams may have the roughest ground; therefore it is expedient that the tangent lines and tangent points be fixed with minuteness, attending particularly to former instructions in chaining, and allowing the hypothenusal difference.

To find the length of a curve, see Problem 65, Part I., and Table, No. 16.

As Table No. 6 contains the first offsets to tangents already calculated, it will take but a small portion of time in the office to prepare a sufficient number of calculations for the day's operation; and although there are very many useful tables published for setting out railway curves, if the preceding examples are carefully attended to it will render those tables unnecessary.

ON SETTING OUT THE WIDTH OF LAND REQUIRED FOR A RAILWAY.

The previous observations were chiefly confined in directions to lay out the centre line of the railway; that being done, it is then necessary to level the line again very carefully, taking the heights from off each stake, which are then numbered consecutively throughout, corresponding to the mileage on the working section, which are plotted from these revised levels.

Another book is then prepared from the above, in which are inserted the heights or depths of the cuttings and embankments, and the corrected half widths, as will be hereafter explained.

When the ground is level there is no difficulty in setting out the widths from the centre line, as both sides are equal.

When the ground is higher on one side of the line than the other, the widths from the centre are then unequal, therefore in setting out the width for an embankment or cutting, not only has their slopes to be considered, but the slope of the natural ground also.

Problem 32.

Example 1, *Fig.* 1, *Plate* 33, represents the cross section of a cutting on level ground.

A B is the horizontal line at right angles with the centre line of the railway, as at C (in this case it is the natural ground); C D is the depth or height of the cutting or embankment, which is constantly varying throughout the length of the line, and is governed entirely by the gradient, or lower line, as shown in the longitudinal section, Fig. 1, Plate 35, called the formation line or level.

In addition to the width of the cutting or embankment, as at G H, there are always six feet on each side for hedge and ditch.

E F, the base or formation level, is the same number of feet wide both in cuttings and embankments; E G and F H are the slopes, which vary according to the nature of the ground, as $\frac{1}{2}$ to 1, 1 to 1, $1\frac{1}{2}$ to 1, 2 to 1, &c.; sometimes the cutting is through rock, and the side made perpendicular, allowing, in addition to the formation width, 2 or 3 feet on each side for drains. In many cases where the ground is loose there are two slopes on each side, as $1\frac{1}{2}$ to 1, and 1 to 1. (See Fig. 5.)

This example represents the most simple case that can occur. The half width of the cutting or distance from the centre C to G and H, the top of the slopes, are each equal to half the width of the base 16′ 6″, added to the width of the slope of $1\frac{1}{2}$ to 1, equal to 30 feet, and 6 feet for the fence, as thus: $16.6 + 30 + 6 = 52.6$; therefore the whole width required at a cutting of 20 feet on the level will be 105 feet. It would be precisely the same for an embankment.

When the formation level is prepared, a layer of gravel, two feet in depth (called ballast), is spread evenly for the reception of the permanent way.

Problem 33.

Fig. 2, *Plate* 33. To set out the widths of a railway cutting when the surface is sloping.

Example 2. This example is more commonly met with in practice than the former. All the dimensions and slopes are

the same, excepting the alterations in their lengths, caused by the natural surface being on the incline, as shown by the line I K passing through the centre of the cutting at C, making the slope G E shorter, and the slope F H longer; consequently the former mode of calculating the widths will not apply to this case.

The following rule will give the corrected widths of land required for the cutting or embankment only; to which must be added 6 feet each side for fences.

At some convenient point within the computed half width place the level staff, as at *i*, and measure with the tape from that point to the centre at C, along the surface of the ground; also measure from the staff at that point the length horizontally to the centre, and enter both into the book.

Then plant the level at an equal distance from the point *i* and the centre C, and observe the difference of level between C and *i*.

Multiply the computed half width *c* C by the measured length *i* C, and reserve the product; then multiply the difference of level *h i* by the ratio of the slopes, and that product subtract from the horizontal length *h* C, will give the divisor for the greater half width C H, and added to *h* C, will give the divisor for the lesser half width C G; then divide the reserved product by the respective divisors, the quotients will be the corrected half widths.

The ratio of a slope is similar to the hypothenuse of a right angled triangle, and is governed by the proportion the base is to the perpendicular: as 1 to 1 the base and perpendicular would be equal, if 1½ to 1 the base would be ½ more, if 2 to 1 the base would be double the perpendicular, and so on.

The computed ½ width C *c* = 46′.5″ × C *i* 32 = 1488 reserved product
The sloping length . C *i* = 32.0
The horizontal length C *h* = 31.5
The difference of level *h i* = 5.0 × slope 1½ = 7.5

$$\left.\begin{aligned} C\,h &= 31.5 - 7.5 = 24.0 \\ C\,h &= 31.5 + 7.5 = 38.0 \end{aligned}\right\}\text{the two divisors}$$

$$\left.\begin{aligned} \text{Then } 1488 \div 24 &= 62 \text{ the length C H} \\ \text{,, } 1488 \div 38 &= 39 \text{ the length C G} \end{aligned}\right\} = \text{the half widths}$$

The manner of plotting this section is as follows, either for a cutting or an embankment:

First draw the centre line, at right angles to which draw the line E F and set off the width of the base; from D set off the depth of the cutting, and draw the line A B; by the protractor plot the angle or angles of inclination, taken at the centre C, and draw the surface line I K, intersecting the two slopes at G H; then will G E F H form the cutting or embankment. This method by geometric construction will be found truly correct if drawn to a large scale; the half widths are then measured from it by the scale.

Or by the above calculation, having plotted the base and slopes, indefinitely, from C, with the distance C G and C H, describe arcs intersecting the slopes at G and H.

Table No. 12 will greatly assist in plotting longitudinal and cross sections.

Problem 34.

Fig. 4, *Plate* 33. To set out the widths when one part is in cutting and the other part embankment.

Example 3. The line A B shows the horizontal line as before. I K the natural surface of the ground, C D the height of the embankment, E F the base or width of the formation level 33 feet, G E *a* represents the embankment, *a* F H the cutting, the slopes $1\frac{1}{2}$ to 1 each side.

This figure frequently occurs, and will require two calculations.

First. For the length D G, apply the same rule as the last example.

The computed $\frac{1}{2}$ width $= b$ D $= 19.5 \times$ G C $38.5 = 750.75$ reserved sum
The surface length . = G C = 38.5
The horizontal length = C c = 36.5
The difference of level = c d = $13.0 \times$ slope $1\frac{1}{2} = 19.5$

Then $365 - 19.5 = 17 \div 750.75 = 44.16$ the half width G C

Second. For the length D H.

Rule. Multiply the half corrected width G C by the difference of the width of the formation level E F and the estimated half width *b* C, and divide the last product by *b* C.

Corrected half width G C = 44.16
Formation level . . E F = 33. 0 − *b* C 19.5 = 13.5
Then 44.16 × 13.5 = 596.160 ÷ 19.5 = 30.52 the half width D H

To find the length from C to *a*.

Multiply the length *d* C = 38.5 by the depth C D = 2; divide that product by *c d* = 13, the quotient will be the length required.

As 38.5 × 2 = 77.0 ÷ 13 = 5.8 the length C *a*

Note.—When the line I K passes through the point D, or centre of the formation level, the cutting and embankment are then equal to one another.

The preceding rules to calculate the correct widths are extremely useful to test any portion that may appear doubtful.

The following is the form of the book to be used in setting out the cuttings and embankments:

WIDTHS OF LAND.

No. of Stake.	Depth of Cutting, or Embankment.	Computed Half Width.	Corrected ½ widths to Edge of Cutting, or to Foot of Embankment.		Whole Width, including the Fences, Six Feet each side.	Angles of Acclivity.	Angles of Declivity.	Length from Centre to Staff.
			North.	South.		North.	South.	
			EMBANKMENT.					feet.
437	17.50	42.75	58.30	34.50	104.80	10° 00′	10° 00′	66
438	15.02	39.03	47.55	33.60	93.15	6 75	5 75	66
439	6.50	26.25	30.00	24.25	66.25	4 25	3 20	100
			CUTTINGS.					
440	1.50	18.75	18.75	18.75	49.50		Level.	
441	4.80	23.70	21.80	26 35	64.15	2 50	2 50	150
442	5.55	24.82	23.40	27.00	62.40	2 50	2 50	75
443	6.80	26.70	25.00	29.20	66.20	3 00	3 00	100

The first column contains the number of the stumps corresponding to the mileage on the plan and section; the second column contains the depths of the cuttings and the heights of embankments; the third column contains the computed half widths, calculated from the second column and the given ratio to a level surface; the fourth and fifth columns contain the corrected half widths of cuttings and embankments; the sixth column contains the whole widths of the land required, including the fences; the seventh and eighth columns contain the angles of acclivity and declivity of the ground each side of

the line; the last column contains the length in feet from the centre line to the staff, right and left, taking the same length each side.

There are many forms of books and systems used for the purpose of setting out widths. Some have as many as seven columns devoted wholly to levels: as centre, north and south, their differences, rise and fall, and differences of level multiplied by ratio.

It is quite evident that some method must be adopted to obtain the natural surface line; the level alone will not give the correct width without calculation, nor the calculation without the level, or some other method of ascertaining the point of intersection with the line of slopes and the slanting surface of the ground.

After the permanent levels have been taken and computed, and the centre line staked out and numbered, the three first columns must be copied into the last described book of widths.

The surveyor then proceeds to take angles of the inclination of the ground throughout the line, wherever the ground changes, which are entered into the columns north and south, opposite the respective number in the first column. When these angles are plotted on the cross sections, the accurate widths may be measured by the scale, and also the quantities of earthwork correctly estimated.

The theodolite is the most perfect instrument that can be used, but where angles are not required to be taken with extreme accuracy, a more portable instrument will answer the purpose, such as the improved quadrant, as described in Part V., Fig. 1, Plate 37.

Cross sections may be taken at every stump if required, the staff being held up at right angles a short distance both sides the main line, and for convenience the distances may be measured by the tape.

PART IV.

CALCULATING QUANTITIES GENERALLY FOR ESTIMATES.

LAND.

The first great item of expense in the construction of a railway is the land, which is generally valued as accommodation land, consequently it will be a much higher price per acre than would be fixed if it were a purchase of extent. Another consideration is always admitted in the value, that is, for severance, notwithstanding that provision is made to compel a company to take more land than required when reduced to a certain small quantity.

The method of ascertaining the quantity has already been described.

The land, timber, and buildings are valued to the landlord.

The tenant has claims for loss of land and other considerations, particularly if he be a leaseholder.

Both of whom have a right to demand free and uninterrupted ways to the lands divided by the railway, by the construction of occupation roads and bridges.

EARTHWORK, OTHERWISE CUTTINGS AND EMBANKMENTS.

The next great items of expense, in some instances, are very heavy, particularly in a rough country, where numerous

bridges, viaducts, and tunnels have to be erected; the latter of which is subject to many unforeseen difficulties, thereby increasing the expense, particularly where land springs or quicksands occur, although every precaution may have been taken to ascertain the true geological strata. Tunnels also considerably increase the expense, by the numerous shafts required when in deep cutting.

The quantities of cuttings and embankments are usually calculated by tables, of which there are several publications by M'Neile, Bidder, Law, &c. &c.

These tables are adapted chiefly to cases where the ground is level; the quantities obtained from them are correct; but when the ground is slanting they will not produce accurate quantities.

It has before been noticed, that when the surface is very irregular it is highly necessary to take numerous cross sections, so that at every change of ground the areas may, by mathematical rules, be obtained with an approximation to truth, and, in many instances, in much less time and trouble than by the tables.

In all rectilineal excavations, such as trenches for foundations, sewers, &c., or any other regular figure, the common rule will produce accurate quantities; as,

Multiply the length by the thickness and the product by the breadth; if the dimensions are taken in feet, to reduce it to cube yards divide the product by 27. A cube yard of earth is equivalent to a load.

This rule is only applicable to such cases where the whole of the sides are parallel to each other; when the cutting or embankment is different at each end, it affects the widths of the slopes, but not the width of the middle part, and therefore requires a different calculation to the former.

It is the opinion amongst many that, by taking the mean depths and mean widths, the correct area is obtained; where the difference is great, the error will be considerable in proportion.

In order that this subject may be useful to all capacities, particularly to those who are not studied in algebra to comprehend the rules given by formulæ, &c., the different rules are calculated arithmetically, shortened only by algebraic symbols.

See Explanation to Tables, page 212.

Problem 1.

Fig. 1, *Plate* 33. Required the content of the cutting or embankment G E F H; the depth 20 feet at both ends, the base E F 33 feet, the slopes G E and E F 1½ to 1, length 66 feet.

By the Table No. 5.

$$\left.\begin{matrix}20\\20\end{matrix}\right\}40 = 48.88 \times 33 = 1613.04 = a,\ b,\ \text{E F}$$

$$\left.\begin{matrix}1 \text{ to } 1 = 977.77\\ \tfrac{1}{2} \text{ to } 1 = 488.88\end{matrix}\right\} = \underline{1466.65} = \text{G}\,a\,\text{E and H b F}$$

$$3079.69 \text{ cube yards}$$

Note.—The same figure inverted would be an embankment.

Problem 2.

The same by the rectangle G K *b* F.

By Mensuration. (See Problem 76, Part I.)

By this, it will be observed, the slope G E forms the diagonal of the rectangle G, *a*, K E; the width of which is added to the rectangle *a*, *b*, E, F, the two being equal to the whole cutting.

$$b\text{ F } 20 \times \text{K F } 63 \times \text{length } 66 = 83160 \div 27 = 3080.7 \text{ cube yards}$$

Problem 3.

The same by the triangle G I H.

Produce the slopes G E and H F, intersecting each other at I, forming the triangle G I H; find the area by Problem 77, Part I., and multiply it by the length. In like manner find the area of the triangle E I F, and multiply by the same length, which deduct from the former; the remainder will be the content of the cutting G E F H.

Given G H = 93 feet, C I = 31 feet
„ E F = 33 „ D I = 11 „

$$93 \times \tfrac{1}{2} \text{ per. } 15.5 = 1441.5 \times 66 = 95139 \div 27 = 3522.96 = \text{G I H}$$
$$33 \times \tfrac{1}{2} \text{ per. } 5.5 = 181.5 \times 66 = 11979 \div 27 = \underline{443.66} = \text{E I F}$$

$$3079.30 \text{ cube yards}$$

Problem 4.

Or take from the table the number opposite the depth for the perpendicular, add to which one half, the sum will be the number of cubic feet.

$$\left.\begin{matrix}31\\31\end{matrix}\right\} 62 = \ 23490 + 1174.5 = 3523.50 = \text{G I H}$$
$$\left.\begin{matrix}11\\11\end{matrix}\right\} 22 = 295.77 + \ 147.89 = \ \underline{443.89} = \text{E I F}$$
$$3079.61 \text{ cube yards}$$

Note.—This rule applies only to cases where the depth of cutting is the same at both ends.

Problem 5.

To find the perpendicular D I when not given.

Divide half the base E F by the ratio of the slopes.

$\frac{1}{2}$ the base $= 16.5 \div 1\frac{1}{2} = 11$ feet

Problem 6.

The same by the wedge. (See Problem 106, Part I., and Fig. 9, Plate 7.)

Supposing the same cutting to be calculated as a wedge, deducting the triangle E I F, the remainder will be the content.

Given H G 93 feet, length from C 66 feet, length from I 66 feet, perpendicular C I 31 feet.

Length from } C 66 + 132 = 198 × C I 31 = 6138 × H G 93 = 570834 ÷ $\frac{1}{6}$ = 95139 = H G E F
And . C 66+132 = 198 × D I 11 = 2178 × E F 33 = 71874 ÷ $\frac{1}{6}$ = 11979 = E I F

Then 95139 − 11979 = 83160 ÷ 27 = 3080 cube yards

Problem 7.

Fig. 5, *Plate* 33. To find the content of the cutting F A C D B E, the two sides having different ratios of slopes, and the same depth at each end.

This cutting will require to be divided, casting each slope separately by any of the preceding rules.

Given C D = 33 feet, H I = 20 feet, and slopes A C and D B 1 to 1
" A B = 73 " I K = 15 " and slopes F A and B E $1\frac{1}{2}$ to 1

By the tables.

$\left.\begin{matrix}20\\20\end{matrix}\right\}$ 40 = 48.88 × 33 = 1613.04 + slopes 1 to 1 = 977.77 = 2590.81 = A B C D

$\left.\begin{matrix}15\\15\end{matrix}\right\}$ 30 = 36.66 × 73 = 2676.18 + „ $1\frac{1}{2}$ to 1 = 825.00 = 3501.18 = A B E F

6091.99 cube yards

For practice, calculate the rectangles F L M B and A O N D.

Problem 8.

Fig. 3, *Plate* 33. To find the content of a cutting, the slopes on one side $1\frac{1}{2}$ to 1, on the opposite side 1 to 1, base 30 feet, depth at each end 20 feet.

First calculate the triangle C G *d*, deducting the triangle D G *c*. Then the triangle F H *d*, deducting the triangle H E *c*. From the table, opposite 60, take out the number in the third column, to which add one-half the same for the small triangle D G *c*, which deduct from the triangle C G *d*, the remainder will give the content of C D *c d*; in like manner proceed with *d* H F, add the two sums together and take half.

$\left.\begin{matrix}30\\30\end{matrix}\right\}$ 60 = 2200 + 1100 = 3300.00 = C D *d* ($1\frac{1}{2}$ to 1)

$\left.\begin{matrix}10\\10\end{matrix}\right\}$ 20 = 244.44 + 122.22 = 366.66 = D G *c*

2933.34 = C D *e f*

$\left.\begin{matrix}35\\35\end{matrix}\right\}$ 70 = 2994.44 = H E *d* (1 to 1)

550.00 = H E *c*

2444.44 C D *e f*

$\left.\begin{matrix}15\\15\end{matrix}\right\}$ = 30 = 550.00 = H E *e* (1 to 1)

Then 2444.44 + 2933.34 = 5377.78 ÷ 2 = 2688.89 cube yards

Problem 9

Fig. 3. The same by the rectangle *a b f e*.

a e 55 × *e d* 20 × 66 = 72600 ÷ 27 = 2689 cube yards

Problem 10.

Fig. 1, *Plate* 33. To find the content of a cutting, the depth at one end 20 feet, the opposite end 10 feet, the base 33 feet, slopes $1\frac{1}{2}$ to 1, and length 66 feet.

By Table No. 5.

$$\left.\begin{array}{l} \left.\begin{matrix}20\\10\end{matrix}\right\} 30 = 550 \;+ 275 \;= 825.00 \\ \text{Diff. } 10 = 20.37 + 10.18 = \;\; 30.55 \end{array}\right\} = 855.55$$

$$\left.\begin{matrix}20\\10\end{matrix}\right\} 30 = 36.66 \times 33 \;.\;.\;.\;.\;. = 1209.70$$

$$\overline{2065.25}$$

Problem 11.

Fig. 4, *Plate* 33, represents an embankment and cutting.

In estimating the quantity of earthwork from the longitudinal section, this would be calculated only as a two-foot embankment, the surface line I K being intersected by the centre line at C; but when the cross section is plotted, it will show a cutting and embankment of unequal quantities.

CUTTING.

$\frac{1}{2}$ per. 1.125 × 23 = 25.775 × 66 = 1701.15 ÷ 27 = 63 cube yards

EMBANKMENT.

$\frac{1}{2}$ per. 4 × G *a* 57 = 228 × 66 = 15048 ÷ 27 = 577 cube yards

Problem 12.

Fig. 2. To find the content of the cutting G E F H, the depths at each end 20 feet, base 33 feet, length 66 feet, and slopes $1\frac{1}{2}$ to 1.

The side-laying surface I K intersects the centre line of railway at C, consequently, in taking the dimensions from the longitudinal section only, that portion of the cross section C *d* H is omitted, and C G *c* will be taken in excess; in this case, a dimension taken in the centre and calculated by the table would be erroneous, therefore measure the height F *g* and calculate G L E F by the table, and the triangle G L H by any of the previous rules.

For proof, calculate the rectangle G *f g* F, and the triangle G L H.

The triangle G I H by mensuration.

G H = 100 feet, and O I = 31 feet

100 × $\frac{1}{2}$ per. 15.5 = 15.500 × 66 = 102300.0 ÷ 27 = 3788.80
Deduct the triangle E I F = 444.00

G E F H = 3344.80

The triangle G L H by mensuration.

G H 100 × ½ per. 6.00 = 606 × 66 = 39600.0 ÷ 27 = 1466.00

The trapezium G L E F.

14.25 } 14.25 } 28.50 = 34.83 × 33	. . . =	1149.39
Slopes 1½ to 1 = 496.52 + 248.26	. =	744.78
		1894.17
Add triangle G L H	. . . .	1466.00
		3360.17 = G E F H

Problem 13.

Fig. 5. The cutting represented by A C D B E is of equal depths at both ends, the slopes A C and D B, 1 to 1, and the slope B E, 1½ to 1, the base 33 feet, and length 66 feet.

This example differs from the former, having two slopes on one side of different ratios; it requires two calculations, which may be varied by any of the preceding examples.

The trapezium A B C D. The triangle A B E.

20 } 20 } 40.00 = 48.88 × 33 = 1613.04 A E 98 × ½ per. 5.5 × 66 = 35574 ÷ 27 = 1317.5'

Slopes 1 to 1 . . . . 977.77

2590.81 = A B C D
1317.55 = A B E

3908.36 = A C D B E

Problem 14.

Fig. 6. The ground in this example falls from E to A and from A to *a*; the depth of cutting at one end is 20 feet, at the other end 10 feet; the base 33 feet, length 66 feet, and slopes 1½ to 1. Width of A E 115 feet, *a e* 79 feet, the perpendicular F G 36.5 feet, and *f g* 25.5 feet, H B 11 feet, and *h b* 8 feet.

The trapezium A B C D and *a b c d* are calculated by Table No. 5.

20 } 10 } 30 = 36.66 × 33	. . . . =	1209.78
Slopes 550 + ½ 275	. . . . . =	825.00
Difference 10 = 61.11 + ½ 30.55	. =	91.66
		2126.44 = A B C D

The triangles A E B and *a e b* are calculated by mensuration.

$115 \times 5.5 \times 66 \div 27$	. . . .	= 1546.1	= A E B
$79 \times 4.0 \times 66 \div 27$	. . . .	= 772.4	= *a e b*
		½) 2318.5	
		1159.25	
		2126.44	= A B C D
Total nearly.	.	= 3285.69	

Problem 15.

Fig. 7, *Plate* 33, represents a longitudinal section with the divisions for each separate casting; the lengths of each are figured on the line A B, representing the formation line; the heights are also figured against the vertical line, measured from the line A B to the surface line. The line C D represents a line drawn from I (longitudinally), the point of intersection of the slopes, as shown in Fig. 1, the depth of D I. When the calculations are taken by the triangle, as G I H, the triangle E I F is deducted; therefore, if the whole length of a cutting is so calculated, then one dimension need only be taken for the deduction, as C *a* D *i*, Fig. 7.

The heights shown on this section refer only to the annexed form of keeping the book for calculations taken from the Table No. 5.

Earthworks for Parliamentary estimates are calculated from the same tables, and from a similar section which represents the central line of railway, without reference to the nature of the ground, which, in many cases, is side-laying, consequently the quantities cannot be correct.

Before the operations to form a railway are commenced, a truly perfect section is made to a larger scale, as shown by Plate 35, and cross sections are taken at every chain where the ground is hilly, and by that means an accurate quantity is obtained, as shown by the last examples.

The tables will only give accurate quantities where the ground is regular; beyond that, other calculations upon mathematical principles are used.

EXPLANATION TO TABLE NO. 5.

The numbers in the first column are the sum of the two heights taken at the end of each division; the second column are the heights added together; the third column are the cube yards of the middle portion of the cutting, equal to the width of the base, taken in lengths of 66 feet and 1 foot in breadth.

Note.—The decimal points for the middle part and slopes are taken from the table annexed.

Calculations to Fig. 7.

Provide a book in the following form: first enter the several heights in the first column, and the sum of them in the second column; then look for that number in the table, against which take out the quantities in the two next columns and enter them in their respective columns in the book.

Where any difference occurs in the two heights as first taken, enter that number in the fifth column, and the difference in cube yards in the sixth column.

CUTTING No. 1.

Depths in feet.	Sums in feet.	Middle cube yards.	Slopes 1 to 1, cube yards.	Difference in feet.	Difference in cube yards.	Lengths in chains and decimals.	Middle cube yards.	Slopes 1 to 1, cube yards.	Difference, cube yards.
$\frac{0}{10}$	10.0	12.22	61.11	10	20.37	1. 0	12.22	61.11	20.37
$\frac{10}{10}$	20.0	24.44	244.44	...	...	2. 0	48.88	488.88	
$\frac{10}{20}$	30.0	36.66	550.00	10	20.37	2. 0	73.32	1100.00	40.74
$\frac{20}{28}$	48.0	58.66	1408.00	8	13.04	2.50	146.65	3520.00	32.60
$\frac{28}{34}$	62.0	75.77	2349.11	6	7.33	2.80	212.16	6577.51	20.52
$\frac{34}{28}$	62.0	75.77	2349.11	6	7.33	1.70	128.81	3993.49	12.46
$\frac{28}{16}$	44.0	53.77	1183.11	12	29.33	2. 0	107.54	2366.22	58.66
$\frac{16}{0}$	16.0	19.55	156.44	16	52.15	1.50	29.33	234.66	78.23
							758.91	18341.87	263.58
								263.58	

$$\left.\begin{array}{r} 18605.45 = 1 \text{ to } 1 \\ 9302.72 = \tfrac{1}{2} \text{ to } 1 \\ \hline 27908.17 = 1\tfrac{1}{2} \text{ to } 1 \end{array}\right\} \text{the slopes}$$

$$758.91 \times 33 = 25044.03 = \left\{ \begin{array}{l} \text{the middle} \\ \text{portion} \end{array} \right.$$

$$\text{Cubic yards} \;.\; 52952\ 20 = \left\{ \begin{array}{l} \text{the whole} \\ \text{cutting} \end{array} \right.$$

The length of each division is entered in the seventh column; the table thus far is dispensed with.

Now multiply the second, third, and fifth columns by the lengths, and enter the products in the three last columns; add them up; multiply the first total by the base, as shown; add the second and third totals together, which give the slopes of 1 to 1; if the slopes are more than 1 to 1, increase that ratio to the given ratio; bring all the totals together, the sum will be in cube yards.

TUNNELS.

Tunnels are avoided as much as possible, but when the ground rapidly increases in height, the open cutting is not continued beyond 50 feet in depth; and, although tunnelling is a very heavy and uncertain expense, it is in many instances the most economical method of continuing a line of railway. Where the tunnel is of great length, it is necessary to sink shafts into it, which answer, in the first place, for the working during the operations, and eventually as a ventilation.

The greatest possible care is required in setting out the line and taking the levels to form a correct section. In a straight tunnel there is no difficulty in setting out and proving the course of operations. It is otherwise when the tunnel is curved, being similar to that of mining.

The working shafts are generally 8 or 9 feet in diameter, and the air shafts about 3 feet in diameter.

Problem 16.

To find the quantity of earthwork, Fig. 1, Plate 34.

First. Calculate the rectilineal part by the rule. (Prob. 100, Mensuration of Solids.)

Second. The semicircular part by the rules of areas of circles and segments. (See Tables, Nos. 8 and 15.)

Required the content of a tunnel of the following dimen-

sions: Length 10 chains, width 36 feet, and 15 feet to the springing of semi-arch, diameter of arch 36 feet.

$$36^2 \times .7854 = 1017.88 \div 2 = 508.94$$
$$\text{Then } 508.94 \times 660 = 335900.40 = \text{the area of semi}$$
$$660 \times 36 \times 15 = 356400 + 335900.40 \div 27 = 25640 \text{ cube yards}$$

To find the area of the semi-arch by the table, take the number opposite $36 = 1017.8 \div 2 = 508.9 =$ the semi-arch.

Required the content of a shaft, diameter 8 feet, depth 15.

$$8 \times .7854 \times 15 = 753.98 \div 27 = 28 \text{ cube yards}$$

BRICKWORK.

Brickwork also forms another considerable item in the estimate of a railway, by the construction of tunnels, bridges, viaducts, retaining walls, culverts, drains, stations, lodges, &c.

In London and many other places, brickwork is calculated by the rod of 272¼ superficial feet (in practice the ¼ is not used).

All thicknesses of work is reduced to 1½ brick, and then it is reduced to rods by dividing the total by 272; 306 cube feet, or 11⅓ cube yards, are equal to one rod of reduced brickwork; to bring it to the standard thickness, multiply by 8 (the number of 1½ inches to a foot) and divide by 9 (the number of 1½ inches in a brick and a half, or 13½ inches).

Engineers always calculate brickwork by the cube yard. According to the London practice there are numerous items allowed in the construction of works, for which consult the Builders' Price Book.

In abstracting quantities, care should be taken to separate every distinction of work, as each will have a different price per yard, such as bricks set in mortar, and other parts in cement. Table or weather bricks, being of a different quality, should also be separated; arches in particular, whether circular, segment, or groined.

The dimensions are entered into the measuring book and squared, therefore each product will either be in square feet or cube feet; they are then abstracted into separate columns under each particular work, and their totals reduced to square yards by dividing by 9, and to cube yards by dividing by 27.

Plans, elevations, and sections must be accurately drawn, and the several dimensions figured therein, as shown by the drawings, Plate 34.

The following is the form of the dimension book: Column 1 contains the number of times any dimension is repeated; column 2, the several dimensions of each part; and column 3, the content of each part when squared. The margin is for a full description of the several parts, as thus:

Required the quantity of excavation and brickwork of the tunnel, Fig. 1.

				Abstract.		
				Walls in mortar.	Arches in mortar.	
	36.0 15.0 660.0	13200. 0	Width Height Length			
	35.0 660.0	11759.18	Diameter in arch Length	14.0 116.4	33.0 83.0	
		24959.18	Cube yards	130.4	116.0	
	7.0 2.0	14.0	Spandrells 14.0			
	1.6 -22.0	33.0	Invert arch 33.0			Cube yards. 130.4 × 600 = 86020 ÷ 27 = 3185.92 116.0 × 600 = 76560 ÷ 27 = 2835.55' Excavation = 24959.18
2	4.4 4.6	39.0	Side walls			
2	5.4 4.0	42.8	Ditto			
2	5.4 3.3	34.8	Ditto 116.4			
2	10.0 2.3	45.0	Arch (semi)			
2	9.6 2.0	38.0	Ditto, ditto 83.0			

Problem 17.

Required the content of two shafts, each 8 feet in diameter inside, 14 inches thick, and 15 feet deep.

In measuring arches the mean diameter must be taken; therefore the diameter would be 8 feet 7 inches.

$$8.58^2 \times .7854 \times 15.0 \times 2 = 1735.80 \div 27 = 64.25 \text{ cube yards}$$

Figs. 2, 3, 4, and 5, Plate 34, represent the plan, elevation, and sections of an arch or bridge; required the content.

4	12.0 6.0 3.0	864.0	Digging and filling in to foundation	4	4.6 4.0 1.3	90.0	Pilasters
2	22.0 6.0 3.0	792.0	Ditto	4	12.0 4.0 1.3	240.0	Ditto
2	4.0 5.0 3.0	120.0	Ditto	4	9.0 4.0 1.0	190.0	Ditto
4	11.0 5.0 3.0	660.0	Brick foundation, ends	2	44.0 1.6 1.6	198.0	Parapets in cement
2	21.6 5.0 3.0	645.0	Ditto, sides	2	43.6 3.0 1.2	609.0	Ditto ditto
2	4.0 4.6 3.0	108.0	Ditto, buttress	4	4.0 1.6 1.3	30.0	Ditto pilasters ditto
4	5.6 1.9 3.0	116.0	Ditto, pilasters	4	3.6 3.0 1.3	52.8	Ditto ditto ditto
4	10.0 4.0 4.0	640.0	Plinths in cement	2	22.0 4.0 4.0	704.0	Side walls, plinth in cement
4	9.6 5.0 3.6	305.0	End walls, mortar	2	22.6 7.0 3.6	1102.6	Ditto, mortar
4	9.6 7.0 3.0	798.0	Ditto ditto	2	23.0 8.6 3.0	1173.0	Ditto, with backing
4	8.0 7.0 2.6	560.0	Ditto ditto	2	4.0 4.0 3.6	112.0	Buttress
4	8.0 4.6 2.0	288.0	Ditto ditto	2	7.0 3.9 3.0	157.6	Ditto
2	9.6 9.6 2.0	181.0	Ditto spandrells	2	7.0 3.6 3.0	147.0	Ditto
2	30.6 2.0 1.0	122.0	Ditto, from walls		32.0 30.0 1.6	1440.0	Arch

(continued)

2	30.0 1.9 1.9	183.10	Deduct springing	2	44.6 1.0 1.0	89.0	Stone string course
4	9.6 1.0 1.0	38.0	Ditto string course	2	43.6 1.7 0.6	137.10	Ditto coping
2	30.0 2.0 1.9	210.0	Stone springing	4	3.6 1.2 1.0	16.4	Ditto caps
4	8.0 1.0 1.0	32.0	Ditto	2	2.9 1.3 2.0	19.2	Ditto key-stones No. 6 centres

Abstract.

Digging.	Brick mortar.	Brick cement.	Arch.	Stone.
864.0 792.0 120.0 27)1776.0 66 yards	660.0 645.0 108.0 116.0 305.0 798.0 560.0 288.0 181.0 122.0 90.0 240.0 190.0 704.0 1102.6 1173.0 112.0 157.6 147.0 7699.0 221.10 27)7477.2 277 yards	640.0 198.0 609.0 30.0 52.8 27)1529.8 37 yards	)1440.0 53 yards **Deduct brick.** 183.10 38.0 221.10	210.0 32.0 89.0 137.10 16.4 19.2 504.4

The Bill.

	£	s	d.
66 cube yards digging to foundation			
277 " brick in mortar			
37 " " in cement			
53 " arch, in mortar			
504 cubic feet masonry			
No. 6. Centres			
60 feet run of 6-inch brick drain-pipe			
60 feet run of 4-inch cast iron pipe, including 4 flanges . .			

In the construction of a railway, provision must be made for drainage and the conveyance of all water-courses, of such

dimensions as the nature of the case demands; the smaller drains may be constructed with earthenware pipes, of various dimensions.

Circular half-brick drains are called barrel drains; these are measured by the foot-run or lineal. As they increase they are then termed culverts or tunnels; these are measured as arches.

When any work is faced with a superior brick and labour, it is measured as such by the foot superficial.

Platforms and ground floors of stations are usually paved, and measured by the yard or foot superficial; they are of great variety in materials, either flat or on edge, or herrin-bone. To these may also be added 10 or 12-inch tiles and encaustic tile paving.

When piers are constructed for the support of iron or wood bridges, partly of brick and stone, the distinction must be made in the abstract.

Piers are generally formed, one part rectilineal, the other by parts of a circle; in measuring they must be taken separately.

MASON'S WORK.

All stone exceeding three inches thick is measured by the cube foot; when faced and laid in regular courses, or rusticated, should be described.

Random work, such as walling, by the cube yard.

Paving and coping are measured by the foot superficial, according to thickness.

Therefore, to get at the real value, each particular description of labour and material must have a separate column in the abstract, each forming a distinct item in the bill.

IRONWORK.

Ironwork of every description is calculated by mensuration in taking the superficial area; accurate drawings, with every requisite dimension marked on them, must be prepared, founded upon the following rules.

Notwithstanding the excellence of many tables published, the following method will in many cases give more accurate results.

The calculations by the Tables Nos. 20, 21, 22, are constructed principally upon the assumption that one cubic foot of wrought iron is equal to 480.62 lbs.

The sectional area in square inches of any tube, bar, or plate of wrought-iron, multiplied by 10, will be the weight in pounds per yard in length.

Thus: an angle iron, whose section is 3.5 square inches, weighs 35 lbs. per yard.

In large masses, or tubes, multiply the sectional area in square inches by the length in feet, and divide by 672 for the weight in tons.

To reduce cubic feet into tons, multiply by $1\frac{1}{2}$, and divide by 7.

A deduction of one-twentieth from these results will give a close approximation for cast-iron.

Five per cent. is allowed for rivets on the amount of tonnage.

The method of casting the weight of heads and nuts is given in Table No. 22.

Table 23 shows the method of calculating girders on the skew.

A superficial foot of wrought-iron one inch thick is equal to 40.32 pounds.

Table 24 shows the weight of a lineal foot of cast-iron pipes, according to its diameter and thickness.

Problem 18.

Required the weight of the cast-iron girder, Figs. 2 and 3, Plate 35, according to the dimensions therein described.

Note.—When any portion of the figure is wider at one end than the other, always take the dimension in the middle.

By Rule 1.

$$\left.\begin{array}{l} A = \overset{\text{in.}}{8} \times \overset{\text{in.}}{1.5} \times 10 = \overset{\text{lbs.}}{120} \times \\ B = 24 \times 2.5 \times 10 = 600 \times \\ C = 20 \times 2.5 \times 10 = 500 \times \end{array}\right\} = 1220 \times 8 = 9760 \text{ pounds}$$

Problem 19.

To find the weight of the angular pieces D and E; D will be twice the quantity, and E twenty times.

First find the superficial area in feet; multiply the product by the proportional thickness it bears to the number of pounds to the square foot, one inch thick, equal to 20, the remainder will be the number of pounds required.

$$\left.\begin{array}{l} D = \left\{\begin{array}{l} \overset{\text{in.}}{20} \times \overset{\text{in.}}{5} \times \overset{\text{times.}}{2} = 200 \\ 8 \times 9 \times 2 = 144 \end{array}\right\} = 344 \\ E = 4 \times 4 \times 20 \qquad\qquad = 320 \end{array}\right\} = 664 \div 144 = \overset{\text{superficial feet.}}{4.61} \times 20 = 92.20 \text{ pounds}$$

$$\begin{array}{r} 92 + 9760 = 9852 \\ \text{less 5 per cent. or } 1/20 = 492 \\ \hline 9360 = \overset{\text{tons.}}{4} \;\; \overset{\text{cwt.}}{3} \;\; \overset{\text{qrs.}}{2} \;\; \overset{\text{lbs.}}{8} \end{array}$$

Three round iron tie-rods, 1¼ inch diameter, 20 feet each, to which will be added 6¼ inches for hexagon heads and nuts, equal to 79 feet.

Problem 20.

The same by the Table No. 22.

Note.—When the dimensions exceed the number in the table, take such numbers as will make up the sum.

$$\begin{array}{l} \text{8 inches wide.} \left\{\begin{array}{l} \left.\begin{array}{l} \overset{\text{in.}}{6} = 10. \\ 2 = 3.34 \end{array}\right\} \tfrac{1}{2} \text{ inch} \\ \left.\begin{array}{l} 6 = 20. \\ 2 = 6.67 \end{array}\right\} 1 \text{ inch} \end{array}\right. \\ \begin{array}{r} \hline 40.01 = 1 \text{ foot} \\ 24 = \text{the length} \\ \hline 16004 \\ 8002 \\ \hline 960.24 = \text{A} \end{array} \end{array} \qquad \begin{array}{l} \text{24 inches wide.} \left\{\begin{array}{l} \left.\begin{array}{r} \overset{\text{in.}}{6} = 20 \times 4 = 80 \\ 80 \end{array}\right\} = 2 \text{ inches} \\ 6 = 10 \times 4 = 40 \quad = \tfrac{1}{2} \quad \text{"} \end{array}\right. \\ \begin{array}{r} \hline 200 \quad = 1 \text{ foot} \\ 24 \quad = \text{the length} \\ \hline 800 \\ 400 \\ \hline 4800 = \text{B} \end{array} \end{array}$$

$$\begin{array}{l} \text{20 inches weight.} \left\{\begin{array}{l} \left.\begin{array}{r} \overset{\text{in.}}{5} = 16.67 \times 4 = 66.68 \\ 66.68 \end{array}\right\} = 1 \text{ inch} \\ 5 = 8.32 \times 4 = 33.28 \quad = \tfrac{1}{2} \text{ inch} \end{array}\right. \\ \begin{array}{r} \hline 166.64 \quad = 1 \text{ foot} \\ 24 \quad = \text{the length} \\ \hline 66656 \\ 33328 \\ \hline 3999.36 \quad = \text{C} \end{array} \end{array}$$

The cornice as before = 92.20.

The result of the two methods proves to be exactly the same; so that by bearing in mind the number of pounds to a yard lineal of inch bar-iron and superficial foot of inch thick iron, in addition to the other short rules, the tables are nearly useless.

Note.—The drawings here introduced are merely made for the purpose of explaining the methods of casting the weight.

Problem 21.

Figs. 4 *and* 5, *Plate* 35, represent half the elevation of the front arch of a cast-iron bridge, and a section through the centre of the same; the dotted lines show the section at B.

Let the weight be calculated from the data that every yard of bar-iron 1 inch square is equal to 10 pounds, and that every superficial foot of wrought-iron 1 inch thick is equal to 40 pounds weight.

First, the bottom flange marked A is 4 inches by $1\frac{1}{2}$ inches, and 9 feet or 3 yards in length, equal to six 1-inch square bars one yard long, which multiplied by the length and then by 10 give the weight; thus:

$$6 \times 3 \times 10 = 180 \text{ pounds}$$

B, B^1, B^2, C, show the front divided into parts, to be calculated to superficial feet; multiply the product by 40, the number of pounds to the inch superficial, will give the weight of that portion, thus:

$$\left.\begin{array}{l} B\ = 2.25 \times 1.\ 5 = \ \ 3.38 \\ B^1 = 7.\ 5 \times \ .42 = \ \ 3.20 \\ B^2 = 7.\ 5 \times \ .43 = \ \ 3.20 \\ C\ = 9.\ 0 \times 2.\ 0 = 18.\ 0 \end{array}\right\} 27.78 \times 40 = 1111.20 \text{ pounds}$$

D, the coping and top of cornice, being separated for the convenience of casting, are attached to the body or face of the arch by screws; the mouldings being girt for the width, multiply by the length, the result gives superficial feet; and that product multiplied as before for $\frac{1}{2}$ inch thick, thus:

$$\left.\begin{array}{ll} \text{Coping and top cornice} & 3.17 \times 9.0 = 28.53 \\ \text{Remainder of ditto} \ \ . & 1.40 \times 9.0 = 12.60 \end{array}\right\} = 41.13 \times 20 = 822.60$$

The fillets round the panel at B, the spandrell, and at B², are cast with the body of the work C; this may be calculated by multiplying the length in lineal yards by 5, equal to half the square inch, thus:

$$\overset{\text{yds.}}{10} \times \overset{\text{lbs.}}{5} = 50 \text{ pounds}$$

The trellis-work F in the spandrells being one inch square, will also be calculated by the lineal yard, multiplied by 10, thus:

$$\left.\begin{array}{l} F = 11 \times 10 \ \ = 110 \\ G = 18 \times \ 7\tfrac{1}{2} = 135 \end{array}\right\} 245 \text{ pounds}$$

The top rail 8 inches wide, 1 inch thick, and 3 yards long, will be equal to $\overset{\text{yds.}}{9} \times 10 = 90$ pounds, thus:

$$180 + 1111.90 + 822.60 + 50 + 245 + 90 = 2499$$

$$\text{Less 5 per cent. } 2500 \div 20 = \overset{\text{lbs.}}{2375} = \text{half the arch}$$

$$2375 \times 2 = \overset{\text{lbs.}}{4750}, \text{ or } \overset{\text{tons.}}{2} \ \ \overset{\text{cwt.}}{2} \ \ \overset{\text{qr.}}{1} \ \ \overset{\text{lbs.}}{18}$$

TIMBER.

The timber that is mostly used for engineering work are balks and planks, and are calculated by the cube foot, or load of 50 feet.

Numerous other works have to be added before a line of railway is in complete working order, as ballast, drainage, retaining walls, fencing, post and rails, permanent way, level crossings, gates, &c.

Stations, lodges, warehouses, engine-houses, and other requisite buildings, are confined chiefly to the architect and builder.

ON TIMBER MEASURING.

The mensuration of timber may be considered a part of the duties of a surveyor or land agent.

The fall of timber is at different periods of the year; elm, beech, ash, &c., when the sap has fallen, oak, on the contrary, when the sap has risen, so that the tree may easily be stripped of its bark, which has been equal to or more than the value of the timber.

In measuring timber there are certain customary laws to be attended to. The merchant, or purchaser, has the power to divide the tree in lengths as he thinks best to his advantage, as a girt may be taken in the middle of a tree, and contain a greater quantity than if it were divided into two or three lengths, and the $\frac{1}{4}$ girt taken in the middle of each.

The usual method of taking the girt is by a strong piece of whipcord, which is folded into four parts and then applied to the foot rule, the measure of which gives the $\frac{1}{4}$ girt; that number, with the length, is entered into the book as one dimension.

There are tapes numbered to show the $\frac{1}{4}$ girt at once; few if any merchants will permit a tape or strap to be used, as it does not compass the tree fairly, and will mostly take a greater circumference; the string may be applied tighter, and benefit the purchaser. There is a practice between the buyer and seller, that, in girting a tree with the cord, if either should break it by over-straining, the fine is a bottle of wine for each breakage.

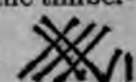
Sometimes the tanner will purchase the bark at the same measure as the timber—that is, by the load of 40 feet; more generally by the yard—that is, the bark is stripped off the tree in yard lengths, and then packed up in piles, the lengths of which are measured lineal.

The merchant, in addition to the body of the tree which is cut off below the crutch, may select two arms, only as much in length as will measure 6 inches $\frac{1}{4}$ girt.

The rest of the tree is called lop and top, which is afterwards cut up into 4 feet lengths, packed up in piles 4 feet high and 8 feet long, which is called a cord of wood, sold as firewood.

There is also another privilege allowed the purchaser; when one of the arms of the oak has a shoulder or knee, he marks off the place where it is to be cut off, and with the timber-knife or rasure cuts the letter K. This piece is used for the keel of a vessel; it is not measured as timber. A value is put on it at the time of measuring, and entered in the book accordingly.

Allowances also have to be made for bark, which cannot always be a fixed rule, as the age and quality of timber varies. The bark in some instances requires more than the usual allowance—elm, beech, ash, and lime (other trees are not considered timber). One inch is sometimes allowed out of one-fourth of the whole circumference; in very young trees not more than one-half the above. Oak, which very seldom happens to be felled with the bark on, the allowance is about $\frac{1}{10}$ or $\frac{1}{12}$ of the circumference.

Every tree is numbered with paint, or generally by the timber-knife or rasure, by lines crossing each other thus: 
every intersection counting as 10; thus, the first line crossing the three lines will read 30, the second line 60, and so on, for the small number is cut separate by the side; if 5 cut a short line on the first cross line which reads as 65, beyond cut other short lines next the 5, say 66, and so on to any extent.

In measuring standing timber, there is very little difficulty in obtaining a close approximation to the true quantity; in this case there is no allowance for bark. By the assistance of a ladder and a 10-foot rod the lengths can be easily obtained; one or two girts by the cord to the body of the tree is sufficient. A good practitioner will guess at the arms near enough for this purpose.

Where young timber will not girt 24 inches they are called saplings, and are generally divided into three classes, first, second, and third, according to their age, and are valued accordingly. Pollards are never taken as timber, and are valued separately, according to their quality.

In some instances the land steward will not allow a pollard on the estate, as they are considered to be an encouragement to the tenant to increase their number by injury to the young timber, the tenant having the privilege to the lop of all pollards.

Some gentlemen take a pride in their estate by encouraging the growth of timber, by lopping off decayed branches, and removing ivy round the trunk which much retards its growth.

In order to know the value of his timber year by year, a book is prepared expressly for the purpose, noting what trees are felled either for sale or repair, and the number planted.

It is remarked by the most eminent arborists that, for every tree fallen, three should be planted to make good for casualties in the young plants.

The following is the method of keeping the book for measuring a fall of timber:

No. of tree.	Side square in inches.	Length in feet.	Quantities in each tree.	Description of each tree.
1	20½	16	ft. in. 46 8	Elm (healthy)

The foregoing observations are intended only as an introduction; practice and custom will be the best master.

PART V.

DESCRIPTION AND USE OF INSTRUMENTS IN SURVEYING AND PLOTTING.

Fig. 1, *Plate* 36. The theodolite is a very superior instrument, and generally considered to be the best for taking vertical and horizontal angles.

The telescope A B is fitted with achromatic glasses,* in order to obtain the greatest magnifying power, near to which are two fine hairs or cobwebs, attached to a diaphragm (Fig. 3, Plate 36), as at A, at right angles to each other. The object-glass (over which is a slide-shade to prevent the sun or rain acting on the glass) is moved by the milled screw *b*, till the distant object and the cross wires appear distinct; underneath is a spirit level D attached to the telescope at one end by a joint, at the other end by a capstan-head screw for adjusting it to the line of collimation. The whole rests on two Y's, and is confined by two clips.

The vertical arc E is fixed to a long axis, sustained and movable on two supporters fixed firmly at right angles to the horizontal plate F; on the upper part of the arc is a bar to which the Y's are fixed. The arc is graduated on one side into

* Achromatic, in optics, without colours, a term first used by M. de la Laude, in his Astronomy, to denote telescopes of a new invention, contrived to remedy aberrations and colours.

Aberration, in astronomy, an apparent motion in the celestial bodies, occasioned by the progressive motion of light and the earth's annual motion in her orbit.

degrees and half-minutes, each way from 0 to 90°, and subdivided by a vernier, as at *d*; to the axis of the arc is attached a movable microscope *e*, to read the angle more distinctly. The other side of the arc is divided into the number of links to be deducted from each chain, according to the angle of inclination or depression to reduce the hypothenusal measure to the horizontal measure. (See Table, No. 12.)

On the other side of the axis, at the point *f*, is a clamp screw to fix the telescope when taking an angle, and a tangent screw, *g*, to give the telescope a gentle motion, and bring the cross wires minutely to the object observed.

The horizontal limb F consists of two plates; the upper plate contains two verniers opposite each other, as at *h* (some instruments have three verniers). Attached to this plate are two spirit levels, *i*, at right angles to each other, with capstan-head screws for adjustment; also a compass or magnetic needle, *k*, having the circumference inside the box divided into 360° and half-minutes, and when not in use a spring is attached to lift off the needle from the point. This plate revolves with the whole of the before-described parts of the instrument on the lower plate, and when required, clamped by the screw *l* to the lower plate, and as before, when taking an angle, the tangent screw *m* will give it a slow and regular motion to bring the cross wires to the objects forming the angle.

The lower plate is chamfered, and divided into 360° and half-minutes; *n* is a microscope for reading the angles more distinct, and revolves round the lower plate; I is a collar that is attached to the main axis, having a clamp screw, G, which fixes the lower plate, and the tangent screw, H, gives it a slow motion.

The parallel plates K K are held together by a long axis, with a ball fitted into a socket upon the lower plate, ground accurately to fit it, attached to a conical axis passing through the upper parallel plate.

The four screws L are for bringing the whole instrument to a perfect level, the vertical arc being first fixed at zero. Then, by

turning the screws contrary ways to each other, bring the head of the instrument to a correct position for taking the observations.

Underneath the lower parallel plate is a female screw, by which it is fixed to the tripod when required, and, when shut up, forms a round staff.

The instrument is neatly packed into a mahogany box, in which is generally fitted a plummet and line used to bring the centre of the theodolite directly over the point of the angle about to be measured; also a turnscrew and pin to alter the screws when any part of the instrument requires adjustment.

There is also an inverted lens of different focus.

The best theodolites have another telescope underneath the lower limb, which moves either vertically or horizontally, and has a clamp and tangent screw to fix it to the object, to detect any error in the observation taken by the other telescope.

Previous to commencing operations with the theodolite, examine well the state of it, and ascertain if it is correct in all its adjustments; if not, the following methods must be adopted:

The first adjustment is of the telescope for collimation* and parallax. Set the instrument up firm, move the eye and object glasses by the screw *b*, till you can see clearly the cross wires and the objects at a distance; then the adjustment for parallax will be perfect.

Then direct the telescope to some sharp object that can be distinctly seen, such as the angle of a building, &c.; fix the telescope, and observe whether the horizontal hair coincides with the object; then turn the telescope round on its axis, and observe if the hair is exactly at the same place; if not, correct half the difference by moving one of the screws, *a*, and tightening the other; then reverse the telescope on the Y's, and if the hair does not coincide with the object, repeat the foregoing operation till in both positions it perfectly coincides with the same object.

* The line of collimation is the line of vision cut by the intersecting point of the cross hairs in the telescope, answering to the visual line, by which we directly point at objects with plain sight.

When the precise situation of the horizontal hair is ascertained, adjust the vertical hair in the same manner, turning the telescope at right angles.

When the two hairs are thus adjusted, reverse the telescope on the Y's, and observe if the intersection of the hairs coincide exactly with the point of the object; the line of collimation will be in adjustment.

The next adjustment is the horizontal and the vertical limb. Set the instrument as nearly level as possible by the legs or tripod; fix the whole by the clamp G, and loosen the upper part of the limb F; fix the vertical arc at zero, turn it round, and bring the telescope over two of the parallel plate screws L; then raise one of the staff screws, and depress the opposite; turn the telescope half round, and repeat the same over the two other screws; continue this operation till the bubble in the level at D remains in the centre of the tube; take the telescope out of the Y's and reverse it end for end. If the bubble remains in the same position as before, it is in good adjustment; if not, that end to which the bubble runs is too high; then, by the capstan screw at the end of the tube regulate half the difference the bubble has moved from the centre, and the other half by turning the screws L between the parallel plates. Return the telescope to its former position on the Y's; if then not correct, the same operation must be repeated both ways until it does; the two levels *i i* must be adjusted at the same time by the capstan screws, and when this adjustment is made satisfactory, the vertical arc is at perfect right angles with the horizontal limb.

As a further proof of the accuracy of the instrument, clamp the upper horizontal limb and loosen the lower plate; then move the instrument slowly round, and if the bubble, D, still retains its position, the two axis on which it revolves are correct; but if the bubble does not retain its position, then some defect exists in the axis, which can only be remedied by the makers.

When all these adjustments are made perfect, the instrument may always be levelled by the staff screws.

The perfect adjustment of the instrument consists of the following particulars:

The horizontal circle must be truly level; the planes of the vertical circle must be truly perpendicular to the horizon; the line of sight or collimation must be exactly in the centre of the circle on which the telescope turns; the spirit level must be parallel to the line of collimation.

There are two methods in measuring an angle.

Theodolite. First: The angle measured from the meridian line to the chain line; second: The angle measured between two fixed objects.

To measure an angle from the meridian.

The theodolite being in perfect adjustment, place it firmly on the ground, and the parallel plates as nearly level as possible, and two staff screws in the direction of one of the chain lines. The centre of the instrument must be exactly over the station peg or angle point, which is proved by suspending a weight or plumb bob.

Fix the index on the vertical arc at zero, then proceed to level the instrument by the four staff screws. Turning them slowly contrary ways, raising one side and depressing the other, until the bubble is in the centre of the level under the telescope; then turn the instrument half round and repeat the same with the other two screws, continuing this operation till the bubble is stationary in the middle.

Then bring the zero of the horizontal limb F to the lower limb at 360°; its opposite will be 180°; and fix them by the clamp screw *l* on the upper plate (in all adjustments the tangent screw is used to move the instrument with greater minuteness).

Move the head of the instrument until the north end of the needle points exactly to 360° on the circle in the box, and fix it by the lower clamp screw G; loosen the clamp screw on the upper plate, turn the telescope until the cross wires cut the object set up on the chain lines, then read the angle on the limb F by the microscope *n*; as, for instance, 12° 40′ N.E.; enter

the same in the field book, adjoining the line for which the angle was taken.

The same angle may be read by the needle if it is correct. Always repeat this operation, and if by the two readings there is a slight difference, take the mean.

When there is more than one instrument used in the survey, care should be taken to observe the difference existing between the needles of each instrument, so that the allowance may be made in plotting; they are frequently very much at variance. This applies to all other instruments having a compass.

When taking an angle, be careful to remove the chains and all other metallic attractions, otherwise the angle will be inaccurate.

To measure an angle between two lines.

The instrument must be fixed and adjusted in the manner before described; set the vernier of the horizontal limb to 360°, and clamp the instrument to the lower limb; bring the whole round until the cross wires of the telescope cut one of the objects set up for the required angle, and fix it by the lower clamp screw, G; then unscrew the clamp screw, *l*, on the upper plate, turn the telescope round until it cuts the other object, and read off the angle; always repeat the operation before moving the instrument.

This method of taking angles applies to the box-sextant, and all instruments not having a compass.

When more angles are required from the same point, they should be read off before moving the instrument.

To take a vertical angle.

The instrument being set firm in the ground, and put into perfect adjustment, raise or depress the telescope until the cross wires intersect the object; then fix the vertical arc by the clamp screw *f*, and bring the telescope minutely to the object by the tangent screw *g*; read off the angle by the microscope *e*. If the angle is below zero, it will be an angle of depression; if above zero, it will be an angle of elevation.

THE DIAPHRAGM.

Fig. 2, *Plate* 36. The diaphragm is a part of the telescope to the theodolite and spirit level. It is a circular ring of brass, made to act freely inside the telescope, and is most accurately divided into four parts (where only two cobwebs are used) at right angles to each other; from each of these points a cobweb is stretched across the ring, forming the vertical and horizontal wires, by which the angles and heights are determined. This ring is attached to the telescope by four screws, *a*, Fig. 1, the adjustment of which has been before described.

It not unfrequently happens to the surveyor to meet with an accident by breaking one or both the cobwebs, which at once destroys the use of the instrument. If unacquainted with the manner of replacing them, and has no other instrument, he is compelled to lose time, whilst it has to go to the makers for reparation; therefore it is well to know how to prevent these inconveniences by being able to repair them.

Find some place where there are cobwebs, wind them singly round a card, the same as winding silk, until you have plenty for the two.

Then take two narrow strips of card to act as weights, and with strong glue, or gum shellac, fasten the ends of the cobweb to each card sufficiently long to hang over the ring on each side; place the cobweb minutely over the two nicks cut on the ring, and drop a small quantity of the gum on the nick to fasten the cobweb; then cut off the two ends; serve the other the same, replace the diaphragm, and adjust it as described.

THE IMPROVED DUMPY LEVEL.

Fig. 3, *Plate* 38. The superiority of this level consists principally in its simplicity and compactness. By adopting an object glass of large aperture and short focal length to the telescope, considerable light and power is obtained. A is the telescope,

having a diaphragm, *a*, with cross wires placed in the usual manner; the internal tube or slide, which carries the eye-piece, B, &c., is nearly equal in length to the external tube or telescope, which, being sprung at its aperture, secures to the slide and its eye-piece a steady and parallel motion, when adjusting for distinct vision of a distant object by the milled screw C; D, the object glass, having a movable slide shade. The spirit level E is placed above the telescope, and attached to two rings passing round it; by the capstan-head screws, C, the air bubble of the level is adjusted for parallelism with the line of collimation.

The parallel plates and screws F are similar in every respect to those of the last-described instrument; *d d* represents the capstan screws for adjusting the telescope, and are protected by cases screwed to the horizontal bar G, which supports the telescope and spirit level; a compass-box is made to be attached to the horizontal bar when required.

The parallel plates are attached to the upper part of the instrument, and the whole is fixed to the tripod by a screw under the lower plate, as in the other instruments.

To examine and correct the collimation according to Mr. Gravatt's method.

On a tolerably level piece of ground drive in three stakes at intervals of about 4 or 5 chains, calling the first *a*, the second *b*, the third *c*.

Place the instrument half way between the stakes *a* and *b*, and read the staff A placed on the stake *a*, and also the staff B on the stake *b*; call the two readings A′ and B′; then, although the instrument be out of adjustment, yet the points read off will be equidistant from the earth's centre, and consequently level. Now remove the instrument to a point half way between *b* and *c*, again read off the staff B, and read also a staff placed on the stake at C, which call staff C (the one before called A being removed into that situation); now, by adding the difference of the reading on B (with its proper sign) to the reading

on C, we get three points, A′ B′ C′, equidistant from the earth's centre, or in the same true level.

Place the instrument at any short distance, say $\frac{1}{2}$ a chain beyond A, and, using the bubble merely to see that you do not disturb the instrument, read all three staves, or, more correctly, get a reading from each of the staffs *a*, *b*, *c*; call the three readings A″, B″, C″. Now, if the stake *b* be half way between *a* and *c*, then ought C″—C′—(A″—A′) be equal to B″—B′—(A″—A′); but if not, alter the screws of the diaphragm, and consequently the horizontal cobweb, or wire, until such be the case; and then the instrument will be adjusted for collimation.

To adjust the spirit bubble. Without removing the instrument, read the staff A, and say it reads A‴; then, adding (A‴—A′ with its proper sign) to B′, we get a value, say B‴. Adjust the instrument by means of the parallel plate screws to read B‴ on the staff B.

Now, by the screws attached to the bubble tube, bring the bubble into the centre of its run. The instrument will now be in complete practical adjustment for level, curvature, and horizontal refraction for any distance not exceeding 10 chains, the maximum error being only $\frac{1}{1000}$th part of a foot.

Example. The instrument being placed half way between two stakes, *a b* (one chain from each other), the staff on *a* or A′ read 6.53, and the staff on *b* or B′ read 3.34, placing the instrument half way between the stakes *b c* (three chains from each); the staff on *b* read 4.01, and the staff on *c* read 5.31.

Hence, taking the stake *a* as the datum, we have—

Stake.	Above datum.
a or A′	= 0.00
b or B′	= 3.19
c or C′	= 1.89

The instrument being now placed at *d* (say 5 feet from *a*, but the closer the better), the staff on *a* or A′ read 4.01, on *b* or B′ 1.03, and on *c* or C′ 3.07. Now, had the instrument been in complete adjustment (under which term curvature and refraction are included), when the reading on staff *a* was 4.01, the

readings on b and c should have been respectively 0.82 and 2.12.

The instrument, therefore, points upwards, the error at b being 0.21, and the error at c 0.95. Now, were the bubble only in error (as is supposed in all other adjustments), the error at c ought to be 4 times as great as at b, but $4 \times 0.21 = 0.84$ only; there is an error, therefore, of $0.95 - 0.84 = 0.11$ not due to the bubble.

For the purpose of correcting this error (and be it remembered, contrary to the former practice, for this purpose only), we must use the capstan-headed screws at the eye end of the telescope, and, neglecting the actual error of the level, we are only to make the error at b one-fourth that at c.

After a few trials, whilst the reading at a contained 4.01, the reading at b became 0.75, and that on c 1.84.

Now $0.82 - 0.75 = 0.07$ and $2.12 - 1.84 = 0.28$
And as $4 \times 0.07 = 0.28$ the telescope is now adjusted for collimation

All that remains to be done is to raise the object end of the telescope by means of the parallel plate screws until the staff at c reads 2.12, and then, by means of the nuts which adjust the bubble tube, to bring the bubble into the centre of its run.

The operation of collimating, when once performed upon levels on Mr. Gravatt's construction, will scarcely ever need being repeated.

Description of the Y level, Fig. 1, Plate 38.

The achromatic telescope A is movable, like the theodolite, and is furnished with two cross wires, vertical and horizontal; the diaphragm (see Theodolite) to which they are fixed is adjusted by the four screws in the telescope. By turning the milled screw B on the side of the telescope, the object glass C (to which is attached a movable brass tube to screen the sun's rays) is moved outwards, having previously adjusted the eye-glass at D to a focus suitable to the observer.

The tube E, with the spirit bubble, is fixed to the telescope by a joint at one end, and a capstan-headed screw at the other,

to raise or depress it for the adjustment—that is, to set it parallel with the optical axis of the telescope.

The telescope is supported and fixed by the Y's.

The lower ends of these supporters move perpendicularly through strong bars; at the end of one of these is a milled nut, F, to bring the instrument accurately to a level, the other supporter having a movable joint (in some of the best instruments a screw is fixed to both of the supporters). Between the two supports is a compass-box, G, divided into 360°, with a spring to throw the needle off its centre when not in use. In some instruments the compass is made to take on and off. The use of the compass is to take the angles or bearings of the line that is being levelled (in the same manner as before described by the theodolite and other instruments), otherwise it has no connexion with levelling.

I is the clamp screw which fixes the telescope steady, and the tangent screw, K, gives the whole a slow motion to bring it nicely to the staff, which prevents handling the level when placed in adjustment. This beautiful adjustment is omitted in other levels.

The whole body, including the parallel plates, is fixed to the tripod, by means of a female screw in the lower plate.

Three adjustments are necessary. 1. To place the intersection of the wires in the telescope, so that they shall coincide with the axis of the cylindric rings on which the telescope turns. 2. To render the spirit level parallel to this axis. 3. To adjust for the horizontal motion, so that the spirit bubble remains stationary in the centre of the tube, whilst the telescope is turned wholly round on its axis.

To adjust the cross wires, or line of collimation.

The eye-glass being adjusted to its focal distance, direct the telescope to some defined object, then turn the telescope round on its dies in the Y's; if the intersection of the wires does not cut precisely the same point, their adjustment is necessary; the wires must be moved one-half the quantity by the four screws

marked *a*, attached to the diaphragm, one of which must be loosened before the other is tightened. The two horizontal screws are to move the vertical wire. The vertical screws are to move the horizontal wire. When the requisite position of the wires is produced, the adjustment of the axis is perfect.

To adjust the spirit level to the line of collimation.

Move the telescope till it lies in the direction of two of the parallel plate screws (the clips which confine the telescope in the Y's being laid open), and by giving motion to the parallel plate screws bring the air bubble to the middle of the tube. Then reverse the telescope in its Y's; should the bubble of air not come to rest in the middle, it then proves that the spirit level is not true to the axis of the tube, and requires adjustment. The end to which the bubble of air goes must be noticed, and the bubble made to return one-half the distance by the parallel plate screws H, and the other half by the screw F. This and the adjustment for the collimation generally require repeated trials before they are completed.

To adjust for the horizontal motion.

The level is said to be completely adjusted when, after the two previous ones, it may be entirely moved round without the bubble changing materially its place. To perform which, bring the telescope over two of the plate screws, and make it level by unscrewing one of the screws while you are screwing up the opposite one, till the air bubble is in the middle and the screws up firm. Then turn the instrument quarter round over the two other plate screws, unscrewing one and screwing up the other, as before; if the bubble remains in the middle all the adjustments are perfect, and none of the screws should be touched, excepting the four plate screws.

It is worthy of notice that all the adjustments of the Y level can be made by the practitioner, particularly the line of collimation, with the spirit level. The glass tube containing the spirits should be ground true, all others are imperfect. The spirit bubble of the Y level can always be proved by moving

it right and left on its axis; if the bubble remains centrical it is true.

In all other levels the spirit level is embedded and fixed by the maker, which prevents proving the accuracy of the glass tube. It is a fact not generally known that the makers, by heating the tube a trifle, will give a slight bend, to cause the bubble to fall more readily to the centre; this can only be proved in the Y level.

In the choice of a spirit level three points in particular should be minutely examined: first, the clearness of all the lens; second, the truth of the glass tube containing the spirits; third, the action of the level on the cone, which should revolve with the greatest accuracy possible; all the adjustments depend on this.

TROUGHTON'S LEVEL.

Fig. 2, *Plate* 38. This level is extremely compact, and when the line of collimation is in perfect adjustment, and with care it is less liable to be deranged than those of a different construction, the spirit level being permanently fixed to the telescope, the whole being confined to the horizontal bar, is attached to the parallel plates by a screw underneath the bar; the parallel plates are separate, and made to screw on to the tripod in the same manner as described on other instruments. The level and parallel plates are neatly fitted into a mahogany box, and the tripod similar in all respects to others.

A, the telescope; B, the spirit level partly let into the telescope, with small screws at each end for adjustment; C, the slide shade over the object glass; D, the achromatic eye-glass, and diaphragm with three cobwebs, two vertical and one horizontal, having capstan-head screws, *c*, for adjustment; E, the compass-box, supported on four pillars to the bar F, on which the whole instrument is fixed; *a* and *b* are two capstan-head screws, for adjusting the telescope, being protected by a cap; G, the parallel plate screws, by which the instrument is brought to a level.

The telescope is constructed on the inverted principle, and shows all objects with great clearness and brilliancy.

The requisite adjustments are the same as those of other instruments; namely, that the line of collimation and spirit level are parallel to each other, that the telescope be perpendicular to the vertical axis, or that the spirit bubble remains in its position in the centre when turned round horizontally on the staff-head.

The adjustment of the level is effected by correcting half the observed error by the capstan screws *a* and *b*, and the other half by the parallel plate screws G.

The spirit level itself has no adjustment, being fixed firmly by the maker.

To prove the line of collimation.

Set up the instrument on some tolerably level ground, and level the telescope by the two parallel plate screws; direct it to a staff held at the same distance (5 or 10 chains); note the height on the staff cut by the horizontal hair, say 6 feet; then measure the height of the instrument to the centre of the eye-glass, say 4 feet, the difference being 2 feet. Now let the instrument and staff change places, and repeat the former operations; if the difference in the two heights is the same the instrument is correct; if otherwise, take half the difference by adjusting the capstan-head screws, and the other half by raising or depressing the horizontal hair, and the other half by the parallel plate screws.

Another method:

Let there be two staves held upright at any convenient distance from each other; call one staff A, the other B; then place the instrument nearly on a line with the staves at 2 or 3 chains' length from A, and having set the instrument level, read off both the staves, and note the same in the book; then remove the instrument about the same distance beyond B, level it, and again read off the staves; compare the former reading; if the differences agree, the instrument is correct; if not, the

adjustments must be corrected in the same manner as before described.

Another method:

Is by means of a sheet of water, and, when practical, is both convenient and accurate. At a distance of a few chains drive two stakes close to the water's edge, so that the top may be even with the water; let the level be set exactly over the stake, and a staff held on the top of the other stake; now measure the height of the telescope to the centre of the eye-glass from the top of the stake, and read the height on the staff cut by the horizontal wire; if the two heights agree, the instrument is perfect; if not, the horizontal wire must be raised or depressed as the error requires, until the heights read exactly the same.

THE CIRCUMFERENTOR.

Fig. 2, *Plate* 36. This instrument has been greatly improved of late years; it is now capable of performing nearly the same operations in surveying as the theodolite, though not with that perfect confidence. In addition to its use in surveying, it is chiefly used in mining operations: called by some dialling a pit, and by others latching. (See Plate 24.)

It consists of a large compass-box, A, divided into 360°, subdivided into minutes, and a vernier, *a*, on the upper part, a magnetic needle, and two sights, B C, perpendicular to the meridian line in the box; in each sight there is a large and small aperture, or slit, one over the other: these are alternate; a fine piece of silk or horse-hair runs through the middle of each aperture, and the circular holes crossed at right angles by another horse-hair.

The two sights are made to fold down for the convenience of packing; underneath the compass-box is a pin to fasten the two plates together at 360°, and a spring to throw the needle off the point to preserve it when not in use.

The instrument is made to turn in a vertical position for

taking angles of altitude and depression. A spirit level, D, is attached to the lower plate.

The cover of the box has two graduated circles, divided into degrees and minutes, similar to that on the arc of the theodolite, for taking vertical angles, and the hypothenusal difference; a fine thread and plummet is suspended, hung on a small pin, which divides the arc according to the angle.

The whole instrument works on a ball and socket, by which it can be placed to take vertical and horizontal angles; it is fixed on a tripod similar to the theodolite.

To measure an angle by the circumferentor.

When taking horizontal angles, place the instrument so that its centre be directly over the station point; fix the two plates by the pin underneath the compass-box, and bring the sights round so that the needle and 360° coincide; fix the whole by the screw F; then release the pin, and by the screw E turn the sights to the object set up; the vernier will give the angle taken from the meridian.

To measure an angle between two lines, fix the instrument as before, and bring the sights to the first object; fix the whole by the screw F; remove the pin, and bring the sights to the second object, and read the angle; the reading of both should agree.

Note.—In mining operations all angles are taken from the meridian.

THE PRISMATIC COMPASS.

Fig. 1, *Plate* 36. This instrument was originally introduced by Mr. Schmalcalder for military operations. A floating card, or silver ring, A, is fixed to the magnetic needle, divided into 360 degrees, and subdivided into 20 minutes. Contrary to other instruments, it is graduated from the south pole of the needle reading west; a prism, B, is fixed on one side of the box, to slide up and down; on the opposite side is a vane, C, having a fine thread down its centre.

In taking angles it is held in the hand, standing in such posi-

tion that the centre of the needle is directly over the station point; it can be made to fix on a stand by a ball and socket, similar to the circumferentor.

This instrument is confined in its use to horizontal angles taken from the meridian, and can be used only on level ground, or filling in portions of a survey not requiring extreme accuracy.

D is a mirror made to slide off and on when required; it is for the purpose of reflecting objects above or below the observer, and may be used with its face upwards or downwards.

The method of taking an angle by it is as follows: First adjust the prism, by sliding it up and down to bring it to a proper focus to read the divisions on the circle underneath distinctly; place the sight vane perpendicular; hold the instrument steadily, and perfectly horizontal; look through the hole and slit in the prism, and bring the thread in the slide vane in a line with the object; at that moment read the angle.

E is a trigger to lift the needle from off the point when not in use; the cover of the box, F, is fixed underneath, and answers to hold the instrument when taking an angle; the slide vane turns down flat on the glass, and the prism turns back; when the cover is placed over the whole is packed in a leather case, making it very portable.

THE PROPORTIONAL COMPASSES.

Fig. 2, *Plate* 37. This is one of the most delicate mathematical instruments in use. It is valuable in reducing plans where there are close towns, or other minute parts. It requires great care, as the slightest injury to any one of the points renders the instrument useless. Independent of its use in reducing plans, there are many other properties to which it can be applied.

The drawings represent one of the best kind, the only difference is the bar having a tangent and clamp screw, by which the points may be moved with the greatest minuteness; the side view shows the bar when not in use.

There is a groove in each shank, with a slide, on which is engraved the index point attached to the milled-head screw; the whole slides regularly in the grooves, when the instrument is shut close and is fixed to any division by the nut and screw, *a*, *b*.

On one side of these grooves are placed two scales, one for lines, the other for circles.

By the scale of lines, a right line may be divided into any number of equal parts contained on the scale.

By the scale of circles, a regular polygon may be inscribed in a circle, provided the sides do not exceed the numbers on the scale.

On the reverse side sometimes are added a scale for superfices and solids.

Examples. To divide a given line into a proposed number.

Shut the compasses, unscrew the milled screw *a*, and move the slider until the line across it coincides with the number of division required, as at 2; then tighten the screw, open the compasses, and with the longer points take the length of the line to be divided; then with the shorter legs you have the reduced length of half.

To use the tangent screw, remove the screw *d*, and place it at *e*; screw the bar firmly, and by turning the screw *c* it will bring the points minutely to the division required. By slackening the screw at *e*, the compasses will be at liberty for other dimensions; the bar also assists in keeping the whole steady.

To inscribe in a circle a regular polygon.

As in the last example, shut the compass, and place the slider to the number of sides required, and on the scale fix the screw *a*; with the longer legs take the radius of the circle.

The line of plans may be practically applied in reducing or enlarging the areas of parallelograms, triangles, circles, &c., in the ratios of from 2 to 10, the numbers on this scale, being the squares of those on that marked "lines" on the other side of the instrument; for example:

Let it be required to enlarge a parallelogram five times its present area, for which purpose the index division should be set against the 5 on the line of plans; then extend the short points to the height of the figure, the long points will indicate the length of that side five times greater; in the same manner take the base of the figure, which, when drawn, will be five times the area.

And by reversing the process a parallelogram will be drawn to $\frac{1}{5}$ of the former area.

In any case of triangles, by taking the lengths of the sides respectively, we obtain either enlarged or reduced, as above shown.

Example. Let it be required to lay down a triangle five times its area.

Extend the short points the length of the base; reverse the points, and mark off the distance; then take the two sides in like manner; describe arcs, and at the intersection draw the lines, which will complete the triangle as required.

In like manner, by reversing the points, it will be reduced $\frac{1}{5}$.

For circles, as their areas are to one another as the squares of their diameters, we have only to take the radius, and the opposite points will indicate the radius either for an enlarged or a reduced area, as may be required.

In using the line of solids.

Let it be required to obtain the dimensions either of a cube or a sphere, which shall be five times the solidity, or, in other words, five times the weight of a given one of the same specific gravity.

Example. The cube weighs one ounce; then if the index be fixed at 5 on the line of solids, and the side of the cube be taken between the short points, the larger points will show the side of a cube weighing five ounces; and, as before shown in the line of plans, we have only to reverse the points in order to reduce the solidity to $\frac{1}{5}$. And in like manner for a sphere, it is only necessary to take its diameter to arrive at corresponding

results; spheres being as the cubes of their diameters, and the numbers on the line of solids are the cubes of those on the line of "lines."

THE BOX-SEXTANT.

Fig. 3, *Plate* 36. This valuable little instrument should always be carried by the surveyor, as it will be found generally useful in filling in the details of small surveys, and particularly in railway practice.

It is constructed on the same principle as Hadley's quadrant or sextant.

It is confined in its practice to level districts, and will only measure angles contained between two lines, which has been fully described in the use of the theodolite. It is used in military, naval, and astronomical operations.

This instrument is enclosed in a brass box about three inches in diameter, and an inch and a half deep. The cover A is made to screw on to the lower part, which answers to hold the instrument when taking an angle; on the top of the cover is engraved a table, No. 18, for calculating heights of buildings, trees, &c.

The instrument, when in use, appears as in the drawing, the telescope H being drawn out; it may be used without the telescope, by drawing the slide G over the hole at H.

B is a silver arc or graduated limb, divided into degrees and 30 minutes; C is a bar having a vernier at the end, divided to read an angle to 1 minute, and moves on the graduated limb B by the milled screw D, which acts on a rack and pinion within; attached to the end of the bar on the inside is a mirror, E, called the index glass, reflecting the distant object set up on the line and angle to be measured; in the line of sight of the telescope, or small hole, is another glass, F, called the horizon glass, the upper half of which is silvered, which must be adjusted perpendicular to the plane, and parallel to the plane of the index glass when the index is at zero.

The eye end of the instrument has a dark glass, which slides over the hole, to be used when required particularly.

For astronomical purposes, there are two coloured glasses made to fold inside; both shades together are proper for the brightest sun, and separately make two more, so that two glasses make three shades.

To adjust the sextant: Fix the zero on the vernier to the zero on the graduated arc B; look through the telescope or hole and the unsilvered part of the glass to some object that is sharp or perpendicular, as the angle of a building; if the reflected object and the real object are in a perfect straight line, or coincide with one another minutely, the adjustment is, so far, true. Then holding the instrument contrary ways or vertical, look through the hole or telescope, as before, at an object that is level, as a window-sill, coping of a building, &c.; if the reflected line and the real object coincide in a perfect straight line, the whole instrument is in adjustment.

If in either case the reflected and real object do not coincide in a direct line, the instrument is not in adjustment; then apply the key marked K, which is screwed into the top for safety, to a small square on the side of the instrument (like to that where a watch is wound up), and turn it gently until the line coincides; this adjusts the mirror horizontally; then apply the key to another square at the top, marked L, turning it gently till the line coincides as before, which will adjust the mirror vertically; when these adjustments are made perfect, and the zeros coincide, the instrument is ready for observation.

The key should always be returned to its place of safety.

A small magnifying-glass, marked I, is attached to the top to read the angle more distinctly.

To take an horizontal angle: Hold the sextant in the left hand, looking through the telescope or hole; turn the screw D by the right hand, and bring the reflected object to coincide with the reflected object on the horizon glass F, and the measure of the angle will be given on the graduated limb B, in the same manner as described by the theodolite.

To take a vertical angle: Hold the instrument in the right

hand, and by the left turn the milled screw, until the reflected object is brought down to coincide with the mark or image of the lower object, and read the instrument as before; to which add the height of the eye from the mark to the ground.

The sextant being set to any angle contained in the table, any height or distance of accessible or inaccessible objects may be obtained in a very simple and expeditious manner.

Make a mark on the object if accessible to the height of the eye; set the index to any angle from the table, and advance or go backwards from the object, until, by reflexion, the top of the object is brought by the mirrors to coincide with the mark first made. If the angle be greater than 45°, multiply the distance to the object by the number in the next column to the angle in the table; if the angle be less than 45° 00′, then divide, and the result will be the height of the object from the mark; to which add the height of the eye.

If the object is inaccessible, set the index to the greatest angle in the table that the least distance from the object will admit of; make a mark as before; move backwards and forwards until the top of the object is reflected to the mark level with the eye. Then set the index to any of the lesser angles; go back in a line with the object, until the top is made to appear on the level with the eye or mark before made; fix here another mark; measure the distance between the two marks set up; this divided by the difference of the corresponding numbers to the angles made use of, the quotient will be the height of the object from the mark; to which add the height of the eye.

If the index is set at 45°, the distance is equal to the height minus the height of the eye.

Also to raise a perpendicular: Set the index at 90°, holding the instrument steady over the point of the angle, and looking at the flag set up on the line on which the perpendicular is to be raised; motion to the man to move gently (right or left), until his flag is reflected on the half mirror, and coincides

exactly with the other flag; then motion to fix the flag, which will be the perpendicular required.

THE PROTRACTOR.

Plate 37. The protractor is used to plot angles taken by the theodolite or other instruments, or for measuring angles that are plotted, and transferring them. It is divided in the same manner as the limb of the theodolite.

They are of various kinds and construction—viz. the ivory protractor A, Fig 3 (such as are generally in a case of instruments).

The semicircular brass protractor B, about five or six inches in diameter.

The semicircular brass protractor. This instrument on extensive surveys is not so convenient as the circular, as the divisions extend only to 180°, so that to plot angles between 180° and 360° it has to be reversed.

The wheel protractor, Fig. 5, is a very useful instrument for measuring angles; it is divided into degrees, minutes, and seconds. The bar A is made to move round the centre, having two verniers; attached to which are two protracting points, *a*, *b*, with adjustment screws, *c*, to bring the two points in a straight line passing through the centre; by means of four fine pins it fixes itself firmly to the paper. All protractors should be engraved with two rows of figures, numbered contrary ways; to some protractors there is attached to the bar A a clamp and tangent screws.

To plot an angle taken from the meridian.

In surveys, when the angles are taken from the meridian line or magnetic needle, the bevel edge of the protractor must always be placed against the meridian line, marking the centre; then prick off as many angles as may be convenient to that part of the survey, noting the number of the station and angle they read. Repeat the same in other parts of the map. (See Plate 22.)

To measure an angle contained between two lines.

Place the protractor so that the bevel edge is against one of the lines and the centre at the station point or meeting of the two lines; read the angle contained between 360°, and the degrees intersected on the edge by the other line will be the angle required.

THE VERNIER.

Figs. 4 *and* 5, *Plate* 36. The vernier or dividing plate, so called from its inventor, Peter Vernier, a gentleman of Franche Comte.

It has been commonly called nonius, from Pedro Nunez, or Peter Nonius, an eminent Portuguese mathematician, being very different from that of the vernier (the term nonius is not now used).

The vernier is a scale made for the purpose of subdividing another scale into certain equal proportional parts to any degree of minuteness.

The vernier is divided into equal parts, one more or less than the scale to which the vernier is attached. In the best instruments they vary in their minuteness or value. They are graduated, sometimes numbering right and left of zero; and in others the numbers are continuous on one side zero.

To find the value of a vernier.

Find the value of each division or subdivision in degrees, minutes, seconds, &c., on the limb; divide the quantity thus found by the number of divisions on the vernier, the quotient will be the value required.

Example 1. The divisions on the lower limb are not subdivided, each being equal to 60 minutes or 1 degree; the vernier has 20 divisions.

Then $60 \div 20 = 3$ minutes, the value to which the vernier reads

Example 2. Each degree is divided into two parts, each part equal to 30 minutes; the vernier is divided into 15 divisions.

Then $30 \div 15 = 2$ minutes value

Example 3. Each degree is subdivided into five parts, or 12 minutes each; reduce the minutes into seconds.

Thus: as $12 \times 60 = 720$ seconds, the vernier has 24 divisions
then $720 \div 24 = 30$ seconds value

Example 4. Each degree is divided into two parts, or 30 minutes; the vernier has 30 divisions.

Then $30 \div 30 = 1$ minute value

Example 5. Each degree is divided into six parts, or 10 minutes each; reduce the minutes into seconds; the vernier has 60 divisions.

Then $10 \times 60 = 600 \div 60 = 10$ seconds value

Example 6. Each degree is divided into three parts, or 20 minutes; the vernier has 20 divisions.

Then $20 \div 20 = 1$ minute value

The nature and use of the vernier is further explained.

Let A B, Fig. 4, represent seven divisions or degrees on the limb, each divided into three parts; therefore, in seven degrees there are twenty-one divisions. C D is the vernier or index, equal to seven degrees, divided into twenty equal parts; it is evident that, since twenty of these answer to twenty-one on the limb, each one of these will exceed each on the limb by $\frac{1}{20}$ part, by 1 minute; therefore two on the vernier will exceed two on the limb by two minutes, three on the vernier by three minutes, and so on.

In taking an observation, suppose zero on the vernier be found between the 23° and 23° 20′, as shown in the figure; then looking for the first coincidence of lines on the limb and vernier, you find it at the sixth division (marked thus *), therefore the angle reads 23 degrees 6 minutes.

The two 10's in the vernier are the only two divisions that coincide at a time with the divisions on the limb, if the instrument is accurately divided.

If the first line or index of the vernier has moved over a space less than half a division of the limb, then the coincidence

will be on the right hand of zero in the vernier; if it has moved more than half, it will be found in the left hand; if it is just half, the coincidence will be at 10′. This shows the reason why the beginning of zero is placed in the centre.

Fig. 5. In the best instruments, the mode of figuring the vernier is usually adopted as shown by the drawing—which is, by taking nineteen divisions on the limb and dividing it into twenty parts for the vernier; consequently, one division on the limb will exceed $\frac{1}{20}$th part on the vernier, reading all one way to the left.

Beginning at zero, it will be found between 20 degrees 40 minutes and 21 degrees; then looking for the coinciding lines on the limb and vernier, it is found to be at the twelfth division on the vernier (marked thus *); therefore the angle reads 20° 52′, that is, by taking 40 seconds on the first degree, and adding 12 minutes on the vernier.

When the index or zero on the vernier coincides with any line on the limb, the angle is read at once; as, for instance, the index of the vernier coincides with the line at 20, then it is 20 degrees; if at the next line, it would be 20 degrees 20 minutes, and so on. As only one line on the vernier can coincide with another line on the limb, that line must decide the angle.

In the best theodolites there are two verniers, and sometimes three; the readings of each should be taken, and, if any difference, the mean difference must be registered as the correct angle.

THE PANTAGRAPH.

Fig. 1, *Plate* 39. The original inventor of this useful instrument is not exactly known; the earliest account is in a small tract, published about the year 1631, by the Jesuit Scheiner, entitled "Pantagraphice sive Ars nova Delineandi." The principles are self-evident to every geometrician; the mechanical construction was first improved by Mr. W. Jones, instrument maker, Holborn, in the year 1750.

Its chief value is in reducing figures, although it may be used in copying plans, &c., of the same scale; it will also enlarge, which is never at any time recommended.

The pantagraph is made of brass, from 12 inches to 4 feet in length, and consists of four flat bars, two of them long and two short. The two longer ones are joined at the end A by a double pivot, which is fixed to one of the bars, and works in two holes placed at the end of the other; under the joint is an ivory wheel, to support this end of the instrument. The two shorter bars are fixed by pivots at E and H, near the middle of the longer bars, and are also joined together in a similar manner at the other end, G; ivory wheels being also fixed under each joint, marked *a*.

By the construction of this instrument the four bars always form a parallelogram.

There is a sliding box on the longer bar, B, and another on the shorter bar, D. These boxes may be fixed at any part of the bars by means of the milled screws; each of the boxes are furnished with a cylindric tube, to carry either the tracing point, the pencil, or fulcrum.

The fulcrum or support, K, is a lead weight, to which is fixed a bright iron pin, *e*; on this the whole instrument moves when in use.

The pencil-holder, tracer, and fulcrum, must in all cases be in a right line, as shown in the drawing marked *b*, so that when they are set to any number, if a fine string be stretched over them, and they do not coincide minutely, there is an error either in the setting or in the graduations.

The long tube *c*, which carries the pencil, moves easily up or down in either tube; there is a fine piece of silk, *f*, fixed to the pencil tube, passing through the holes in the three small knobs to the tracer point *d*, where it may, if necessary, be fastened. By pulling this string the pencil is lifted up occasionally, and thus prevented from making false or improper marks upon the copy.

There is a cup at the top of the pencil-holder for receiving an additional weight when required to make a stronger mark on the paper.

The greatest possible nicety is required in fixing a short piece of pencil in the tube, so that when it is brought to a fine point, by revolving it in the tube, it should make a point, and not a circle. This should be adjusted very truly before commencing to use the instrument.

Fasten down upon a smooth board a sheet of paper under the pencil D, to receive the reduced plan, and fix the original plan under the tracer at C.

Then, with a steady hand, carefully move the tracing point C over the outlines of the plan, and the pencil will describe exactly the same figure as the original, but half the size, or rather it is quarter the size, although half the scale.

In the same manner for any other proportion, by setting the two sockets to the number of the required proportion.

If the original should be so large that the instrument will not extend over it at one operation, on the original plan draw a line, or make three or four points, and mark the same also on the reduced plan.

The fulcrum and copy may then be moved into such situations as to admit copying the remaining part of the original, first observing that, when the tracing point is applied to the lines or points marked on the original, the pencil must also fall on the corresponding lines and marks made on the copy.

In very large plans, the better method is to divide the original plan into equal portions by lines, as much as the instrument can easily travel over, and make the copy on tracing paper first, then transfer it to the paper prepared for the reduced plan.

As a general rule, the following instructions should be observed:

On the two divided bars which carry the sliding tubes appropriated to the fulcrum and pencil, are usually 21 divisions; those numbered $\frac{1}{3}$, $\frac{1}{4}$, up to $\frac{1}{12}$, are to be used with the fulcrum in the tube B bar, the pencil in the D bar, the tracer being fixed at C.

The other fractions, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, downwards to $\frac{11}{12}$, are used with the fulcrum at D, and the pencil at B.

The above directions are to be followed when a plan is to be reduced; but when it is required to be enlarged, the pencil and tracer must be changed.

Example 1. It is required to reduce a plan in the ratio of 10 to 11. The fulcrum at D being set to the divisions marked 10 to 11, and the pencil at B at the corresponding division 10, 11.

Example 2. It is required to reduce a plan to $\frac{1}{3}$ of its size; place the slide on each of the divided bars at $\frac{1}{3}$, the pencil placed in D slide, the fulcrum in B slide.

Example 3. To copy the same size, but reversed, place the two sockets at $\frac{1}{2}$, the fulcrum at D, and the pencil at B.

Example 4. To reduce 6 chains to 8, place the two sockets at $\frac{3}{4}$, the fulcrum at D, and the pencil at C.

Frequently plans have to be reduced when none of the divisions on the bars will answer the purpose; then shift the sockets by repeated trials until the proper division is found, taking care always to prove that the three sockets are all directly in a right line.

Where a mass of buildings has to be reduced, the proportional compass will be found to be a most valuable auxiliary.

THE EIDOGRAPH.

Fig. 2, *Plate* 36. The construction of this instrument is different to the pantagraph, but its use is similar.

A B C, the three subdivided arms; the pencil, D, has three small weights at top; E, a fine silk cord from the tracer at G to the crank at F, which lifts the pencil when required; H, the lead weight or fulcrum, on which the whole instrument moves; I is an elastic cord attached to the two pulley wheels K and L; M is a screw to regulate the elastic cord; N is a movable slide, with a vernier or index plate; *a* and *b* are two holes in the pulleys to set the divisions on the bars A and B.

To set the instrument, to reduce, or enlarge to any required proportion.

Take the sum and difference of the fractional terms; then, as the sum is to the difference, so is 100 to the number required, and to this number the arms and centre bar are to be set.

For example. Let it be required to reduce one-third.

$$3 + 1 = 4; \; 3 - 1 = 2; \text{ then as } 4 : 2 :: 100 : 50$$

The arm carrying the tracer is to be lengthened to division 50. The centre bar is to be set to the division 50, on the pencil side of O, and the arm carrying the pencil is to be shortened to division 50.

The instrument thus set will give a tracing with the pencil one-third of the size of that traced by the tracer. This arrangement is to be reversed when the instrument is required to give an enlarged tracing of any subject.

The only adjustment which may be put out is the parallelism of the bars passing under the wheels. This is to be corrected as follows: Place all the verniers to O, and, with the arms at right angles to the centre bar, make a mark with the tracer and pencil points; then wheel the instrument half round, and placing the tracer into the mark made by the pencil, the pencil should fall into the mark made by the tracer; if it does not, half the error is to be corrected by letting out and taking up the spring passing over the wheels, by means of the screws in the middle of the wires.

See Table, No. 17, for Reducing and Enlarging.

H. S. MERRETT'S PATENT IMPROVED QUADRANT.*

Fig. 1, *Plate* 37. The simplicity and portability of this instrument will make it particularly useful to the surveyor or engineer where angles of inclination or declination are required for practical purposes, in the same manner as the theodolite.

It is made of boxwood, having two arcs of 90 degrees, divided into degrees and half degrees, being sufficiently near for the purposes to which it would be required.

There are two tables, Nos. 12 and 18, engraved on it. One

* This instrument can only be obtained of the author.

to ascertain the height of any object, as a building, tree, &c., according to the angles marked thereon, the same as the box-sextant; the other table is for shortening the hypothenusal line to the horizontal line, when required in surveying hilly districts, the same as the theodolite.

There are also all the angles of slopes usually adopted in railway cuttings and embankments; and may be applied with great advantage by the engineer in setting out the widths of land in sloping ground, or in taking cross sections, and also in proving the accuracy of the slopes, and in measuring embanking and cuttings to ascertain the quantity of earthwork.

It is governed in its operation by a spirit level on the top, having two sights, which may be used either for angles of acclivity or declivity.

The arc is divided both ways. To read the angle of depression, place the eye at *a*; and for inclination, place the eye at *b*.

The drawing represents it fixed on a tripod, which is desirable when extreme accuracy is required; but for ordinary purposes a rod, having a spike at one end, a ferrule at the other, about 4′ 6″ high, in which it may revolve. The staff from which the angle is measured must be of the same height as the instrument.

When taking an angle: first bring zero to the bevelled edge at *d*, and fix it by the clamp screw, *c*; then, placing it on the tripod or rod, bring it to a level by the spirit bubble, and turn the instrument a quarter round; when adjusted, fix it in the position required by the screw *e*, then unclamp the screw *c*, and proceed to take the angle required.

Fig. 1, *Plate* 38. To find the difference between the hypothenuse and the base.

First fix the instrument at A and the staff at B, the angle is found to be 23 degrees, and the length 7 chains; look in the table for the nearest angle, the next column will show the number of links to be deducted, equal to 8 links; which, multiplied by the length, gives 56 links to be deducted, as, $700 - 56 = 644$ links, the true horizontal measure from A to C.

In practice, such an inclination should be avoided.

Fig. 2, *Plate* 40. To obtain a cross section.

Adjust the quadrant as before described, and fix it on the centre line at A; send the staff forward to B and C; take the angles, and measure the distances as before described.

If it be required to extend the section, then, without moving the instrument, take the angles, A D and A E, and their distances; this may be repeated any number of times, by which a very correct profile of the ground can be plotted.

Fig. 3, *Plate* 40. To find the quantity of earthwork in an embankment.

Fix the instrument at A, and adjust as before described; send forward a staff of equal height, and fix it at C; take the angle, and measure the distance in feet and inches (read the angle from the upper circle, it will be the complement of the angle A C D; if the angle was read at C, take the lower circle).

Plot the angle by the complement, and mark off the distance A C; in like manner take the other slope, D I.

Having the correct width of the formation level, A D, and the depth of the embankment, H G, from the longitudinal section, or from the permanent level book, the line F G C can be obtained very nearly, which will form the cross section, A C G F D, from which the area may be accurately calculated.

The same method may be applied to the cuttings.

It is also useful in proving the slopes whilst forming.

By the same instrument the widths of ground on each side the centre line may be staked out also.

Fig. 4, *Plate* 40. To determine the height of a building, monument, &c.

Fix and adjust the instrument as before; when perfectly level, make a mark on the building as at *a*; take the angle, which is equal to 45 degrees, and measure the length of the base line from the centre of the instrument to the centre line of the monument, as shown by the dotted line at *b*; plot the angle, and by the scale mark off the length of the base line;

raise a perpendicular from *b*; measure by the scale from *a* to *c*; to which add the height from *a* to *b*, the product will be the height required.

Note.—In all cases, when the angle is 45 degrees, the base is equal to the height.

Fig. 5, *Plate* 40. To find the height of a building, &c., when the same is inaccessible.

First take the angle A B C = 26° 34′; set up a mark at D level with the line A C; measure the distance from A to D = 54 feet; then fix the instrument at D from the same level as A C, and measure the angle B D C = 63° 26′, being the complement to A B C; plot these angles, and from the intersection at B let fall a perpendicular to *b*; by the scale measure the height B C; to which add the height *a b*; the sum will be the whole height.

The difference of the height *a b*, taken from the height of the instrument at A, will give the inclination of the ground.

Fig. 6, *Plate* 40. To determine the height of a statue which is to be erected on a column at any altitude above the level of the eye.

First draw the line A B, and, perpendicular to it, the line B E; mark off the height of the column B D = 50 feet, and the length A B = 70 feet; on the line B E mark off 6 feet at C, the height a statue would appear level with the eye.

Draw the lines A C and A D; then with the radius and centre, A, describe the arc *a b c d*; with the distance *a b*, mark off the same distance from *c*, *d*; through *d*, draw the line A *d* E, intersecting the perpendicular line B E; measure by scale the length D E = 9′ 9″, which will be the height required for a statue to appear by the visual angle as the natural height.

By the same rule, the height of the statue if erected may be ascertained, having the difference of the two angles at *c* and *d*; transfer the same angles from *a* to *b*, and mark the height at *c*.

COMPUTATION SCALE.

Fig. 1, *Plate* 41. This scale is in fact two plotting scales, but divided different to the ordinary plotting scales, the chains counting right and left from the centre, and one side being double that of the other, purposely for calculating quantities, according to the rules given at page 57 for finding the area of a triangle.

Before the introduction of the improved calculating scale, this was considered the most perfect system in obtaining a true quantity; as, for example:

Suppose the plan to be plotted to 4 chains to the inch, the irregular fences equalised, and the whole field brought into straight lines, and then into triangles and trapeziums.

That part of the scale divided into 40 parts is applied to all the bases; then, by applying the two zeros exactly on the base, and brought to the vertex of the triangle, the side that is divided into 20 parts gives half the perpendicular at once, instead of having to divide the whole perpendicular.

When calculating a trapezoid, take the sum of the perpendiculars and multiply by the base.

DESCRIPTION AND USE OF THE IMPROVED COMPUTATION SCALE.

Fig. 2, *Plate* 41. This scale is usually made to 3 chains to the inch. The drawing shows only a portion of the scale. The large figures from O to 6, on the upper side, denote 6 acres; and from 6 to 12, on the lower side, 12 acres; each division is subdivided into 4 parts or 4 roods, and are numbered accordingly 1, 2, 3. The perches are engraved on the ivory scale attached to the movable metal frame, divided into 4 parts, each part equal in value to 10 poles; each part is subdivided into 5, equal to 2 poles each. Those on the right hand read with the acres and roods on the upper part of the scale; those on the left hand read with the acres and roods on the lower side of the scale.

The transparent horn is fixed to the metal frame; the horizontal lines engraved on it, parallel to the edge of the scale, are one chain apart; the vertical line is directly over the zero on the ivory scale, which divides the fence line between the two horizontal lines; and in like manner the two oblique lines are used to divide the fence lines that more immediately run in that direction.

When using the scale, first prepare a piece of tracing paper by drawing parallel lines one chain apart; lay the tracing paper so that any two lines touch two opposite corners of the field, as at A and B, Fig. 82, Plate 6. Keep the tracing paper firmly fixed to prevent its shifting; bring the vertical line to zero on the scale, and place that line on the fence at *a*, dividing it equally, taking care the two horizontal lines on the horn are over the first two lines on the tracing paper; hold the scale firm, and move the metal frame to the left until the vertical line equally divides the fence at *b*; then move the whole, being careful not to move the metal frame; then place the vertical line on the next parallel line at *c*, dividing the fence as before; hold the scale firm, and move the frame to *d*, dividing that fence equally; so proceed on with every parallel until the metal frame stops at 6 acres, which may be in a part of the parallel line, as shown at C; at this point make a mark on the tracing directly over the vertical line; without moving the metal frame, bring the vertical line to the fence at *e*, on the same parallel, and move the frame to the right hand to the mark made at C; without touching the metal frame, bring the whole back to *f* and slide it to *g*, and so on to the last fence at *h*; at this point read the quantity from the scale.

Should the quantity exceed 12 acres, make a mark as before at C, and keep a memorandum, and then proceed as at the commencement, which must be added to the last quantity.

The quantity as shown on the scale is the content of a field calculated on the upper side of the scale, 3A. 3R. 16P.; but if it returned the reading on the lower side would be 6A. 0R. 24P.;

if the scale had been expended once before, then it would be 16A. 0R. 24P., and so on.

H. S. MERRETT'S PATENT IMPROVED COMPUTATION SCALE.*

Fig. 3, *Plate* 41. The difference between this and the last described scale is in making the same metal frame applicable to other scales, as 3, 4, 5 chains to the inch.

For which purpose three scales, 3, 4, and 5, to the inch are divided in like manner, and made to fit the same metal frame, which is made to hold horns divided in like manner to their respective scales.

Instead of the poles being divided on the ivory scale, as in the former example, they are divided on the scale with the acres, roods, and perches.

The screw in the metal frame acts as a clamp.

It is more desirable to divide the whole value of the scale to a decimal number, as 10 acres.

In using these scales, remember always to keep the side C D of the metal frame for reading the quantities, that being a continuation of the vertical line on the horn.

It is used precisely in the same manner as before described.

In all cases where accuracy is required the method described at page 68 and Fig. 83 is recommended as the best.

Where very irregular boundaries occur the computation scales are then most desirable.

THE DIAGONAL SCALE.

Fig. 4, *Plate* 41. This scale is seldom used, excepting for large works requiring great accuracy.

If the larger divisions be accounted as units, the first subdivision will be a tenth part of a unit, and the second, marked by the diagonal line upon the parallel lines, a hundredth part of a unit.

* To be obtained only of the author.

But if we suppose the larger divisions to be tens, the first subdivisions will be units, and the second tenths; if the larger are hundreds, then will the first be tens and the second units.

The numbers, therefore, 576, 57.6, and 5.76, are all expressible by the same extent of the compasses; thus, setting one foot in the number 5 of the larger divisions, extend the other along the sixth parallel to the seventh diagonal.

For if the five larger divisions be taken for 500, seven of the first subdivisions will be 70, which, upon the sixth parallel taking in six of the second subdivisions for units, make the whole number equal 576.

Or if the five larger divisions be taken for five tens, or 50, seven of the first subdivisions will be seven units, and the six second subdivisions upon the sixth parallel will be six-tenths of a unit.

If the five larger divisions be only esteemed as five units, then will the seven first subdivisions be seven-tenths, and the six second subdivisions the six-hundredth part of a unit.

Example 1. To take off the number 4.79.

Set one foot of the compasses on the point where the fourth vertical line cuts the seventh parallel line, and extend the other foot to the point where the ninth diagonal cuts the seventh horizontal line.

Example 2. To take off the number 76.4.

Observe the points where the sixth horizontal cuts the seventh vertical and fourth diagonal line; the extent between these points will represent the number.

In the first example, each primary division is taken for one; in the second, it is taken for ten.

Example 3. To lay down a line of 7.85 chains.

Set one point of the compasses where the eighth parallel (counting upwards) cuts the seventh vertical line, and extend the other point to the intersection of the same eighth parallel with the fifth diagonal.

Set off the extent of 7.85 thus found on the line.

Example 4. To measure by the diagonal scale a line that is already drawn.

Take the extent of the line in the compasses, place one foot on the first vertical line, that will bring the other foot among the diagonals; move both feet upwards until one of them falls into the point where the diagonal from the nearest tenth cuts the same parallel as is cut by the other on the vertical line; then one foot shares the chain, and the other the hundredth part, or odd links. Then if one foot is on the eighth diagonal of the fourth parallel, while the other is on the same parallel, but coincides with the twelfth vertical, we have 12 chains 48 links, or 12.48 links.

PLOTTING SCALES.

Plotting scales are made either of ivory or boxwood: the ivory is preferable, the divisions being clearer; they are generally 12 inches long, and are distinguished by the number of chains to the inch; they are numbered numerically, and subdivided into 10 parts, each part equal to 10 links.

There should never be two scales on the same ivory, and the zeros always at the same end; the scale can then be used right or left.

It is required frequently to have a scale of feet; those scales should be separate from the general plotting scales. It will be found convenient to have a set of 6-inch scales, similar to Fig. 1, Plate 41, also a set of off-set scales, with zero in the middle.

STRAIGHT EDGE.

However simple it may appear, it is very difficult to obtain a straight edge, or long rule, of great length. It is one of the most important parts on laying down the principal lines of an extensive survey; equal to that in chaining a straight line in the field.

To prove the accuracy of a straight edge before using it, lay it on the paper prepared for the survey, fix two needles, place the edge against them and draw a very fine pencil line,

mark on it the place of the needles, then reverse the rule, and draw another line; if correct, there will be a parallel line equal the thickness of the needles. When it is not in use it should be hung up out of the sun or draughts of wind, both having an influence to make it imperfect.

By stretching a fine thread from point to point will also prove a line, but there is much difficulty in pricking off a line from a thread; under all circumstances, a steel straight edge is preferable.

THE LEVEL STAFF.

***Fig.* 4, *Plate* 38. The best staves for strength and accuracy are those that are made solid, about 16 or 18 feet long and about $\frac{1}{32}$ inches wide, divided into three lengths for convenience of carriage, having two strong ferrule joints, the bottom shod with iron.**

Each foot is divided in 10 parts, and each part into 5 equal or two hundredths, which are alternately painted black and white.

Every $\frac{5}{10}$ are changed from right to left; the figures denoting feet are painted red, or by a circle, as large as the staff will admit, leaving the figure white.

The staff should be made thicker in the middle and gradually diminish, most at the top, and made of sufficient strength to prevent its bending in windy weather, as in taking the levels very frequently considerable difference is made by the curvature of the staff, which cannot be detected by the person taking the levels: a slight staff is at all times objectionable.

THE SECTOR.

Figs. 5 *and* 6, *Plate* 41. Amidst the variety of mathematical instruments that have been contrived to facilitate the art of drawing, there is none so extensive in its use, or of such general application, as the sector.

It is a universal scale, uniting, as it were, angles and parallel lines, the rule and the compasses, which are the only means used in geometry for measuring, whether in theory or practice.

The real inventor of this valuable instrument is unknown, yet of so much merit has the invention appeared, that it was claimed by Galileo and disputed by nations.

This instrument derives its name from the tenth definition of the third book of Euclid, where he defines the sector of a circle.

It is formed of two equal rules, called legs. These legs are movable about the centre, C, of a joint, *d e f*, and will, consequently, by their different openings, represent every possible variety of plane angles. The distance of the extremity of these are the subtenses or chords of the arches they describe.

Sectors are made of different sizes; it is denominated by the length of the sector when shut. Those usually placed in a case of instruments are 6-inch sectors. The most perfect are made of brass; the larger it is the more correct. Two sectors in many cases are exceedingly useful, if not absolutely necessary.

The sectoral lines or scales, Fig. 3, are graduated from the centre, and are—1. Two scales of equal parts, one on each leg, marked LIN, or L; each of these scales, from the great extensiveness of its use, is called the line of lines. 2. Two lines of chords, marked CHO, or C. 3. Two lines of secants, marked SEC, or S. A line of polygons, marked POL. Upon the other face, Fig. 4, the sectoral lines are—1. Two lines of sines, marked SIN, or S. 2. Two lines of tangents, marked TAN, or T. 3. Between the line of tangents and sines there is another line of tangents to a less radius to supply the defect of the former, and extending from 45° to 75°.

Each pair of these lines (except the line of polygons) is so

adjusted as to make equal angles at the centre, and consequently at whatever distance the sector is opened the angles will always be respectively equal; that is, the distance between 10 and 10 on the line of lines will be equal to 60 and 60 on the line of chords, 90 and 90 on the line of sines, and 45 and 45 on the line of tangents.

Beside the sectoral scales, there are others on each face placed parallel to the outward edges, and used as those of the common plain scales.

There are on the face of Fig. 3: 1. A line of inches. 2. A line of latitudes. 3. A line of hours. 4. A line of inclination of meridians. 5. A line of chords.

On the face of Fig. 4, three logarithmic scales—namely, one of numbers, one of sines, and one of tangents; these are used when the sector is fully opened, the legs forming one line.

To read and estimate the division on the sectoral lines.

The value of the divisions on most of the lines is determined by the figures adjacent to them; these proceed by tens, which constitute the divisions of the first order, and are numbered accordingly; but the value of the divisions on the line of lines distinguished by figures is entirely arbitrary, and may represent any value that is given to them; hence the figures 1, 2, 3, 4, &c., may denote either 10, 20, 30, 40, &c., or 100, 200, 300, and so on.

The line of lines is divided into ten equal parts, 1, 2, 3, to 10; these may be called divisions of the first order; each of these are again subdivided into ten other equal parts, which may be called divisions of the second order; and each of these is divided into two equal parts, forming divisions of the third order.

The divisions on all the scales are contained between four parallel lines; those of the first order extend to the most distant, those of the third to the least, and those of the second to the intermediate parallel.

When the whole line of lines represents 100, the divisions of the first order, or those to which the figures are annexed, represent tens; those of the second order, units; those of the third order, the halves of these units. If the whole line represents ten, then the divisions of the first order are units; those of the second, tenths; and the third, twentieths.

In the line of tangents, the divisions to which the numbers are affixed are the degrees expressed by those numbers; every fifth degree is denoted by a line somewhat longer than the rest; between every number and each fifth degree there are four divisions longer than the intermediate adjacent ones; these are whole degrees; the shorter ones are those of the third order, or 30 minutes.

From the centre to 60 degrees the line of sines is divided like the line of tangents; from 60 to 70, it is divided only to every degree; from 70 to 80, to every two degrees; from 80 to 90, the divisions must be estimated by the eye.

The divisions on the line of chords are to be estimated in the same manner as the tangents.

The lesser line of tangents is graduated every two degrees from 45° to 50°, but from 50 to 60 to every degree, from 60 to the end to half degrees.

The line of secants from 0 to 10 is to be estimated by the eye; from 20 to 50, it is divided to every two degrees; from 50 to 60, to every degree; and from 60 to the end, to every half degree.

Of the general law or foundation of sectoral lines.

Let C A, C B, Fig. 7, Plate 41, represent a pair of sectoral lines (*ex gr.* those of the line of lines) forming the angle A C B; divide each of these lines into four equal parts in the points H D F A, I E G B; draw the lines H I, D E, F G, A B; then, because C A, C B are equal, their sections are also equal, the triangles are equiangular, having a common angle at C and equal angles at the base; and therefore the sides about the equal angles will be proportionals, for as C H is to C A so is

H I to A B, and, therefore, as C A is to C H so is A B to H I, and, consequently, as C H is to H I so is C A to A B; and thence, if C H be one-fourth of C A, H I will be one-fourth of A B, and so on of all other sections.

Hence, as in all the operations on the sectoral lines, there are two triangles, both isosceles and equiangled; isosceles because the pair of sectoral lines are equal by construction, and equiangled because of the common angle at the centre; the sides encompassing the equal angles are, therefore, proportional.

Hence, also, if the lines C A, C B represent the line of chords, sines, or tangents—that is, if C A, A B be the radius, and the line C F the chord, sine, or tangent of any proposed number of degrees, then the line F G will be the chord, sine, or tangent of the same number of degrees to the radius A B.

Of the general mode of using sectoral lines.

It is necessary to explain in this place a few terms either used by other writers in their description of the sector, or such as we may occasionally use ourselves.

The solution of the question on the sector is said to be simple when the work is begun and ended on the same line; compound, when the operation begins on one line and finishes on the other.

The operation varies also by the manner in which the compasses are applied to the sector. If a measure be taken on any of the sectoral lines beginning at the centre, it is called a lateral distance; but if the measure be taken from any point in one line, to its corresponding point on the line of the same denomination on the other leg, it is called the transverse or parallel distance.

The divisions of each sectoral line are bounded by three parallel lines; the innermost of these is that on which the points of the compasses are to be placed, because this alone is the line which goes to the centre, and is alone, therefore, the sectoral line.

We shall now proceed to give a few general instances of the

manner of operating with the sector, and then proceed to practical geometry, exemplified in the progress of the work.

Multiplication of the Line of Lines.

Make the lateral distance of one of the factors the parallel distance of 10, then the parallel distance of the other factor is the product.

Example. Multiply 5 by 6. Extend the compasses from the centre of the sector to 5 on the primary divisions, and open the sector until the distance becomes the parallel distance from 10 to 10 on the same divisions; then the parallel distance from 6 to 6 extended from the centre of the sector shall reach to 3, which is now to be reckoned 30; at the same opening of the sector the parallel distance of 7 shall reach from the centre to 35; that of 8 shall reach from the centre to 40, &c.

Division by the Line of Lines.

Make the lateral distance of the dividend the parallel distance of the divisor, the parallel distance of 10 is the quotient.

Thus, to divide 30 by 5, make the lateral distance of 30—viz. 3 on the primary divisions the parallel distance of 5 on the same division; then the parallel distance of 10 extended from the centre shall reach to 6.

Proportion by the Line of Lines.

Make the lateral distance of the second term the parallel distance of the first term; the parallel distance of the third term is the fourth proportional.

Example. To find a fourth proportional to 8, 4, and 6.

Take the lateral distance of 4, and make it the parallel distance of 8; then the parallel distance of 6 extended from the centre shall reach the fourth proportional, 3.

In the same manner a third proportional is found to two numbers. Thus: to find a third proportional to 8 and 4.

The sector remaining as in the former example, the parallel

distance of 4 extended from the centre shall reach to the third proportional, 2.

In all these cases, if a number to be made a parallel distance be too great for the sector, some aliquot part of it is to be taken, and the answer multiplied by the number by which the first number was divided. Thus: if it were required to find a fourth proportional to 4, 8, and 6, because the lateral distance of the second term 8 cannot be made the parallel distance of the first term 4, take the lateral distance of 4, viz. half of 8, and make it the parallel distance of the first term 4; then the parallel distance of the third term 6 shall reach from the centre to 6, viz. half of 12. Any other aliquot part of a number may be used in the same way. In like manner, if the number proposed be too small to be made the parallel distance, it may be multiplied by some number, and the answer divided by the same.

To Protract Angles by the Line of Chords.

Case 1. When the given degrees are under 60.

1. With any radius A B (Fig. 8, Plate 41), on A as a centre, describe the arc B G.

2. Make the same radius a transverse distance between 60 and 60 on the line of chords.

3. Take out the transverse distance of the given degrees, and lay this on the arc from B towards G, which will mark out the angular distance required.

Case 2. When the given degrees are more than 60.

Open the sector and describe the arc as before; take $\frac{1}{2}$ or $\frac{1}{3}$ of the given degrees, and take the transverse distance of this $\frac{1}{2}$ or $\frac{1}{3}$; lay it off from B towards G twice, if the degrees were halved; three times, if the third was used as a transverse distance.

Case 3. When the required angle is less than 6 degrees.

Suppose 3: 1. Open the sector to the given radius, and describe the arc as before. 2. Set off the radius from B to C. 3. Set off the chord of 57 degrees backwards from C to *f*, which will give the arc *f* B of three degrees.

Some uses of the Sectoral Scale of Sines, Tangents, and Secants.

Given the radius of a circle (suppose equal to 2 inches), required the sine and tangent of 28° 30′ to that radius.

Solution. Open the sector so that the transverse distance of 90 and 90 on the sines, or of 45 and 45 on the tangents, may be equal to the given radius, viz. 2 inches; then will the transverse distance of 38° 30′, taken from the sines, be the length of that sine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the secant of 28° 30′ was required?

Make the given radius, 2 inches, a transverse distance to O and O, at the beginning of the line of secants; and then take the transverse distance of the degrees wanted, viz. 28° 30′.

A tangent greater than 45° (suppose 60°) is found thus:

Make the given radius (suppose 2 inches) a transverse distance to 45 and 45 at the beginning of the scale of upper tangents; and then the required number 60° 00′ may be taken from this scale.

The scales of upper tangents and secants do not run quite to 76 degrees; and as the tangent and secant may be sometimes wanted to a greater number of degrees than can be introduced on the sector, they may be readily found by the help of the annexed table of the natural tangents and secants of the degrees above 75; the radius of the circle being unity.

Degrees.	Nat. tangent.	Nat. secant.
76	4.011	4.133
77	4.331	4.445
78	4.701	4.810
79	5.144	5.241
80	5.671	5.759
81	6.314	6.392
82	7.115	7.185
83	8.144	8.205
84	9.514	9.567
85	11.430	11.474
86	14.301	14.335
87	19.081	19.107
88	28.636	28.654
89	57.290	57.300

Measure the radius of the circle used upon any scale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or secant sought, to be taken from the same scale of equal parts.

Example. Required the length of the tangent and se-

cant of 80 degrees to a circle, whose radius, measured on a scale of 25 parts to an inch, is 47½ of those parts?

	Tan.	Sec.
Against 80° stands . . .	5.671	5.759
The radius is	47.5	47.5
	28355	28795
	39697	40313
	22684	23036
	269.3725	273.5525

So the length of the tangent on the twenty-five scale will be 269⅓, and that of the secant about 273½. Or thus: the tangent of any number of degrees may be taken from the sector at once, if the radius of the circle can be made a transverse distance to the complement of those degrees on the lower tangent.

Example. To find the tangent of 78 degrees to a radius of 2 inches.

Make 2 inches a transverse distance to 12 degrees on the lower tangents; then the transverse distance of 45 degrees will be the tangent of 78 degrees.

In like manner the secant of any number of degrees may be taken from the sines, if the radius of the circle can be made a transverse distance to the co-sine of those degrees; thus making 2 inches a transverse distance to the sine of 12 degrees; then the transverse distance of 90 and 90 will be the secant of 78 degrees.

From hence it will be easy to find the degrees answering to a given line, expressing the length of a tangent or secant, which is too long to be measured on those scales when the sector is set to the given radius.

Thus, for a Tangent,

Make the given line a transverse distance to 45 and 45 on the lower tangents, and take the given radius and apply it to the lower tangents, and the degrees where they become a transverse distance will be the co-tangent of the degrees answering to the given line.

And for a Secant,

Make the given line a transverse distance to 90 and 90 on the sines; then the degrees answering to the given radius, applied as a transverse distance on the sines, will be the co-sine of the degrees answering to the given secant line.

Given the length of the sine, tangent, or secant of any number of degrees, to find the length of the radius to that sine, tangent, or secant.

Make the given length a transverse distance to its given degrees on its respective scale; then,

In the sines, the transverse distance of 90 and 90 will be the radius sought.

In the lower tangents, the transverse distance of 45 and 45 near the end of the sector will be the radius sought.

In the upper tangents, the transverse distance of 45 and 45, taken towards the centre of the sector on the line of the upper tangents, will be the centre sought.

In the secant, the transverse distance of 0 and 0, or the beginning of the secants near the centre of the sector, will be the radius sought.

Given the radius, and any line representing a sine, tangent, or secant, to find the degrees corresponding to that line.

Solution. Set the sector to the given radius, according as a sine, or tangent, or secant is concerned.

Take the given line between the compasses, apply the two feet transversely to the scale concerned, and slide the feet along until they both rest on like divisions on both legs; then will those divisions show the degrees and parts corresponding to the given line.

To find the length of a versed sine to a given number of degrees and a given radius.

Make the transverse distance of 90 and 90 on the sines equal to the given radius; take the transverse distance of the sine complement of the given degrees. If the given degrees are less than 90°, the difference between the sine complement and

the radius gives the versed sine. If the given degrees are more than 90°, the sum of the sine complement and the radius give the versed sine.

To open the legs of the sector, so that the corresponding double scales of lines, chords, sines, and tangents may make each a right angle.

On the lines. Make the lateral distance 10 a distance between eight on one leg and six on the other leg.

On the sines. Make the lateral distance 90 a transverse distance from 45 to 45, or from 40 to 50, or from 30 to 60, or from the sine of any number of degrees to their complement.

Or on the sines. Make the lateral distance of 45, or transverse distance between 30 and 30.

PART VI.

TABLE I.

LOGARITHMS OF NUMBERS.

EXPLANATION.

The first column shows the number.
The second column the logarithm to that number.
The third column shows the difference between the two logarithms adjoining.

Nos.	Log.		Nos.	Log.		Nos.	Log.	Diff.
1	0.000000		51	1.707570		101	2.004321	4279
2	0.301030		2	1.716003		2	2.008600	4237
3	0.477121		3	1.724276		3	2.012837	4196
4	0.602060		4	1.732394		4	2.017033	4156
5	0.698970		5	1.740363		5	2.021189	4117
6	0.778151		6	1.748188		6	2.025306	4078
7	0.845098		7	1.755875		7	2.029384	4040
8	0.903090		8	1.763428		8	2.033424	4002
9	0.954243		9	1.770852		9	2.037426	3967
10	1.000000		60	1.778151		110	2.041393	3930
1	1.041393		1	1.785330		1	2.045323	3895
2	1.079181		2	1.792392		2	2.049218	3880
3	1.113943		3	1.799341		3	2.053078	3827
4	1.146128		4	1.806180		4	2.056905	3793
5	1.176091		5	1.812913		5	2.060698	3760
6	1.204120		6	1.819544		6	2.064458	3728
7	1.230449		7	1.826075		7	2.068186	3692
8	1.255273		8	1.832509		8	2.071882	3665
9	1.278754		9	1.838849		9	2.075547	3634
20	1.301030		70	1.845098		120	2.079181	3604
1	1.322219		1	1.851258		1	2.082785	3574
2	1.342423		2	1.857332		2	2.086360	3545
3	1.361728		3	1.863323		3	2.089905	3517
4	1.380211		4	1.869232		4	2.093422	3488
5	1.397940		5	1.875061		5	2.096910	3460
6	1.414973		6	1.880814		6	2.100371	3433
7	1.431364		7	1.886491		7	2.103804	3406
8	1.447158		8	1.892095		8	2.107210	3380
9	1.462398		9	1.897627		9	2.110590	3353
30	1.477121		80	1.903090		130	2.113943	3328
1	1.491362		1	1.908485		1	2.117271	3303
2	1.505150		2	1.913814		2	2.120574	3278
3	1.518514		3	1.919078		3	2.123852	3253
4	1.531479		4	1.924279		4	2.127105	3229
5	1.544068		5	1.929419		5	2.130334	3205
6	1.556303		6	1.934498		6	2.133539	3182
7	1.568202		7	1.939519		7	2.136721	3158
8	1.579784		8	1.944483		8	2.139879	3136
9	1.591065		9	1.949390		9	2.143015	3113
40	1.602060		90	1.954243		140	2.146128	3091
1	1.612784		1	1.959041		1	2.149219	3069
2	1.623249		2	1.963788		2	2.152288	3048
3	1.633468		3	1.968483		3	2.155336	3026
4	1.643453		4	1.973128		4	2.158362	3006
5	1.653213		5	1.977724		5	2.161368	2985
6	1.662758		6	1.982271		6	2.164353	2964
7	1.672098		7	1.986772		7	2.167317	2945
8	1.681241		8	1.991226		8	2.170262	2924
9	1.690196		9	1.995635		9	2.173186	2905
50	1.698970		100	2.000000		150	2.176091	2886

Nos.	Log.	Diff.	Nos.	Log.	Diff.	Nos.	Log.	Diff.
151	2.178977	2867	201	2.303196	2155	251	2.399674	1727
2	2.181844	2847	2	2.305351	2145	2	2.401401	1720
3	2.184691	2830	3	2.307496	2134	3	2.403121	1713
4	2.187521	2811	4	2.309630	2124	4	2 404834	1706
5	2.190332	2793	5	2.311754	2113	5	2.406540	1700
6	2.193125	2775	6	2.313867	2103	6	2.408240	1693
7	2.195900	2757	7	2.315970	2093	7	2.409933	1687
8	2.198657	2740	8	2.318063	2083	8	2.411620	1680
9	2.201397	2723	9	2.320146	2073	9	2.413300	1673
160	2.204120	2706	210	2.322219	2063	260	2.414973	1668
1	2.206826	2689	1	2.324282	2054	1	2.416641	1660
2	2.209515	2673	2	2.326336	2044	2	2.418301	1655
3	2.212188	2656	3	2.328380	2034	3	2.419956	1648
4	2.214844	2640	4	2.330414	2024	4	2.421604	1642
5	2.217484	2624	5	2.332438	2016	5	2.423246	1636
6	2.220108	2608	6	2.334454	2006	6	2.424882	1629
7	2.222716	2593	7	2.336460	1996	7	2.426511	1624
8	2.225309	2578	8	2.338456	1988	8	2.428135	1618
9	2.227887	2562	9	2.340444	1979	9	2.429753	1611
170	2.230449	2547	220	2.342423	1969	270	2.431364	1605
1	2.232996	2532	1	2.344392	1961	1	2.432969	1600
2	2.235528	2518	2	2.346353	1952	2	2.434569	1594
3	2.238046	2503	3	2.348305	1943	3	2.436163	1588
4	2.240549	2489	4	2.350248	1935	4	2.437751	1582
5	2.243038	2475	5	2.352183	1925	5	2.439333	1576
6	2.245513	2460	6	2.354108	1918	6	2.440909	1571
7	2.547973	2447	7	2.356026	1909	7	2.442480	1565
8	2.250420	2463	8	2.357935	1900	8	2.444045	1559
9	2.252853	2420	9	2.359835	1893	9	2.445604	1554
180	2.255273	2406	230	2.361728	1884	280	2.447158	1548
1	2.257679	2392	1	2.363612	1876	1	2.448706	1543
2	2.260071	2380	2	2.365488	1868	2	2.450249	1537
3	2.262451	2367	3	2.367356	1860	3	2.451786	1532
4	2.264818	2354	4	2.369216	1852	4	2.453318	1527
5	2.267172	2341	5	2.371068	1844	5	2.454845	1521
6	2.269513	2329	6	2.372912	1836	6	2.456366	1516
7	2.271842	2316	7	2.374748	1829	7	2.457882	1510
8	2.274158	2304	8	2.376577	1821	8	2.459392	1506
9	2.276462	2292	9	2.378398	1813	9	2.460898	1500
190	2.278754	2279	240	2.380211	1806	290	2.462398	1495
1	2.281033	2268	1	2.382017	1798	1	2.463893	1490
2	2.283301	2256	2	2.383815	1796	2	2.465383	1485
3	2.285557	2245	3	2.385606	1784	3	2.466868	1479
4	2.287802	2233	4	2.387390	1776	4	2.468347	1475
5	2.290035	2221	5	2.389166	1769	5	2.469822	1470
6	2.292256	2210	6	2.390935	1762	6	2.471292	1464
7	2.294466	2199	7	2.392697	1755	7	2.472756	1460
8	2.296665	2188	8	2.394452	1747	8	2.474216	1455
9	2.298853	2177	9	2.396199	1741	9	2.475671	1450
200	2.301030	2166	250	2.397940	1734	300	2.477121	1445

Nos.	Log.	Diff.	Nos.	Log.	Diff.	Nos.	Log.	Diff.
301	2.478566	1441	351	2.545307	1236	401	2.603144	1082
2	2.480007	1436	2	2.546543	1232	2	2.604226	1079
3	2.481443	1431	3	2.547775	1228	3	2.605305	1076
4	2.482874	1426	4	2.549003	1225	4	2.606381	1074
5	2.484300	1421	5	2.550228	1222	5	2.607455	1071
6	2.485721	1417	6	2.551450	1218	6	2.608526	1068
7	2.487138	1413	7	2.552668	1215	7	2.609594	1066
8	2.488551	1407	8	2.553883	1211	8	2.610660	1063
9	2.489958	1404	9	2.555094	1209	9	2.611723	1061
310	2.491362	1398	360	2.556303	1204	410	2.612784	1058
1	2.492760	1395	1	2.557507	1202	1	2.613842	1055
2	2.494155	1389	2	2.558709	1198	2	2.614897	1053
3	2.495544	1386	3	2.559907	1194	3	2.615950	1050
4	2.496930	1381	4	2.561101	1192	4	2.617000	1048
5	2.498311	1376	5	2.562293	1188	5	2.618048	1045
6	2.499687	1372	6	2.563481	1185	6	2.619093	1043
7	2.501059	1368	7	2.564666	1182	7	2.620136	1040
8	2.502427	1364	8	2.565848	1178	8	2.621176	1038
9	2.503791	1359	9	2.567026	1176	9	2.622214	1035
320	2.505150	1355	370	2.568202	1172	420	2.623249	1033
1	2.506505	1351	1	2.569374	1169	1	2.624282	1030
2	2.507856	1347	2	2.570543	1166	2	2.625312	1028
3	2.509203	1342	3	2.571709	1163	3	2.626340	1026
4	2.510545	1338	4	2.572872	1159	4	2.627367	1023
5	2.511883	1335	5	2.574031	1157	5	2.628389	1021
6	2.513218	1330	6	2.575188	1153	6	2.629410	1018
7	2.514548	1326	7	2.576341	1151	7	2.630428	1016
8	2.515874	1322	8	2.577492	1147	8	2.631444	1013
9	2.517196	1318	9	2.578639	1144	9	2.632457	1011
330	2.518514	1314	380	2.579784	1141	430	2.633468	1006
1	2.519828	1310	1	2.580925	1138	1	2.634477	1007
2	2.521138	1306	2	2.582063	1136	2	2.635484	1004
3	2.522444	1302	3	2.583199	1133	3	2.636488	1002
4	2.523746	1299	4	2.584331	1130	4	2.637490	999
5	2.525045	1294	5	2.585461	1126	5	2.638489	997
6	2.526339	1291	6	2.586587	1124	6	2.639486	995
7	2.527630	1287	7	2.587711	1121	7	2.640481	993
8	2.528917	1283	8	2.588832	1118	8	2.641474	991
9	2.530200	1279	9	2.589950	1115	9	2.642465	988
340	2.531479	1275	390	2.591065	1112	440	2.643453	986
1	2.532754	1272	1	2.592177	1109	1	2.644439	983
2	2.534026	1268	2	2.593286	1107	2	2.645422	982
3	2.535294	1264	3	2.594393	1103	3	2.646404	979
4	2.536558	1261	4	2.595496	1101	4	2.647383	977
5	2.537819	1257	5	2.596597	1098	5	2.648360	975
6	2.539076	1253	6	2.597695	1096	6	2.649335	972
7	2.540329	1250	7	2.598791	1092	7	2.650307	971
8	2.541579	1246	8	2.599883	1090	8	2.651278	968
9	2.542825	1243	9	2.600973	1087	9	2.652246	967
350	2.544068	1239	400	2.602060	1084	450	2.653213	964

Nos.	Log.	Diff.	Nos.	Log.	Diff.	Nos.	Log.	Diff.
451	2.654177	961	501	2.699838	866	551	2.741152	787
2	2.655138	960	2	2.700704	864	2	2.741939	786
3	2.656098	958	3	2.701568	863	3	2.742725	785
4	2.657056	955	4	2.702431	860	4	2.743510	783
5	2.658011	954	5	2.703291	860	5	2.744293	782
6	2.658965	951	6	2.704151	857	6	2.745075	780
7	2.659916	949	7	2.705008	856	7	2.745855	779
8	2.660865	948	8	2.705864	854	8	2.746634	778
9	2.661813	945	9	2.706718	852	9	2.747412	776
460	2.662758	943	510	2.707570	851	560	2.748188	775
1	2.663701	941	1	2.708421	849	1	2.748963	773
2	2.664642	939	2	2.709270	847	2	2.749736	772
3	2.665581	937	3	2.710117	846	3	2.750508	771
4	2.666518	935	4	2.710963	844	4	2.751279	769
5	2.667453	933	5	2.711807	843	5	2.752048	768
6	2.668386	931	6	2.712650	841	6	2.752816	767
7	2.669317	929	7	2.713491	839	7	2.753583	765
8	2.670246	927	8	2.714330	837	8	2.754348	764
9	2.671173	925	9	2.715167	836	9	2.755112	763
470	2.672098	923	520	2.716003	835	570	2.755875	761
1	2.673021	921	1	2.716838	833	1	2.756636	760
2	2.673942	919	2	2.717671	831	2	2.757396	759
3	2.674861	917	3	2.718502	829	3	2.758155	757
4	2.675778	916	4	2.719331	828	4	2.758912	756
5	2.676694	913	5	2.720159	827	5	2.759668	754
6	2.677607	911	6	2.720986	825	6	2.760422	754
7	2.678518	910	7	2.721811	823	7	2.761176	752
8	2.679428	908	8	2.722634	822	8	2.761928	751
9	2.680336	905	9	2.723456	820	9	2.762679	749
480	2.681241	904	530	2.724276	819	580	2.763428	748
1	2.682145	902	1	2.725095	817	1	2.764176	747
2	2.683047	900	2	2.725912	815	2	2.764923	746
3	2.683947	898	3	2.726727	814	3	2.765669	744
4	2.684845	897	4	2.727541	813	4	2.766413	743
5	2.685742	894	5	2.728354	811	5	2.767156	742
6	2.686636	893	6	2.729165	809	6	2.767898	740
7	2.687529	891	7	2.729974	808	7	2.768638	739
8	2.688420	889	8	2.730782	807	8	2.769377	738
9	2.689309	887	9	2.731589	805	9	2.770115	737
490	2.690196	885	540	2.732394	803	590	2.770852	735
1	2.691081	884	1	2.733197	802	1	2.771587	735
2	2.691965	882	2	2.733999	801	2	2.772322	733
3	2.692847	880	3	2.734800	799	3	2.773055	731
4	2.693727	878	4	2.735599	798	4	2.773786	730
5	2.694605	877	5	2.736397	796	5	2.774517	729
6	2.695482	874	6	2.737193	794	6	2.775246	728
7	2.696356	873	7	2.737987	798	7	2.775974	727
8	2.697229	872	8	2.738781	791	8	2.776701	726
9	2.698101	870	9	2.739572	790	9	2.777427	724
500	2.698970	867	550	2.740363	789	600	2.778151	723

Nos.	Log.	Diff.	Nos.	Log.	Diff.	Nos.	Log.	Diff.
601	2.778874	722	651	2.813581	666	701	2.845718	619
2	2.779596	721	2	2.814248	665	2	2.846337	618
3	2.780317	720	3	2.814913	664	3	2.846955	617
4	2.781037	718	4	2.815578	663	4	2.847573	616
5	2.781755	718	5	2.816241	662	5	2.848189	615
6	2.782473	716	6	2.816904	661	6	2.848805	614
7	2.783189	715	7	2.817565	660	7	2.849419	614
8	2.783904	713	8	2.818225	659	8	2.850033	613
9	2.784617	712	9	2.818885	658	9	2.850646	612
610	2.785330	711	660	2.819544	657	710	2.851258	611
1	2.786041	710	1	2.820202	656	1	2.851870	610
2	2.786751	709	2	2.820858	655	2	2.852480	609
3	2.787460	708	3	2.821514	654	3	2.853090	608
4	2.788168	707	4	2.822168	653	4	2.853698	608
5	2.788875	706	5	2.822822	652	5	2.854306	607
6	2.789581	704	6	2.823474	651	6	2.854913	606
7	2.790285	703	7	2.824126	650	7	2.855519	605
8	2.790988	702	8	2.824777	649	8	2.856124	604
9	2.791691	701	9	2.825426	648	9	2.856729	603
620	2.792392	700	670	2.826075	647	720	2.857333	602
1	2.793092	699	1	2.826723	646	1	2.857935	602
2	2.793790	698	2	2.827369	645	2	2.858537	601
3	2.794488	697	3	2.828015	644	3	2.859138	600
4	2.795185	695	4	2.828660	643	4	2.859739	599
5	2.795880	694	5	2.829304	642	5	2.860338	599
6	2.796574	693	6	2.829947	641	6	2.860937	597
7	2.797268	692	7	2.830589	640	7	2.861534	597
8	2.797960	991	8	2.831230	639	8	2.862131	596
9	2.798651	690	9	2.831870	638	9	2.862728	595
630	2.799341	689	680	2.832509	638	730	2.863323	594
1	2.800029	688	1	2.833147	636	1	2.863917	594
2	2.800717	687	2	2.833784	636	2	2.864511	593
3	2.801404	685	3	2.834421	635	3	2.865104	592
4	2.802089	684	4	2.835056	634	4	2.865696	591
5	2.802774	683	5	2.835691	633	5	2.866287	591
6	2.803457	682	6	2.836324	632	6	2.866878	590
7	2.804139	681	7	2.836957	631	7	2.867468	589
8	2.804821	680	8	2.837588	630	8	2.868056	588
9	2.805501	679	9	2.838219	629	9	2.868644	588
640	2.806180	678	690	2.838849	628	740	2.869232	586
1	2.806858	677	1	2.839478	628	1	2.869818	586
2	2.807535	676	2	2.840106	627	2	2.870404	585
3	2.808211	675	3	2.840733	626	3	2.870989	584
4	2.808886	674	4	2.841360	625	4	2.871573	583
5	2.809560	673	5	2.841985	624	5	2.872156	583
6	2.810232	672	6	2.842609	623	6	2.872739	582
7	2.810904	671	7	2.843233	622	7	2.873321	581
8	2.811575	670	8	2.843855	622	8	2.873902	580
9	2.812245	668	9	2.844477	621	9	2.874482	579
650	2.812913	667	700	2.845098	620	750	2.875061	578

Nos.	Log.	Diff.	Nos.	Log.	Diff.	Nos.	Log.	Diff.
751	2.875640	578	801	2.903633	541	851	2.929930	510
2	2.876218	577	2	2.904175	541	2	2.930440	509
3	2.876795	576	3	2.904716	540	3	2.930949	509
4	2.877371	576	4	2.905256	540	4	2.931458	508
5	2.877947	575	5	2.905796	539	5	2.931966	508
6	2.878522	574	6	2.906335	539	6	2.932474	507
7	2.879096	573	7	2.906874	538	7	2.932981	506
8	2.879669	573	8	2.907412	537	8	2.933487	506
9	2.880242	572	9	2.907949	536	9	2.933993	505
760	2.880814	571	810	2.908485	536	860	2.934499	504
1	2.881385	570	1	2.909021	535	1	2.935003	504
2	2.881955	569	2	2.909556	535	2	2.935507	504
3	2.882525	568	3	2.910091	534	3	2.936011	503
4	2.883093	568	4	2.910625	533	4	2.936514	502
5	2.883661	567	5	2.911158	532	5	2.937016	502
6	2.884229	567	6	2.911690	532	6	2.937518	501
7	2.884795	566	7	2.912222	531	7	2.938019	501
8	2.885361	565	8	2.912753	531	8	2.938520	500
9	2.885926	564	9	2.913284	530	9	2.939020	499
770	2.886491	563	820	2.913814	529	870	2.939519	499
1	2.887054	563	1	2.914343	529	1	2.940018	499
2	2.887617	562	2	2.914872	528	2	2.940517	497
3	2.888180	561	3	2.915400	527	3	2.941014	497
4	2.888741	561	4	2.915927	527	4	2.941511	496
5	2.889302	560	5	2.916454	526	5	2.942008	496
6	2.889862	559	6	2.916980	526	6	2.942504	496
7	2.890421	559	7	2.917506	525	7	2.943000	495
8	2.890980	558	8	2.918031	524	8	2.943495	494
9	2.891538	557	9	2.918555	523	9	2.943989	494
780	2.892095	556	830	2.919078	523	880	2.944483	493
1	2.892651	556	1	2.919601	522	1	2.944976	493
2	2.893207	555	2	2.920123	522	2	2.945469	492
3	2.893762	554	3	2.920645	521	3	2.945961	491
4	2.894316	554	4	2.921166	520	4	2.946452	491
5	2.894870	553	5	2.921687	519	5	2.946943	490
6	2.895423	552	6	2.922206	519	6	2.947434	490
7	2.895975	551	7	2.922725	518	7	2.947924	489
8	2.896526	551	8	2.923244	518	8	2.948413	489
9	2.897077	550	9	2.923762	517	9	2.948902	488
790	2.897627	549	840	2.924279	517	890	2.949390	488
1	2.898177	548	1	2.924796	516	1	2.949878	487
2	2.898725	548	2	2.925312	516	2	2.950365	487
3	2.899273	547	3	2.925828	514	3	2.950852	486
4	2.899821	546	4	2.926342	515	4	2.951338	485
5	2.900367	546	5	2.926857	513	5	2.951823	485
6	2.900913	545	6	2.927370	513	6	2.952308	484
7	2.901458	545	7	2.927883	513	7	2.952792	484
8	2.902003	544	8	2.928396	511	8	2.953276	483
9	2.902547	543	9	2.928908	511	9	2.953760	483
800	2.903090	542	850	2.929419	510	900	2.954243	482

Nos.	Log.	Diff.	Nos.	Log.	Diff.	Nos.	Log.	Diff.
901	2.954725	482	936	2.971276	464	971	2.987219	447
2	2.955207	481	7	2.971740	463	2	2.987666	447
3	2.955688	480	8	2.972203	463	3	2.988113	446
4	2.956168	481	9	2.972666	462	4	2.988559	446
5	2.956649	479	940	2.973128	462	5	2.989005	445
6	2.957128	479	1	2.973590	461	6	2.989450	445
7	2.957607	479	2	2.974051	461	7	2.989895	444
8	2.958086	478	3	2.974512	460	8	2.990339	444
9	2.958564	477	4	2.974972	460	9	2.990783	443
910	2.959041	477	5	2.975432	459	980	2.991226	443
1	2.959518	477	6	2.975891	459	1	2.991669	442
2	2.959995	476	7	2.976350	458	2	2.992111	443
3	2.960471	475	8	2.976808	458	3	2.992554	441
4	2.960946	475	9	2.977266	458	4	2.992995	441
5	2.961421	474	950	2.977724	457	5	2.993436	441
6	2.961895	474	1	2.978181	456	6	2.993877	440
7	2.962369	474	2	2.978637	456	7	2.994317	440
8	2.962843	473	3	2.979093	455	8	2.994757	439
9	2.963316	472	4	2.979548	455	9	2.995196	439
920	2.963788	472	5	2.980003	455	990	2.995635	438
1	2.964260	471	6	2.980458	454	1	2.996074	438
2	2.964731	471	7	2.980912	454	2	2.996512	437
3	2.965202	470	8	2.981366	453	3	2.996949	437
4	2.965672	470	9	2.981819	452	4	2.997386	437
5	2.966142	469	960	2.982271	452	5	2.997823	436
6	2.966611	469	1	2.982723	452	6	2.998259	436
7	2.967080	468	2	2.983175	451	7	2.998695	436
8	2.967548	468	3	2.983626	451	8	2.999131	435
9	2.968016	467	4	2.984077	450	9	2.999566	434
930	2.968483	467	5	2.984527	450	1000	3.000000	
1	2.968950	466	6	2.984977	449			
2	2.969416	466	7	2.985426	449			
3	2.969882	465	8	2.985875	449			
4	2.970347	465	9	2.986324	448			
935	2.970812	464	970	2.986772	447			

TABLE II.—LOGARITHMIC SINES AND CO-SINES.

Explanation to the Tables of Sines and Tangents.

The top column shows the minutes; the left hand column the number of degrees; the co-sines and co-tangents read upwards.

Logaritmic Sines.														
Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
0	0	7.162696	7.463726	7.639816	7.764754	7.861162	7.940842	8.007787	8.065776	8.116926	8.162681	8.204070	8.241855	89
1	8.241855	8.276614	8.308794	8.338753	8.366777	8.393101	8.417919	8.441394	8.463665	8.484848	8.505045	8.524343	8.542819	88
2	8.542819	8.560540	8.577566	8.593948	8.609734	8.624965	8.639680	8.653911	8.667689	8.681043	8.693998	8.706577	8.718800	87
3	8.718800	8.730688	8.742259	8.753528	8.764511	8.775223	8.785675	8.795881	8.805852	8.815599	8.825130	8.834456	8.843585	86
4	8.843585	8.852525	8.861283	8.869868	8.878285	8.886542	8.894643	8.902596	8.910403	8.918073	8.925609	8.933015	8.940296	85
5	8.940296	8.947456	8.954499	8.961429	8.968249	8.974962	8.981573	8.988083	8.994497	9.000816	9.007044	9.013182	9.019235	84
6	9.019235	9.025203	9.031089	9.036896	9.042625	9.048279	9.053859	9.059367	9.064806	9.070176	9.075480	9.080719	9.085895	83
7	9.085895	9.091008	9.096062	9.101056	9.105992	9.110873	9.115698	9.120469	9.125187	9.129854	9.134470	9.139037	9.143555	82
8	9.143555	9.148026	9.152451	9.156830	9.161164	9.165454	9.169702	9.173908	9.178072	9.182196	9.186280	9.190325	9.194332	81
9	9.194332	9.198302	9.202235	9.206131	9.209992	9.213818	9.217609	9.221367	9.225092	9.228784	9.232444	9.236073	9.239670	80
10	9.239670	9.243237	9.246775	9.250282	9.253761	9.257211	9.260633	9.264027	9.267395	9.270735	9.274049	9.277337	9.280599	79
11	9.280599	9.283836	9.287048	9.290236	9.293400	9.296539	9.299655	9.302749	9.305819	9.308869	9.311893	9.314897	9.317879	78
12	9.317879	9.320840	9.323780	9.326700	9.329599	9.332478	9.335337	9.338176	9.340996	9.343797	9.346579	9.349343	9.352088	77
13	9.352088	9.354815	9.357524	9.360215	9.362889	9.365546	9.368185	9.370808	9.373414	9.376003	9.378577	9.381134	9.383675	76
14	9.383675	9.386201	9.388711	9.391206	9.393685	9.396150	9.398600	9.401035	9.403455	9.405862	9.408254	9.410632	9.412996	75
15	9.412996	9.415347	9.417684	9.420007	9.422318	9.424615	9.426899	9.429170	9.431429	9.433675	9.435908	9.438129	9.440338	74
16	9.440338	9.442535	9.444720	9.446893	9.449054	9.451204	9.453342	9.455469	9.457584	9.459688	9.461782	9.463864	9.465935	73
17	9.465935	9.467996	9.470046	9.472086	9.474115	9.476133	9.478142	9.480140	9.482128	9.484107	9.486075	9.488034	9.489982	72
18	9.489982	9.491922	9.493851	9.495772	9.497682	9.499584	9.501476	9.503360	9.505234	9.507099	9.508956	9.510803	9.512642	71
19	9.512642	9.514472	9.516294	9.518107	9.519911	9.521707	9.523495	9.525275	9.527046	9.528810	9.530565	9.532312	9.534052	70
20	9.534052	9.535783	9.537507	9.539223	9.540931	9.542632	9.544325	9.546011	9.547689	9.549360	9.551024	9.552680	9.554329	69
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.
Logarithmic Co-sines.														

TABLE III.—*continued.*

LOGARITHMIC TANGENTS.

Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
21	9.584177	9.586062	9.587941	9.589814	9.591681	9.593542	9.595398	9.597247	9.599091	9.600929	9.602761	9.604588	9.606410	68
22	9.606410	9.608225	9.610036	9.611841	9.613641	9.615435	9.617224	9.619008	9.620787	9.622561	9.624330	9.626093	9.627852	67
23	9.627852	9.629806	9.631355	9.633099	9.634838	9.636572	9.638302	9.640027	9.641747	9.643463	9.645174	9.646881	9.648583	66
24	9.648583	9.650281	9.651974	9.653663	9.655348	9.657028	9.658704	9.660376	9.662043	9.663707	9.665366	9.667021	9.668673	65
25	9.668673	9.670320	9.671963	9.673602	9.657237	9.676869	9.678496	9.680120	9.681740	9.683356	9.684968	9.686577	9.688182	64
26	9.688182	9.689782	9.691381	9.692975	9.694566	9.696153	9.697736	9.699316	9.700893	9.702466	9.704036	9.705603	9.707166	63
27	9.707166	9.708726	9.710282	9.711836	9.713386	9.714933	9.716477	9.718017	9.719555	9.721089	9.722621	9.724149	9.725674	62
28	9.725674	9.727197	9.728716	9.730233	9.731746	9.733257	9.734764	9.736269	9.737771	9.739271	9.740767	9.742261	9.743752	61
29	9.743752	9.745240	9.746726	9.748209	9.749689	9.751167	9.752642	9.754115	9.755585	9.757052	9.758517	9.759979	9.761439	60
30	9.761439	9.762897	9.764352	9.765805	9.767255	9.768703	9.770149	9.771592	9.773033	9.774471	9.775908	9.777342	9.778774	59
31	9.778774	9.780203	9.781631	9.783056	9.784479	9.785900	9.787319	9.788736	9.790151	9.791564	9.792974	9.794383	9.795789	58
32	9.795789	9.797194	9.798596	9.799997	9.801396	9.802793	9.804187	9.805580	9.806971	9.808361	9.809748	9.811134	9.812517	57
33	9.812517	9.813899	9.815280	9.816658	9.818035	9.819410	9.820783	9.822155	9.823524	9.824893	9.826259	9.827624	9.828987	56
34	9.828987	9.830349	9.831709	9.833068	9.834425	9.835780	9.837134	9.838487	9.839837	9.841187	9.842535	9.843882	9.845227	55
35	9.845227	9.846571	9.847913	9.849254	9.850593	9.851931	9.853268	9.854603	9.855938	9.857270	9.858602	9.859932	9.861261	54
36	9.861261	9.862589	9.863915	9.865240	9.866564	9.867887	9.869209	9.870529	9.871849	9.873167	9.874484	9.875800	9.877114	53
37	9.877114	9.878428	9.879741	9.881052	9.882363	9.883672	9.884981	9.886288	9.887594	9.888900	9.890204	9.891507	9.892810	52
38	9.892810	9.894111	9.895412	9.896712	9.898010	9.899308	9.900605	9.901901	9.903197	9.904491	9.905785	9.907077	9.908369	51
39	9.908369	9.909660	9.910951	9.912240	9.913529	9.914817	9.916105	9.917391	9.918677	9.919962	9.921247	9.922530	9.923814	50
40	9.923814	9.925096	9.926378	9.927659	9.928940	9.930220	9.931499	9.932778	9.934056	9.935334	9.936611	9.937887	9.939163	49
41	9.939163	9.940439	9.941714	9.942988	9.944262	9.945535	9.946808	9.948081	9.949353	9.950625	9.951896	9.953167	9.954437	48
42	9.954437	9.955708	9.956978	9.958247	9.959516	9.960784	9.962053	9.963320	9.964588	9.965856	9.967123	9.968389	9.969656	47
43	9.969656	9.970922	9.972188	9.973454	9.974720	9.975984	9.977250	9.978515	9.979780	9.981044	9.982309	9.983573	9.984837	46
44	9.984837	9.986101	9.987365	9.988629	9.989893	9.991156	9.992420	9.993683	9.994947	9.996210	9.997473	9.998737	10.000000	45
45	10.000000	10.001263	10.002527	10.003790	10.005053	10.006317	10.007580	10.008844	10.010107	10.011371	10.012635	10.013899	10.015163	44
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.

LOGARITHMIC CO-TANGENTS.

TABLE III.—*continued.*

LOGARITHMIC TANGENTS.

Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
46	10.015163	10.016427	10.017691	10.018956	10.020220	10.021485	10.022750	10.024015	10.025281	10.026546	10.027812	10.029078	10.030344	43
47	10.030344	10.031611	10.032878	10.034145	10.035412	10.036680	10.037948	10.039216	10.040485	10.041754	10.043023	10.044293	10.045563	42
48	10.045563	10.046833	10.048104	10.049375	10.050647	10.051919	10.053192	10.054465	10.055738	10.057012	10.058287	13.059562	10.060837	41
49	10.060837	10.062113	10.063390	10.064667	10.065944	10.067222	10.068501	10.069781	10.071060	10.072341	10.073622	10.074904	10.076187	40
50	10.076187	10.077470	10.078753	10.080038	10.081323	10.082609	10.083896	10.085183	10.086471	10.087760	10.089049	10.090340	10.091631	39
51	10.091631	10.092923	10.094216	10.095509	10.096803	10.098098	10.099395	10.100692	10.101990	10.103288	10.104588	10.105889	10.107190	38
52	10.107190	10.108493	10.109796	10.111100	10.112406	10.113712	10.115020	10.116328	10.117638	10.118948	10.120259	10.121572	10.122886	37
53	10.122886	10.124200	10.125516	10.126833	10.128151	10.129471	10.130791	10.132113	10.133436	10.134760	10.136085	10.137412	10.138739	36
54	10.138739	10.140068	10.141398	10.142730	10.144062	10.145397	10.146732	10.148069	10.149407	10.150746	10.152087	10.153430	10.154773	35
55	10.154773	10.156118	10.157465	10.158813	10.160162	10.161513	10.162366	10.164220	10.165575	10.166932	10.168291	10.169651	10.171013	34
56	10.171013	10.172376	10.173741	10.175107	10.176476	10.177846	10.179217	10.180590	10.181965	10.183342	10.184721	10.186101	10.187483	33
57	10.187483	10.188866	10.190252	10.191639	10.193029	10.194420	10.195813	10.197208	10.198604	10.200033	10.201404	10.202806	10.204211	32
58	10.204211	10.205617	10.207026	10.208437	10.209849	10.211264	10.212681	10.214100	10.215521	10.216944	10.218869	10.219797	10.221226	31
59	10.221226	10.222658	10.224092	10.225529	10.226967	10.228408	10.229852	10.231297	10.232745	10.234195	10.235648	10.237163	10.238561	30
60	10.238561	10.240021	10.241483	10.242948	10.244415	10.245885	10.247358	10.248833	10.250311	10.251791	10.253274	10.254760	10.256248	29
61	10.256248	10.257739	10.259233	10.260729	10.262229	10.263731	10.265236	10.266743	10.268254	10.269768	10.271284	10.272803	10.274326	28
62	10.274326	10.275851	10.277379	10.278911	10.280445	10.281983	10.283523	10.285067	10.286614	10.288164	10.289719	10.291274	10.292834	27
63	10.292834	10.294397	10.295964	10.297534	10.299107	10.300684	10.302264	10.303847	10.305434	10.307025	10.308619	10.310217	10.311818	26
64	10.311818	10.313423	10.315032	10.316644	10.318260	10.319880	10.321504	10.323131	10.324763	10.326398	10.328037	10.329680	10.331328	25
65	10.331328	10.332979	10.334634	10.336293	10.337957	10.339624	10.341296	10.342972	10.344652	10.346337	10.348026	10.349719	10.351417	24
66	10.351417	10.353119	10.354826	10.356537	10.358253	10.359973	10.361698	10.363428	10.365162	10.366902	10.368646	10.370394	10.372148	23
67	10.372148	10.373907	10.375670	10.377439	10.379213	10.380992	10.382776	10.384565	10.386359	10.388159	10.389964	10.391775	10.393590	22
68	10.393590	10.395412	10.397239	10.399071	10.400909	10.402753	10.404603	10.406458	10.408319	10.410186	10.412059	10.413938	10.415823	21
69	10.415823	10.417714	10.419611	10.421514	10.423424	10.425340	10.427262	10.429191	10.431127	10.433068	10.435017	10.436972	10.438934	20
70	10.438934	10.440903	10.442879	10.444861	10.446851	10.448848	10.450851	10.452862	10.454881	10.456906	10.458939	10.460980	10.463028	19
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.

LOGARITHMIC CO-TANGENTS.

TABLE III.—*continued.*

LOGARITHMIC TANGENTS.

Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
71	10.463028	10.465084	10.467147	10.469219	10.471298	10.473385	10.475480	10.477583	10.479695	10.481815	10.483943	10.486079	10.488224	18
72	10.488224	10.490378	10.492540	10.494711	10.496891	10.499080	10.501278	10.503485	10.505701	10.507927	10.510162	10.512407	10.514661	17
73	10.514661	10.516925	10.519199	10.521483	10.523777	10.526081	10.528395	10.530720	10.533055	10.535401	10.537758	10.540125	10.542504	16
74	10.542504	10.544893	10.547294	10.549706	10.552130	10.554565	10.557012	10.559471	10.561941	10.564424	10.566920	10.569427	10.571948	15
75	10.571948	10.574481	10.577027	10.579585	10.582158	10.584743	10.587342	10.589955	10.592581	10.595222	10.597876	10.600545	10.603229	14
76	10.603229	10.605927	10.608641	10.611369	10.614112	10.616872	10.619646	10.622437	10.625244	10.628067	10.630906	10.633763	10.636636	13
77	10.636636	10.639526	10.642434	10.645360	10.648303	10.651265	10.654245	10.657243	10.660261	10.663298	10.666354	10.669430	10.672526	12
78	10.672526	10.675642	10.678778	10.681936	10.685115	10.688315	10.691537	10.694782	10.698049	10.701338	10.704651	10.707987	10.711348	11
79	10.711348	10.714732	10.718142	10.721576	10.725036	10.728521	10.732033	10.735572	10.739138	10.742731	10.746352	10.750002	10.753681	10
80	10.753681	10.757390	10.761128	10.764897	10.768698	10.772529	10.776394	10.780290	10.784221	10.788185	10.792184	10.796218	10.800288	9
81	10.800288	10.804394	10.808538	10.812720	10.816941	10.821201	10.825501	10.829843	10.834226	10.838653	10.843123	10.847637	10.852196	8
82	10.852196	10.856804	10.861458	10.866161	10.870913	10.875716	10.880571	10.885479	10.890441	10.895458	10.900532	10.905665	10.910856	7
83	10.910856	10.916109	10.921424	10.926803	10.932248	10.937760	10.943341	10.948992	10.954716	10.960515	10.966391	10.972345	10.978380	6
84	10.978380	10.984498	10.990702	10.996993	11.003876	11.009851	11.016423	11.023094	11.029867	11.036746	11.043733	11.050832	11.058048	5
85	11.058048	11.065384	11.072844	11.080433	11.088154	11.096013	11.104016	11.112167	11.120471	11.128936	11.137567	11.146372	11.155356	4
86	11.155356	11.164528	11.173897	11.183471	11.193258	11.203269	11.213519	11.224005	11.234754	11.245773	11.257078	11.268683	11.280604	3
87	11.280604	11.292861	11.305471	11.318456	11.331840	11.345648	11.359907	11.374648	11.389906	11.405717	11.422123	11.439172	11.456916	2
88	11.456916	11.475414	11.494733	11.514950	11.536151	11.558440	11.581932	11.606766	11.633106	11.661144	11.691116	11.723309	11.758079	1
89	11.758079	11.795874	11.837273	11.883037	11.934195	12.992191	12.059142	12.138326	12.235239	12.360180	12.536273	12.837304	Infinite	0
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.

LOGARITHMIC CO-TANGENTS.

TABLE IV.—NATURAL SINES AND NATURAL CO-SINES.

Natural Sines.														
Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
0	0	.001454	.002909	.004363	.005818	.007272	.008727	.010181	.011635	.013090	.014544	.015998	.017452	89
1	.017452	.018907	.020361	.021815	.023269	.024723	.026177	.027631	.029085	.030539	.031992	.033446	.034900	88
2	.034899	.036353	.037807	.039260	.040713	.042166	.043619	.045072	.046526	.047978	.049431	.050884	.052336	87
3	.052336	.053788	.055241	.056693	.058145	.059597	.061049	.062500	.063952	.065403	.066854	.068306	.069757	86
4	.069757	.071207	.072658	.074109	.075559	.077009	.078459	.079909	.081359	.082808	.084258	.085707	.087156	85
5	.087156	.088605	.090053	.091502	.092950	.094398	.095846	.097293	.098741	.100188	.101635	.103082	.104529	84
6	.104529	.105975	.107421	.108867	.110313	.111758	.113203	.114648	.116093	.117537	.118982	.120426	.121869	83
7	.121869	.123313	.124756	.126199	.127642	.129084	.130526	.131968	.133410	.134851	.136292	.137733	.139173	82
8	.139173	.140613	.142053	.143493	.144932	.146371	.147809	.149248	.150686	.152123	.153561	.154998	.156435	81
9	.156435	.157871	.159307	.160743	.162178	.163613	.165048	.166482	.167916	.169350	.170783	.172216	.173648	80
10	.173648	.175080	.176512	.177944	.179375	.180805	.182236	.183665	.185095	.186524	.187953	.189381	.190810	79
11	.190809	.192237	.193664	.195090	.196517	.197943	.199368	.200793	.202218	.203642	.205066	.206489	.207912	78
12	.207912	.209334	.210756	.212178	.213599	.215019	.216440	.217859	.219279	.220697	.222116	.223534	.224951	77
13	.224951	.226368	.227784	.229200	.230616	.232030	.233445	.234859	.236273	.237686	.239098	.240510	.241922	76
14	.241921	.243334	.244743	.246153	.247563	.248971	.250380	.251788	.253195	.254602	.256008	.257414	.258819	75
15	.258819	.260224	.261628	.263031	.264434	.265837	.267238	.268640	.270040	.271440	.272840	.274239	.275637	74
16	.275637	.277035	.278432	.279829	.281225	.282621	.284015	.285410	.286803	.288196	.289589	.290981	.292372	73
17	.292372	.293762	.295152	.296542	.297930	.299318	.300706	.302093	.303479	.304864	.306249	.307633	.309017	72
18	.309017	.310410	.311782	.313164	.314545	.315925	.317305	.318684	.320062	.321440	.322816	.324193	.325568	71
19	.325568	.326943	.328317	.329691	.331063	.332436	.333807	.335178	.336548	.337917	.339285	.340653	.342020	70
20	.342020	.343387	.344752	.346117	.347481	.348845	.350207	.351569	.352931	.354291	.355651	.357010	.358368	69
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.
Natural Co-sines.														

TABLE IV.—*continued.*

NATURAL SINES.

Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
21	.358368	.359725	.361082	.362438	.363793	.365148	.366501	.367854	.369206	.370557	.371908	.373258	.374607	68
22	.374607	.375955	.377302	.378649	.379994	.381339	.382683	.384027	.385369	.386711	.388052	.389392	.390731	67
23	.390731	.392070	.393407	.394744	.396080	.397415	.398749	.400082	.401415	.402747	.404078	.405408	.406737	66
24	.406737	.408065	.409392	.410719	.412045	.413369	.414693	.416016	.417339	.418660	.419980	.421300	.422618	65
25	.422618	.423936	.425253	.426569	.427884	.429198	.430511	.431823	.433135	.434446	.435755	.437063	.438371	64
26	.438371	.439678	.440984	.442289	.443593	.444896	.446198	.447499	.448799	.450099	.451397	.452694	.453991	63
27	.453991	.455286	.456580	.457874	.459166	.460458	.461749	.463038	.464327	.465615	.466901	.468187	.469472	62
28	.469472	.470755	.472038	.473320	.474600	.475880	.477159	.478436	.479713	.480989	.482263	.483537	.484810	61
29	.484810	.486081	.487352	.488621	.489890	.491157	.492424	.493689	.494953	.496217	.497479	.498740	.500000	60
30	.500000	.501260	.502517	.503774	.505030	.506285	.507539	.508791	.510043	.511293	.512543	.513791	.515038	59
31	.515038	.516284	.517529	.518773	.520016	.521258	.522499	.523738	.524977	.526214	.527450	.528685	.529919	58
32	.529919	.531152	.532384	.533615	.534844	.536072	.537300	.538526	.539751	.540975	.542197	.543419	.544639	57
33	.544639	.545858	.547076	.548293	.549509	.550724	.551937	.553149	.554360	.555570	.556779	.557987	.559193	56
34	.559193	.560398	.561602	.562805	.564007	.565207	.566406	.567604	.568801	.569997	.571191	.572384	.573576	55
35	.573576	.574767	.575957	.577145	.578332	.579518	.580703	.581886	.583069	.584250	.585429	.586608	.587785	54
36	.587785	.588961	.590136	.591310	.592482	.593653	.594823	.595991	.597159	.598325	.599489	.600653	.601815	53
37	.601815	.602976	.604136	.605294	.606451	.607607	.608761	.609915	.611067	.612217	.613367	.614515	.615662	52
38	.615662	.616807	.617951	.619094	.620236	.621376	.622515	.623652	.624789	.625924	.627057	.628189	.629320	51
39	.629320	.630450	.631578	.632705	.633831	.634955	.636078	.637200	.638320	.639439	.640557	.641673	.642788	50
40	.642788	.643901	.645013	.646124	.647233	.648341	.649448	.650554	.651657	.652760	.653861	.654961	.656059	49
41	.656059	.657156	.658252	.659346	.660439	.661530	.662620	.663709	.664796	.665882	.666966	.668020	.669131	48
42	.669131	.670211	.671290	.672367	.673443	.674517	.675590	.676662	.677732	.678801	.679868	.680934	.681998	47
43	.681998	.683061	.684123	.685183	.686242	.687299	.688355	.689409	.690462	.691513	.692563	.693611	.694658	46
44	.694658	.695704	.696748	.697791	.698832	.699871	.700909	.701946	.702981	.704015	.705047	.706078	.707107	45
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.

NATURAL CO-SINES.

TABLE IV.—*continued.*

NATURAL SINES.

Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
45	.707107	.708135	.709161	.710185	.711209	.712230	.713251	.714269	.715286	.716302	.717316	.718329	.719340	44
46	.719340	.720349	.721357	.722364	.723369	.724372	.725374	.726375	.727374	.728371	.729367	.730361	.731354	43
47	.731354	.732345	.733335	.734322	.735309	.736294	.737277	.738259	.739239	.740218	.741195	.742171	.743145	42
48	.743145	.744117	.745088	.746057	.747025	.747991	.748956	.749919	.750880	.751840	.752798	.753755	.754471	41
49	.754710	.755663	.756615	.757565	.758514	.759461	.760406	.761350	.762292	.763233	.764171	.765109	.766044	40
50	.766044	.766979	.767911	.768842	.769771	.770699	.771625	.772549	.773472	.774393	.775312	.776230	.777146	39
51	.777146	.778060	.778973	.779885	.780794	.781702	.782608	.783513	.784416	.785317	.786217	.787115	.788011	38
52	.788011	.788905	.789798	.790690	.791579	.792467	.793353	.794238	.795121	.796002	.796882	.797759	.778636	37
53	.798636	.799510	.800383	.801254	.802123	.802991	.803857	.804721	.805584	.806445	.807304	.808161	.809017	36
54	.809017	.809871	.810723	.811574	.812423	.813270	.814116	.814959	.815801	.816642	.817480	.818317	.819152	35
55	.819152	.819985	.820817	.821647	.822475	.823302	.824126	.824949	.825770	.826590	.827407	.828223	.829038	34
56	.829038	.829850	.830661	.831470	.832277	.833082	.833886	.834688	.835488	.836286	.837083	.837878	.838671	33
57	.838671	.839462	.840251	.841039	.841825	.842609	.843391	.844172	.844951	.845728	.846503	.847276	.848048	32
58	.848048	.848818	.849586	.850352	.851117	.851879	.852640	.853399	.854156	.854912	.855666	.856417	.857167	31
59	.857167	.857916	.858662	.859406	.860149	.860890	.861629	.862366	.863102	.863836	.864567	.865297	.866025	30
60	.866025	.866752	.867476	.868199	.868920	.869639	.870356	.871071	.871784	.872496	.873206	.873914	.874620	29
61	.874620	.875324	.876026	.876727	.877425	.878122	.878817	.879510	.880201	.880891	.881578	.882264	.882948	28
62	.882948	.883629	.884310	.884988	.885664	.886338	.887011	.887682	.888350	.889017	.889682	.890345	.891007	27
63	.891007	.891666	.892323	.893979	.893633	.894284	.894934	.895582	.896229	.896873	.897515	.898156	.898794	26
64	.898794	.899431	.900065	.900698	.901329	.901958	.902585	.903211	.903834	.904455	.905075	.905692	.906308	25
65	.906308	.906922	.907533	.908143	.908751	.909357	.909961	.910564	.911164	.911762	.912358	.912953	.913545	24
66	.913545	.914136	.914725	.915312	.915896	.916479	.917060	.917639	.918216	.918791	.919364	.919936	.920505	23
67	.920505	.921072	.921638	.922201	.922762	.923322	.923880	.924435	.924989	.925541	.926090	.926638	.927184	22
68	.927184	.927728	.928270	.928810	.929348	.929884	.930418	.930950	.931950	.932008	.932534	.933058	.933580	21
69	.933580	.934101	.934619	.935135	.935650	.936162	.936672	.937181	.937687	.938191	.938694	.939194	.939693	20
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.

NATURAL CO-SINES.

TABLE IV.—*continued.*

NATURAL SINES.

Deg.	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′	60′	
70	.939693	.940189	.940684	.941176	.941667	.942155	.942642	.943126	.943609	.944089	.944568	.945044	.945519	19
71	.945519	.945991	.946462	.946930	.947397	.947861	.948324	.948784	.949243	.949699	.950154	.950606	.951057	18
72	.951057	.951505	.951951	.952396	.952838	.953279	.953717	.954153	.954588	.955020	.955450	.955879	.956305	17
73	.956305	.956729	.957151	.957571	.957990	.958406	.958818	.959232	.959642	.960050	.960456	.960856	.961262	16
74	.961262	.961662	.962059	.962455	.962849	.963241	.963031	.964018	.964404	.964787	.965169	.965548	.965926	15
75	.965926	.966301	.966675	.967046	.967415	.967783	.968148	.968511	.968872	.969231	.969588	.969943	.970296	14
76	.970296	.970647	.970995	.971342	.971687	.972029	.972370	.972708	.973045	.973379	.973712	.974042	.974370	13
77	.974370	.974696	.975020	.975342	.975662	.975980	.976296	.976610	.976922	.977231	.977539	.977844	.978148	12
78	.978148	.978449	.978748	.979046	.979341	.979634	.979925	.980214	.980501	.980785	.981068	.981349	.981627	11
79	.981627	.981904	.982178	.982450	.982721	.982989	.983255	.983519	.983781	.984041	.984299	.984554	.984808	10
80	.984808	.985059	.985309	.985556	.985801	.986045	.986286	.986525	.986762	.986996	.987229	.987460	.987688	9
81	.987688	.987915	.988139	.988362	.988582	.988800	.989016	.989230	.989442	.989651	.989859	.990065	.990268	8
82	.990268	.990469	.990669	.990866	.991061	.991254	.991445	.991634	.991820	.992005	.992187	.992368	.992546	7
83	.992546	.992722	.992896	.993069	.993239	.993406	.993572	.993736	.993897	.994056	.994214	.994369	.994522	6
84	.994522	.994673	.994822	.994969	.995113	.995256	.995396	.995535	.995671	.995805	.995937	.996067	.996195	5
85	.996195	.996320	.996444	.996566	.996685	.996802	.996917	.997030	.997141	.997250	.997357	.997462	.997564	4
86	.997564	.997665	.997763	.997859	.997953	.998045	.998135	.998223	.998308	.998392	.998473	.998552	.998630	3
87	.998630	.998705	.998778	.998848	.998917	.998984	.999048	.999111	.999171	.999229	.999285	.999339	.999391	2
88	.999391	.999441	.999488	.999534	.999577	.999618	.999657	.999694	.999729	.999762	.999793	.999821	.999848	1
89	.999848	.999872	.999894	.999914	.999932	.999948	.999962	.999974	.999983	.999991	.999996	.999999	1.000000	0
	60′	55′	50′	45′	40′	35′	30′	25′	20′	15′	10′	5′	0′	Deg.

NATURAL CO-SINES.

TABLE V.

TABLES for calculating Cuttings and Embankments 1 chain or 66 feet in length, Slopes 1 to 1, and Breadth 1 foot.

Depths in feet.	Middle breadth 1 foot.	Slopes 1 to 1.	Depths in feet.	Middle breadth 1 foot.	Slopes 1 to 1.	Depths in feet.	Middle breadth 1 foot.	Slopes 1 to 1.
1	1.22	0.61	51	62.33	1589.50	101	123.44	6233.94
2	2.44	2.44	52	63.55	1652.44	102	124.66	6358.00
3	3.66	5.50	53	64.77	1716.61	103	125.88	6483.27
4	4.88	9.77	54	66.00	1782.00	104	127.11	6609.77
5	6.11	15.27	55	67.22	1848.61	105	128.33	6737.50
6	7.33	22.00	56	68.44	1916.44	106	129.55	6866.44
7	8.55	29.94	57	69.66	1985.50	107	130.77	6996.61
8	9.77	39.11	58	70.88	2055.77	108	132.00	7128.00
9	11.00	49.50	59	72.11	2127.27	109	133.22	7260.61
10	12.22	61.11	60	73.33	2200.00	110	134.44	7394.44
11	13.44	73.94	61	74.55	2273.94	111	135.66	7529.50
12	14.66	88.00	62	75.77	2349.11	112	136.88	7665.77
13	15.88	103.27	63	77.00	2425.50	113	138.11	7803.27
14	17.11	119.77	64	78.22	2503.11	114	139.33	7942.00
15	18.33	137.50	65	79.44	2581.94	115	140.55	8081.94
16	19.55	156.44	66	80.66	2662.00	116	141.77	8223.11
17	20.77	176.61	67	81.88	2743.27	117	143.00	8365.50
18	22.00	198.00	68	83.11	2825.77	118	144.22	8509.11
19	23.22	220.61	69	84.33	2909.50	119	145.44	8653.94
20	24.44	244.44	70	85.55	2994.44	120	146.66	8800.00
21	25.66	269.50	71	86.77	3080.61	121	147.88	8947.27
22	26.88	295.77	72	88.00	3168.00	122	149.11	9095.77
23	28.11	323.27	73	89.22	3256.61	123	150.33	9245.50
24	29.33	352.00	74	90.44	3346.44	124	151.55	9396.44
25	30.55	381.94	75	91.66	3437.50	125	152.77	9548.61
26	31.77	413.11	76	92.88	3529.77	126	154.00	9702.00
27	33.00	445.50	77	94.11	3623.27	127	155.22	9856.61
28	34.22	479.11	78	95.33	3718.00	128	156.44	10012.44
29	35.44	513.94	79	96.55	3813.94	129	157.66	10169.50
30	36.66	550.00	80	97.77	3911.11	130	158.88	10327.77
31	37.88	587.27	81	99.00	4009.50	131	160.11	10487.27
32	39.11	625.77	82	100.22	4109.11	132	161.33	10648.00
33	40.33	665.50	83	101.44	4209.94	133	162.55	10809.94
34	41.55	706.44	84	102.66	4312.00	134	163.77	10973.11
35	42.78	748.61	85	103.88	4415.27	135	165.00	11137.50
36	44.00	792.00	86	105.11	4519.77	136	166.22	11303.11
37	45.22	836.61	87	106.33	4625.50	137	167.44	11469.94
38	46.44	882.44	88	107.55	4732.44	138	168.66	11638.00
39	47.66	929.50	89	108.77	4840.61	139	169.88	11807.27
40	48.88	977.77	90	110.00	4950.00	140	171.11	11977.77
41	50.11	1027.27	91	111.22	5060.61	141	172.33	12149.50
42	51.33	1078.00	92	112.44	5172.44	142	173.55	12322.44
43	52.55	1129.27	93	113.66	5285.50	143	174.77	12496.61
44	53.77	1183.11	94	114.88	5399.77	144	176.00	12672.00
45	55.00	1237.50	95	116.11	5515.27	145	177.22	12848.61
46	56.22	1293.11	96	117.33	5632.00	146	178.24	13026.44
47	57.44	1349.94	97	118.55	5749.94	147	179.66	13205.50
48	58.66	1408.00	98	119.77	5869.11	148	180.88	13385.77
49	59.88	1467.27	99	121.00	5989.50	149	182.11	13567.27
50	61.11	1527.77	100	122.22	6111.11	150	183.33	13750.00

AUXILIARY to Table V.—The Difference in Cube Yards to be added to the Slopes.

Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.
1	0.20	39	309.83	77	1207.76	115	2693.98
2	0.81	40	325.92	78	1239.33	116	2741.04
3	1.83	41	342.42	79	1271.31	117	2788.50
4	3.26	42	359.33	80	1303.70	118	2836.37
5	5.09	43	376.65	81	1336.50	119	2884.65
6	7.33	44	394.37	82	1369.70	120	2933.33
7	9.98	45	412.50	83	1403.31	121	2982.42
8	13.04	46	431.04	84	1437.33	122	3031.92
9	16.50	47	449.98	85	1471.76	123	3081.83
10	20.37	48	469.33	86	1506.59	124	3132.15
11	24.65	49	489.09	87	1541.83	125	3182.87
12	29.33	50	509.26	88	1577.48	126	3234.00
13	34.42	51	529.83	89	1613.54	127	3285.54
14	39.92	52	550.81	90	1650.00	128	3337.48
15	45.83	53	572.20	91	1686.87	129	3389.83
16	52.15	54	594.00	92	1724.15	130	3442.59
17	58.87	55	616.20	93	1761.83	131	3495.76
18	66.00	56	638.81	94	1799.92	132	3549.33
19	73.54	57	661.83	95	1838.42	133	3603.31
20	81.48	58	685.26	96	1877.33	134	3657.70
21	89.83	59	709.09	97	1916.65	135	3712.50
22	98.59	60	733.33	98	1956.37	136	3767.70
23	107.76	61	757.98	99	1996.50	137	3823.31
24	117.33	62	783.04	100	2037.04	138	3879.33
25	127.31	63	808.50	101	2077.98	139	3935.76
26	137.70	64	834.37	102	2119.33	140	3992.59
27	148.50	65	860.65	103	2161.09	141	4049.83
28	159.70	66	887.33	104	2203.26	142	4107.48
29	171.31	67	914.42	105	2245.83	143	4165.54
30	183.33	68	941.92	106	2288.81	144	4224.00
31	195.76	69	969.83	107	2332.20	145	4282.87
32	208.59	70	998.15	108	2376.00	146	4342.15
33	221.83	71	1026.87	109	2420.20	147	4401.83
34	235.48	72	1056.00	110	2464.81	148	4461.92
35	249.54	73	1085.54	111	2509.83	149	4522.42
36	264.00	74	1115.48	112	2555.26	150	4583.33
37	278.87	75	1145.83	113	2601.09		
38	294.15	76	1176.59	114	2647.33		

See explanation at p. 212.

TABLE VI.

TABLE of Offsets for Railway Curves, showing the set off at the end of the first Chain from the Tangent Point.

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
5	79.200	12	33.000	19	20.842	26	15.231
6	66.000	13	30.462	20	19.800	27	14.667
7	56.871	14	28.286	21	19.857	28	14.143
8	49.500	15	26.400	22	18.000	29	13.655
9	44.000	16	24.750	23	17.217	30	13.200
10	39.600	17	23.291	24	16.500	31	12.744
11	36.000	18	22.000	25	15.840	32	12.376

(continued)

TABLE VI.—*continued.*

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
33	12.000	58	6.828	86	4.605	190	2.084
34	11.647	59	6.712	88	4.500	195	2.031
35	11.314	60	6.600	90	4.400	200	1.980
36	11.000	61	6.492	92	4.304	210	1.886
37	10.703	62	6.387	94	4.213	220	1.800
38	10.421	63	6.286	96	4.125	230	1.722
39	10.154	64	6.186	98	4.041	240	1.650
40	9.900	65	6.092	100	3.960	250	1.584
41	9.659	66	6.000	105	3.771	260	1.523
42	9.429	67	5.910	110	3.600	270	1.467
43	9.209	68	5.824	115	3.444	280	1.414
44	9.000	69	5.739	120	3.300	290	1.366
45	8.800	70	5.657	125	3.168	300	1.320
46	8.609	71	5.574	130	3.046	310	1.277
47	8.426	72	5.500	135	2.933	320	1.238
48	8.250	73	5.425	140	2.829	330	1.200
49	8.082	74	5.351	145	2.731	340	1.165
50	7.920	75	5.280	150	2.640	350	1.131
51	7.765	76	5.211	155	2.555	360	1.100
52	7.615	77	5.143	160	2.475	370	1.070
53	7.472	78	5.077	165	2.400	380	1.042
54	7.333	79	5.013	170	2.329	390	1.015
55	7.200	80	4.950	175	2.623	396	1.000
56	7.071	82	4.829	180	2.200		
57	6.947	84	4.714	185	2.141		

Note.—This table is used in setting out curves from a tangent; the first offset only is given; the following offsets are easily found by multiplying the first offset by the square of each succeeding chain. As for example: 66.0 × 9 = 594.0 ÷ 12 = 48.66 feet. See Fig. 2, Plate 31.

TABLE VII.

TABLE showing the first Angle from the Tangent Line, according to the Radius.

Radius in chains.	Angles.	Radius in chains.	Angles.	Radius in chains.	Angles.	Radius in chains.	Angles.
	° ′ ″		° ′ ″		° ′ ″		° ′ ″
5	5 44 22	22	1 18 8	39	0 44 4	56	0 30 42
6	4 46 49	23	1 14 44	40	0 42 58	57	0 30 9
7	4 5 46	24	1 11 37	41	0 41 55	58	0 29 38
8	3 35 00	25	1 8 46	42	0 40 56	59	0 29 8
9	3 11 5	26	1 6 7	43	0 39 58	60	0 28 39
10	2 51 58	27	1 3 40	44	0 39 4	61	0 28 11
11	2 36 19	28	1 1 23	45	0 38 12	62	0 27 43
12	2 23 17	29	0 59 16	46	0 37 22	63	0 27 17
13	2 12 15	30	0 57 18	47	0 36 34	64	0 26 51
14	2 2 48	31	0 55 27	48	0 35 49	65	0 26 27
15	1 54 37	32	0 53 43	49	0 35 5	66	0 26 3
16	1 47 27	33	0 52 5	50	0 34 23	67	0 25 39
17	1 41 7	34	0 50 33	51	0 33 42	68	0 25 17
18	1 35 30	35	0 49 7	52	0 33 3	69	0 24 55
19	1 30 29	36	0 47 45	53	0 32 26	70	0 24 33
20	1 25 57	37	0 46 27	54	0 31 50	71	0 24 13
21	1 21 52	38	0 45 14	55	0 31 15	72	0 23 52

(*continued*)

AUXILIARY to Table V.—The Difference in Cube Yards to be added to the Slopes.

Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.
1	0.20	39	309.83	77	1207.76	115	2693.98
2	0.81	40	325.92	78	1239.33	116	2741.04
3	1.83	41	342.42	79	1271.31	117	2788.50
4	3.26	42	359.33	80	1303.70	118	2836.37
5	5.09	43	376.65	81	1336.50	119	2884.65
6	7.33	44	394.37	82	1369.70	120	2933.33
7	9.98	45	412.50	83	1403.31	121	2982.42
8	13.04	46	431.04	84	1437.33	122	3031.92
9	16.50	47	449.98	85	1471.76	123	3081.83
10	20.37	48	469.33	86	1506.59	124	3132.15
11	24.65	49	489.09	87	1541.83	125	3182.87
12	29.33	50	509.26	88	1577.48	126	3234.00
13	34.42	51	529.83	89	1613.54	127	3285.54
14	39.92	52	550.81	90	1650.00	128	3337.48
15	45.83	53	572.20	91	1686.87	129	3389.83
16	52.15	54	594.00	92	1724.15	130	3442.59
17	58.87	55	616.20	93	1761.83	131	3495.76
18	66.00	56	638.81	94	1799.92	132	3549.33
19	73.54	57	661.83	95	1838.42	133	3603.31
20	81.48	58	685.26	96	1877.33	134	3657.70
21	89.83	59	709.09	97	1916.65	135	3712.50
22	98.59	60	733.33	98	1956.37	136	3767.70
23	107.76	61	757.98	99	1996.50	137	3823.31
24	117.33	62	783.04	100	2037.04	138	3879.33
25	127.31	63	808.50	101	2077.98	139	3935.76
26	137.70	64	834.37	102	2119.33	140	3992.59
27	148.50	65	860.65	103	2161.09	141	4049.83
28	159.70	66	887.33	104	2203.26	142	4107.48
29	171.31	67	914.42	105	2245.83	143	4165.54
30	183.33	68	941.92	106	2288.81	144	4224.00
31	195.76	69	969.83	107	2332.20	145	4282.87
32	208.59	70	998.15	108	2376.00	146	4342.15
33	221.83	71	1026.87	109	2420.20	147	4401.83
34	235.48	72	1056.00	110	2464.81	148	4461.92
35	249.54	73	1085.54	111	2509.83	149	4522.42
36	264.00	74	1115.48	112	2555.26	150	4583.33
37	278.87	75	1145.83	113	2601.09		
38	294.15	76	1176.59	114	2647.33		

See explanation at p. 212.

TABLE VI.

TABLE of Offsets for Railway Curves, showing the set off at the end of the first Chain from the Tangent Point.

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
5	79,200	12	33.000	19	20.842	26	15.231
6	66.000	13	30.462	20	19.800	27	14.667
7	56.871	14	28.286	21	19.857	28	14.143
8	49.500	15	26.400	22	18.000	29	13.655
9	44.000	16	24.750	23	17.217	30	13.200
10	39.600	17	23.291	24	16.500	31	12.744
11	36.000	18	22.000	25	15.840	32	12.376

(continued)

TABLE VI.—*continued.*

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
33	12.000	58	6.828	86	4.605	190	2.084
34	11.647	59	6.712	88	4.500	195	2.031
35	11.314	60	6.600	90	4.400	200	1.980
36	11.000	61	6.492	92	4.304	210	1.886
37	10.703	62	6.387	94	4.213	220	1.800
38	10.421	63	6.286	96	4.125	230	1.722
39	10.154	64	6.186	98	4.041	240	1.650
40	9.900	65	6.092	100	3.960	250	1.584
41	9.659	66	6.000	105	3.771	260	1.523
42	9.429	67	5.910	110	3.600	270	1.467
43	9.209	68	5.824	115	3.444	280	1.414
44	9.000	69	5.739	120	3.300	290	1.366
45	8.800	70	5.657	125	3.168	300	1.320
46	8.609	71	5.574	130	3.046	310	1.277
47	8.426	72	5.500	135	2.933	320	1.238
48	8.250	73	5.425	140	2.829	330	1.200
49	8.082	74	5.351	145	2.731	340	1.165
50	7.920	75	5.280	150	2.640	350	1.131
51	7.765	76	5.211	155	2.555	360	1.100
52	7.615	77	5.143	160	2.475	370	1.070
53	7.472	78	5.077	165	2.400	380	1.042
54	7.333	79	5.013	170	2.329	390	1.015
55	7.200	80	4.950	175	2.623	396	1.000
56	7.071	82	4.829	180	2.200		
57	6.947	84	4.714	185	2.141		

Note.—This table is used in setting out curves from a tangent; the first offset only is given; the following offsets are easily found by multiplying the first offset by the square of each succeeding chain. As for example: 66.0 × 9 = 594.0 ÷ 12 = 48.66 feet. See Fig. 2, Plate 31.

TABLE VII.

TABLE showing the first Angle from the Tangent Line, according to the Radius.

Radius in chains.	Angles. ° ′ ″	Radius in chains.	Angles. ° ′ ″	Radius in chains.	Angles. ° ′ ″	Radius in chains.	Angles. ° ′ ″
5	5 44 22	22	1 18 8	39	0 44 4	56	0 30 42
6	4 46 49	23	1 14 44	40	0 42 58	57	0 30 9
7	4 5 46	24	1 11 37	41	0 41 55	58	0 29 38
8	3 35 00	25	1 8 46	42	0 40 56	59	0 29 8
9	3 11 5	26	1 6 7	43	0 39 58	60	0 28 39
10	2 51 58	27	1 3 40	44	0 39 4	61	0 28 11
11	2 36 19	28	1 1 23	45	0 38 12	62	0 27 43
12	2 23 17	29	0 59 16	46	0 37 22	63	0 27 17
13	2 12 15	30	0 57 18	47	0 36 34	64	0 26 51
14	2 2 48	31	0 55 27	48	0 35 49	65	0 26 27
15	1 54 37	32	0 53 43	49	0 35 5	66	0 26 3
16	1 47 27	33	0 52 5	50	0 34 23	67	0 25 39
17	1 41 7	34	0 50 33	51	0 33 42	68	0 25 17
18	1 35 30	35	0 49 7	52	0 33 3	69	0 24 55
19	1 30 29	36	0 47 45	53	0 32 26	70	0 24 33
20	1 25 57	37	0 46 27	54	0 31 50	71	0 24 13
21	1 21 52	38	0 45 14	55	0 31 15	72	0 23 52

(*continued*)

Auxiliary to Table V.—The Difference in Cube Yards to be added to the Slopes.

Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.
1	0.20	39	309.83	77	1207.76	115	2693.98
2	0.81	40	325.92	78	1239.33	116	2741.04
3	1.83	41	342.42	79	1271.31	117	2788.50
4	3.26	42	359.33	80	1303.70	118	2836.37
5	5.09	43	376.65	81	1336.50	119	2884.65
6	7.33	44	394.37	82	1369.70	120	2933.33
7	9.98	45	412.50	83	1403.31	121	2982.42
8	13.04	46	431.04	84	1437.33	122	3031.92
9	16.50	47	449.98	85	1471.76	123	3081.83
10	20.37	48	469.33	86	1506.59	124	3132.15
11	24.65	49	489.09	87	1541.83	125	3182.87
12	29.33	50	509.26	88	1577.48	126	3234.00
13	34.42	51	529.83	89	1613.54	127	3285.54
14	39.92	52	550.81	90	1650.00	128	3337.48
15	45.83	53	572.20	91	1686.87	129	3389.83
16	52.15	54	594.00	92	1724.15	130	3442.59
17	58.87	55	616.20	93	1761.83	131	3495.76
18	66.00	56	638.81	94	1799.92	132	3549.33
19	73.54	57	661.83	95	1838.42	133	3603.31
20	81.48	58	685.26	96	1877.33	134	3657.70
21	89.83	59	709.09	97	1916.65	135	3712.50
22	98.59	60	733.33	98	1956.37	136	3767.70
23	107.76	61	757.98	99	1996.50	137	3823.31
24	117.33	62	783.04	100	2037.04	138	3879.33
25	127.31	63	808.50	101	2077.98	139	3935.76
26	137.70	64	834.37	102	2119.33	140	3992.59
27	148.50	65	860.65	103	2161.09	141	4049.83
28	159.70	66	887.33	104	2203.26	142	4107.48
29	171.31	67	914.42	105	2245.83	143	4165.54
30	183.33	68	941.92	106	2288.81	144	4224.00
31	195.76	69	969.83	107	2332.20	145	4282.87
32	208.59	70	998.15	108	2376.00	146	4342.15
33	221.83	71	1026.87	109	2420.20	147	4401.83
34	235.48	72	1056.00	110	2464.81	148	4461.92
35	249.54	73	1085.54	111	2509.83	149	4522.42
36	264.00	74	1115.48	112	2555.26	150	4583.33
37	278.87	75	1145.83	113	2601.09		
38	294.15	76	1176.59	114	2647.33		

See explanation at p. 212.

TABLE VI.

Table of Offsets for Railway Curves, showing the set off at the end of the first Chain from the Tangent Point.

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
5	79,200	12	33.000	19	20.842	26	15.231
6	66.000	13	30.462	20	19.800	27	14.667
7	56.871	14	28.286	21	19.857	28	14.143
8	49.500	15	26.400	22	18.000	29	13.655
9	44.000	16	24.750	23	17.217	30	13.200
10	39.600	17	23.291	24	16.500	31	12.744
11	36.000	18	22.000	25	15.840	32	12.376

(*continued*)

TABLE VI.—*continued.*

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
33	12.000	58	6.828	86	4.605	190	2.084
34	11.647	59	6.712	88	4.500	195	2.031
35	11.314	60	6.600	90	4.400	200	1.980
36	11.000	61	6.492	92	4.304	210	1.886
37	10.703	62	6.387	94	4.213	220	1.800
38	10.421	63	6.286	96	4.125	230	1.722
39	10.154	64	6.186	98	4.041	240	1.650
40	9.900	65	6.092	100	3.960	250	1.584
41	9.659	66	6.000	105	3.771	260	1.523
42	9.429	67	5.910	110	3.600	270	1.467
43	9.209	68	5.824	115	3.444	280	1.414
44	9.000	69	5.739	120	3.300	290	1.366
45	8.800	70	5.657	125	3.168	300	1.320
46	8.609	71	5.574	130	3.046	310	1.277
47	8.426	72	5.500	135	2.933	320	1.238
48	8.250	73	5.425	140	2.829	330	1.200
49	8.082	74	5.351	145	2.731	340	1.165
50	7.920	75	5.280	150	2.640	350	1.131
51	7.765	76	5.211	155	2.555	360	1.100
52	7.615	77	5.143	160	2.475	370	1.070
53	7.472	78	5.077	165	2.400	380	1.042
54	7.333	79	5.013	170	2.329	390	1.015
55	7.200	80	4.950	175	2.623	396	1.000
56	7.071	82	4.829	180	2.200		
57	6.947	84	4.714	185	2.141		

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5	5 44 22	22	1 18 8	39	0 44 4	56	0 30 42
6	4 46 49	23	1 14 44	40	0 42 58	57	0 30 9
7	4 5 46	24	1 11 37	41	0 41 55	58	0 29 38
8	3 35 00	25	1 8 46	42	0 40 56	59	0 29 8
9	3 11 5	26	1 6 7	43	0 39 58	60	0 28 39
10	2 51 58	27	1 3 40	44	0 39 4	61	0 28 11
11	2 36 19	28	1 1 23	45	0 38 12	62	0 27 43
12	2 23 17	29	0 59 16	46	0 37 22	63	0 27 17
13	2 12 15	30	0 57 18	47	0 36 34	64	0 26 51
14	2 2 48	31	0 55 27	48	0 35 49	65	0 26 27
15	1 54 37	32	0 53 43	49	0 35 5	66	0 26 3
16	1 47 27	33	0 52 5	50	0 34 23	67	0 25 39
17	1 41 7	34	0 50 33	51	0 33 42	68	0 25 17
18	1 35 30	35	0 49 7	52	0 33 3	69	0 24 55
19	1 30 29	36	0 47 45	53	0 32 26	70	0 24 33
20	1 25 57	37	0 46 27	54	0 31 50	71	0 24 13
21	1 21 52	38	0 45 14	55	0 31 15	72	0 23 52

(*continued*)

AUXILIARY to Table V.—The Difference in Cube Yards to be added to the Slopes.

Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.	Diff.	Cube yards.
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4	3.26	42	359.33	80	1303.70	118	2836.37
5	5.09	43	376.65	81	1336.50	119	2884.65
6	7.33	44	394.37	82	1369.70	120	2933.33
7	9.98	45	412.50	83	1403.31	121	2982.42
8	13.04	46	431.04	84	1437.33	122	3031.92
9	16.50	47	449.98	85	1471.76	123	3081.83
10	20.37	48	469.83	86	1506.59	124	3132.15
11	24.65	49	489.09	87	1541.83	125	3182.87
12	29.33	50	509.26	88	1577.48	126	3234.00
13	34.42	51	529.83	89	1613.54	127	3285.54
14	39.92	52	550.81	90	1650.00	128	3337.48
15	45.83	53	572.20	91	1686.87	129	3389.83
16	52.15	54	594.00	92	1724.15	130	3442.59
17	58.87	55	616.20	93	1761.83	131	3495.76
18	66.00	56	638.81	94	1799.92	132	3549.33
19	73.54	57	661.83	95	1838.42	133	3603.31
20	81.48	58	685.26	96	1877.33	134	3657.70
21	89.83	59	709.09	97	1916.65	135	3712.50
22	98.59	60	733.33	98	1956.37	136	3767.70
23	107.76	61	757.98	99	1996.50	137	3823.31
24	117.33	62	783.04	100	2037.04	138	3879.33
25	127.31	63	808.50	101	2077.98	139	3935.76
26	137.70	64	834.37	102	2119.33	140	3992.59
27	148.50	65	860.65	103	2161.09	141	4049.83
28	159.70	66	887.33	104	2203.26	142	4107.48
29	171.31	67	914.42	105	2245.83	143	4165.54
30	183.33	68	941.92	106	2288.81	144	4224.00
31	195.76	69	969.83	107	2332.20	145	4282.87
32	208.59	70	998.15	108	2376.00	146	4342.15
33	221.83	71	1026.87	109	2420.20	147	4401.83
34	235.48	72	1056.00	110	2464.81	148	4461.92
35	249.54	73	1085.54	111	2509.83	149	4522.42
36	264.00	74	1115.48	112	2555.26	150	4583.33
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TABLE VI.

TABLE of Offsets for Railway Curves, showing the set off at the end of the first Chain from the Tangent Point.

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
5	79.200	12	33.000	19	20.842	26	15.231
6	66.000	13	30.462	20	19.800	27	14.667
7	56.571	14	28.286	21	18.857	28	14.143
8	49.500	15	26.400	22	18.000	29	13.655
9	44.000	16	24.750	23	17.217	30	13.200
10	39.600	17	23.291	24	16.500	31	12.744
11	36.000	18	22.000	25	15.840	32	12.376

(continued)

TABLE VI.—*continued.*

Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.	Radius in chains.	Offsets in inches and decimals.
33	12.000	58	6.828	86	4.605	190	2.084
34	11.647	59	6.712	88	4.500	195	2.031
35	11.314	60	6.600	90	4.400	200	1.980
36	11.000	61	6.492	92	4.304	210	1.886
37	10.703	62	6.387	94	4.213	220	1.800
38	10.421	63	6.286	96	4.125	230	1.722
39	10.154	64	6.186	98	4.041	240	1.650
40	9.900	65	6.092	100	3.960	250	1.584
41	9.659	66	6.000	105	3.771	260	1.523
42	9.429	67	5.910	110	3.600	270	1.467
43	9.209	68	5.824	115	3.444	280	1.414
44	9.000	69	5.739	120	3.300	290	1.366
45	8.800	70	5.657	125	3.168	300	1.320
46	8.609	71	5.574	130	3.046	310	1.277
47	8.426	72	5.500	135	2.933	320	1.238
48	8.250	73	5.425	140	2.829	330	1.200
49	8.082	74	5.351	145	2.731	340	1.165
50	7.920	75	5.280	150	2.640	350	1.131
51	7.765	76	5.211	155	2.555	360	1.100
52	7.615	77	5.143	160	2.475	370	1.070
53	7.472	78	5.077	165	2.400	380	1.042
54	7.333	79	5.013	170	2.329	390	1.015
55	7.200	80	4.950	175	2.623	396	1.000
56	7.071	82	4.829	180	2.200		
57	6.947	84	4.714	185	2.141		

Note.—This table is used in setting out curves from a tangent; the first offset only is given; the following offsets are easily found by multiplying the first offset by the square of each succeeding chain. As for example: 66.0 × 9 = 594.0 ÷ 12 = 48.66 feet. See Fig. 2, Plate 31.

TABLE VII.

TABLE showing the first Angle from the Tangent Line, according to the Radius.

Radius in chains.	Angles. ° ′ ″	Radius in chains.	Angles. ° ′ ″	Radius in chains.	Angles. ° ′ ″	Radius in chains.	Angles. ° ′ ″
5	5 44 22	22	1 18 8	39	0 44 4	56	0 30 42
6	4 46 49	23	1 14 44	40	0 42 58	57	0 30 9
7	4 5 46	24	1 11 37	41	0 41 55	58	0 29 38
8	3 35 00	25	1 8 46	42	0 40 56	59	0 29 8
9	3 11 5	26	1 6 7	43	0 39 58	60	0 28 39
10	2 51 58	27	1 3 40	44	0 39 4	61	0 28 11
11	2 36 19	28	1 1 23	45	0 38 12	62	0 27 43
12	2 23 17	29	0 59 16	46	0 37 22	63	0 27 17
13	2 12 15	30	0 57 18	47	0 36 34	64	0 26 51
14	2 2 48	31	0 55 27	48	0 35 49	65	0 26 27
15	1 54 37	32	0 53 43	49	0 35 5	66	0 26 3
16	1 47 27	33	0 52 5	50	0 34 23	67	0 25 39
17	1 41 7	34	0 50 33	51	0 33 42	68	0 25 17
18	1 35 30	35	0 49 7	52	0 33 3	69	0 24 55
19	1 30 29	36	0 47 45	53	0 32 26	70	0 24 33
20	1 25 57	37	0 46 27	54	0 31 50	71	0 24 13
21	1 21 52	38	0 45 14	55	0 31 15	72	0 23 52

(*continued*)

TABLE VII.—*continued.*

Radius in chains.	Angles.	Radius in chains.	Angles.	Radius in chains.	Angles.	Radius in chains.	Angles.
	° ′ ″		° ′ ″		° ′ ″		° ′ ″
73	0 23 33	95	0 18 6	145	0 11 51	195	0 8 49
74	0 23 14	100	0 17 11	150	0 11 28	200	0 8 36
75	0 22 55	105	0 16 22	155	0 11 5	210	0 8 11
76	0 22 37	110	0 15 38	160	0 10 45	215	0 8 0
77	0 22 19	115	0 14 57	165	0 10 25	220	0 7 49
78	0 22 2	120	0 14 19	170	0 10 7	225	0 7 38
79	0 21 45	125	0 13 46	175	0 9 47	230	0 7 28
80	0 21 29	130	0 13 13	180	0 9 33	235	0 7 19
85	0 20 13	135	0 12 44	185	0 9 17	240	0 7 10
90	0 19 6	140	0 12 17	190	0 9 3		

Note.—The use of this table is to set out curves by the method shown by Fig. 5, Plate 31. Ten chains being the limit, the following are known by multiplying the first angle by the number of chains; as for example, 5° 44′ 22″ × 2 = 11° 28′ 44″.

TABLE VIII.

AREAS of Circles either in inches, feet, or yards; if the diameter be taken in inches, the area will be in inches; if in feet, then the area will be in feet; if in yards, the area is yards. See Areas, p. 61.

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
1	.7854	8	50.26	15	176.71	22	380.13
1/4	1.23	1/4	53.46	1/4	182.65	1/4	388.82
1/2	1.77	1/2	56.74	1/2	188.69	1/2	397.60
3/4	2.40	3/4	60.17	3/4	194.82	3/4	406.49
2	3.14	9	63.62	16	201.06	23	415.47
1/4	3.98	1/4	67.20	1/4	207.39	1/4	424.55
1/2	4.90	1/2	70.88	1/2	213.82	1/2	433.73
3/4	5.94	3/4	74.66	3/4	220.35	3/4	443.01
3	7.06	10	78.54	17	226.98	24	452.39
1/4	8.29	1/4	82.52	1/4	233.70	1/4	461.86
1/2	9.62	1/2	86.59	1/2	240.53	1/2	471.43
3/4	11.04	3/4	92.70	3/4	247.45	3/4	481.10
4	12.56	11	95.03	18	254.47	25	490.87
1/4	14.18	1/4	99.40	1/4	261.58	1/4	500.74
1/2	15.90	1/2	103.87	1/2	268.80	1/2	510.70
3/4	17.72	3/4	108.43	3/4	276.11	3/4	520.77
5	19.64	12	113.09	19	283.53	26	530.93
1/4	21.65	1/4	117.86	1/4	291.04	1/4	541.18
1/2	23.76	1/2	122.72	1/2	298.64	1/2	551.54
3/4	25.97	3/4	127.66	3/4	306.35	3/4	562.00
6	28.27	13	132.73	20	314.16	27	572.55
1/4	30.68	1/4	137.88	1/4	322.06	1/4	583.20
1/2	33.18	1/2	143.14	1/2	330.06	1/2	593.95
3/4	35.78	3/4	148.49	3/4	338.36	3/4	604.80
7	38.48	14	153.94	21	346.36	28	615.75
1/4	41.28	1/4	159.48	1/4	354.65	1/4	626.79
1/2	44.18	1/2	165.13	1/2	363.05	1/2	637.94
3/4	47.17	3/4	170.87	3/4	371.54	3/4	649.18

(*continued*)

TABLE VIII.—*continued.*

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
29	660.52	44	1520.5	59	2733.9	74	4300.8
¼	671.95	¼	1537.8	¼	2757.2	¼	4329.9
½	683.49	½	1555.0	½	2780.5	½	4359.1
¾	695.12	¾	1572.8	¾	2803.9	¾	4388.4
30	706.86	45	1590.4	60	2827.4	75	4417.8
¼	718.69	¼	1608.1	¼	2851.0	¼	4447.3
½	730.61	½	1625.9	½	2874.7	½	4476.9
¾	742.64	¾	1643.9	¾	2898.5	¾	4506.6
31	754.76	46	1661.9	61	2922.4	76	4536.4
¼	766.99	¼	1680.0	¼	2946.4	¼	4566.3
½	779.31	½	1698.2	½	2970.5	½	4596.3
¾	791.73	¾	1716.5	¾	2994.7	¾	4626.4
32	804.24	47	1734.9	62	3019.1	77	4656.6
¼	816.86	¼	1753.4	¼	3043.4	¼	4686.9
½	829.57	½	1772.0	½	3067.9	½	4717.3
¾	842.39	¾	1790.7	¾	3092.5	¾	4747.8
33	855.30	48	1809.5	63	3117.2	78	4778.3
¼	868.30	¼	1828.4	¼	3142.0	¼	4809.0
½	881.41	½	1847.4	½	3166.9	½	4839.8
¾	894.61	¾	1866.5	¾	3191.9	¾	4870.8
34	907.92	49	1885.7	64	3217.0	79	4901.6
¼	921.32	¼	1905.0	¼	3242.1	¼	4932.7
½	934.82	½	1924.4	½	3267.4	½	4963.9
¾	948.41	¾	1943.9	¾	3292.8	¾	4995.2
35	962.11	50	1963.5	65	3318.3	80	5026.5
¼	975.90	¼	1983.2	¼	3343.9	¼	5058.0
½	989.80	½	2002.9	½	3369.5	½	5089.5
¾	1003.8	¾	2022.8	¾	3395.3	¾	5121.2
36	1017.8	51	2042.8	66	3421.2	81	5153.0
¼	1032.0	¼	2062.9	¼	3447.1	¼	5184.8
½	1046.3	½	2083.0	½	3473.2	½	5216.8
¾	1060.7	¾	2103.3	¾	3499.4	¾	5248.8
37	1075.2	52	2123.7	67	3525.6	82	5281.0
¼	1089.8	¼	2144.2	¼	3552.0	¼	5313.2
½	1104.4	½	2164.7	½	3578.4	½	5345.6
¾	1119.2	¾	2185.4	¾	3605.0	¾	5378.0
38	1134.1	53	2206.2	68	3631.6	83	5410.6
¼	1149.1	¼	2227.0	¼	3658.4	¼	5443.2
½	1164.1	½	2248.0	½	3685.3	½	5476.0
¾	1179.3	¾	2269.0	¾	3712.2	¾	5508.8
39	1194.6	54	2290.2	69	3739.2	84	5541.7
¼	1209.9	¼	2311.4	¼	3766.4	¼	5574.8
½	1225.4	½	2332.8	½	3793.6	½	5607.9
¾	1240.9	¾	2354.3	¾	3821.0	¾	5641.1
40	1256.6	55	2375.8	70	3848.4	85	5674.5
¼	1272.4	¼	2397.4	¼	3875.9	¼	5707.9
½	1288.2	½	2419.2	½	3903.6	½	5741.4
¾	1304.2	¾	2441.0	¾	3931.3	¾	5775.1
41	1320.2	56	2463.0	71	3959.2	86	5808.8
¼	1336.4	¼	2485.0	¼	3987.1	¼	5842.6
½	1352.6	½	2507.1	½	4015.1	½	5876.5
¾	1369.0	¾	2529.4	¾	4043.2	¾	5910.5
42	1385.4	57	2551.7	72	4071.5	87	5944.7
¼	1402.0	¼	2574.2	¼	4099.8	¼	5978.9
½	1418.6	½	2596.7	½	4128.2	½	6013.2
¾	1435.3	¾	2619.3	¾	4156.7	¾	6047.6
43	1452.2	58	2642.0	73	4185.4	88	6082.1
¼	1469.1	¼	2664.9	¼	4214.1	¼	6116.7
½	1486.1	½	2687.8	½	4242.9	½	6151.4
¾	1503.3	¾	2710.8	¾	4271.8	¾	6186.2

(*continued*)

TABLE VIII.—*continued.*

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
89	6221.1	92	6647.6	95	7088.2	98	7542.9
¼	6256.1	¼	6683.8	¼	7125.6	¼	7581.5
½	6291.2	½	6720.0	½	7163.0	½	7620.1
¾	6326.4	¾	6756.4	¾	7200.6	¾	7658.8
90	6361.7	93	6792.9	96	7238.2	99	7697.7
¼	6397.1	¼	6829.5	¼	7275.9	¼	7736.6
½	6432.6	½	6866.1	½	7313.8	½	7775.6
¾	6468.2	¾	6902.9	¾	7351.7	¾	7814.7
91	6503.8	94	6939.8	97	7389.8	100	7854.0
¼	6539.6	¼	6976.7	¼	7427.9	¼	7893.3
½	6575.5	½	7013.8	½	7466.2	½	7932.7
¾	6611.5	¾	7050.9	¾	7504.5	¾	7972.2

Note.—The above table may be used for larger circles in the following manner: For circles from 100 to 200 diameter, divide the diameter by 2, and find a diameter in the above table which corresponds to the quotient, then multiply the area opposite to it by 4, which will give the area required; and for circles from 200 to 400 diameter, divide the diameter by 4, and multiply the area opposite to the quotient by 16. Example: To find the area of a circle 162½ diameter: $\frac{162\frac{1}{2}}{2} =$ 81¼ area 5184.8 × 4 = 20739.2 the area of the circle required.

TABLE IX.

DIFFERENCE between the apparent and true Level.

Corrections in decimals of feet.		Corrections in decimals of links.		Corrections in decimals of miles.	
Distance in feet.	Curvature and Refraction.	Distance in links.	Curvature and Refraction.	Distance in miles.	Curvature and Refraction.
100	.00020	100	.00009	¼	.0357
150	.00046	150	.00021	½	.1430
200	.00083	200	.00036	¾	.3216
250	.00128	250	.00056	1	.5717
300	.00184	300	.00081	1½	1.2854
350	.00251	350	.00110	2	2.2869
400	.00328	400	.00143	2½	3.5733
450	.00415	450	.00181	3	5.1469
500	.00513	500	.00224	3½	7.0035
550	.00621	550	.00270	4	9.1474
600	.00738	600	.00321	4½	11.5775
650	.00866	650	.00377	5	14.2929
700	.01005	700	.00438	5½	17.2945
750	.01153	750	.00562	6	20.5817
800	.01312	800	.00572	6½	24.1551
850	.01481	850	.00645	7	28.0143
900	.01661	900	.00723	7½	32.1591
950	.01851	950	.00806	8	36.5883
1000	.02059	1000	.00893	8½	41.3066
1050	.02261	1050	.00985	9	46.3089
1100	.02481	1100	.01081	9½	51.5975
1150	.02712	1150	.01181	10	57.1714
1200	.02953	1200	.01287	11	69.1774
1250	.03204	1250	.01395	12	82.3269
1300	.03465	1300	.01509	13	96.6197

(*continued*)

TABLE IX.—*continued.*

Corrections in decimals of feet.		Corrections in decimals of links.		Corrections in decimals of miles.	
Distance in feet.	Curvature and Refraction.	Distance in links.	Curvature and Refraction.	Distance in miles.	Curvature and Refraction.
1350	.03738	1350	.01628	14	112.0560
1400	.04019	1400	.01751	15	128.6357
1450	.04311	1450	.01878	16	146.3589
1500	.04614	1500	.02010	17	165.2254
1550	.04927	1550	.02146	18	185.2359
1600	.05250	1600	.02287	19	206.3889
1650	.05583	1650	.02432	20	228.6857
1700	.05926	1700	.02582		
1750	.06280	1750	.02736		
1800	.06645	1800	.02895		
1850	.07161	1850	.03058		
1900	.07403	1900	.03225		
1950	.07798	1950	.03397		
2000	.08203	2000	.03573		

TABLE X.

TABLE to reduce Links into Feet, Inches, and Decimals.

Links.	Ft. in. d.	Links.	Ft. in. d.	Links.	Ft. in. d.	Links.	Ft. in. d.
1	0 7 92	26	17 1 92	51	33 7 92	76	50 1 92
2	1 3 84	27	17 9 84	52	34 3 84	77	50 9 84
3	1 11 76	28	18 5 76	53	34 11 76	78	51 5 76
4	2 7 68	29	19 1 68	54	35 7 68	79	52 1 68
5	3 3 60	30	19 9 69	55	36 3 60	80	52 9 60
6	3 11 52	31	20 5 52	56	36 11 52	81	53 5 52
7	4 7 44	32	21 1 44	57	37 7 44	82	54 1 44
8	5 3 36	33	21 9 36	58	38 3 36	83	54 9 36
9	5 11 28	34	22 5 28	59	38 11 28	84	55 5 28
10	6 7 20	35	23 1 20	60	39 7 20	85	56 1 20
11	7 3 12	36	23 9 12	61	40 3 12	86	56 9 12
12	7 11 04	37	24 5 4	62	40 11 4	87	57 5 4
13	8 6 96	38	25 1 96	63	41 6 96	88	58 0 96
14	9 2 88	39	25 9 88	64	42 2 88	89	58 8 88
15	9 10 80	40	26 4 80	65	42 10 80	90	59 4 80
16	10 6 72	41	27 0 72	66	43 6 72	91	60 0 72
17	11 2 64	42	27 8 64	67	44 2 64	92	60 8 64
18	11 10 56	43	28 4 56	68	44 10 56	93	61 4 56
19	12 6 48	44	29 0 48	69	45 6 48	94	62 0 48
20	13 2 40	45	29 8 40	70	46 2 40	95	62 8 40
21	13 10 32	46	30 4 32	71	46 10 32	96	63 4 32
22	14 6 24	47	31 0 24	72	47 6 24	97	64 0 24
23	15 2 16	48	31 8 16	73	48 2 16	98	64 8 16
24	15 10 08	49	32 4 8	74	48 10 8	99	65 4 8
25	16 6 0	50	33 0 0	75	49 6 0	100	66 0 0

TABLE XI.

TABLE to reduce Decimals to Roods and Perches.

Poles.	Dec.	1 R.	2 R.	3 R.	Poles.	Dec.	1 R.	2 R.	3 R.
1	.006	.256	.506	.756	21	.131	.381	.631	.881
2	.012	.262	.512	.762	22	.137	.387	.637	.887
3	.018	.268	518	.768	23	.143	.393	.643	.893
4	.025	.275	.525	.775	24	.150	.400	.650	.900
5	.031	.281	.531	.781	25	.156	.406	.656	.906
6	.037	.287	.537	.787	26	.162	.412	.662	.912
7	.043	.293	.543	.793	27	.168	.418	.668	.918
8	.050	.300	.550	.800	28	.175	.425	.675	.925
9	.056	.306	.556	.806	29	.181	.431	.681	.931
10	.062	.312	.562	.812	30	.187	.437	.687	.937
11	.068	.318	.568	.818	31	.193	.443	.693	.943
12	.075	.325	.575	.825	32	.200	.450	.700	.950
13	.081	.331	.581	.831	33	.206	.456	.706	.956
14	.087	.337	.587	.837	34	.212	.462	.712	.962
15	.093	.343	.593	.843	35	.218	.468	.718	.968
16	.100	.350	.600	.850	36	.225	.475	.725	.975
17	.106	.356	.606	.856	37	.231	.481	.731	.981
18	.112	.362	.612	.862	38	.237	.487	.737	.987
19	.118	.368	.618	.868	39	.243	.493	.743	.993
20	.125	.375	.625	.875	40	.250	.500	.750	

TABLE XII.

TABLE for reducing the Hypothenuse to the Horizontal Measure.

Deg. Min.	Links.	Deg. Min.	Links.	Deg. Min.	Links.	Rise in feet for 1 chain.	Reduction for 1 chain in links and decimals.
4 3	¼	19 57	6	32 20	15½	1	0.01
5 45	½	20 40	6½	32 50	16	2	0.04
6 38	⅔	21 35	7	33 25	16½	3	0.11
7 1	¾	22 20	7½	33 55	17	4	0.19
8 7	1	23 5	8	34 25	17½	5	0.27
9 40	1¼	23 45	8½	34 55	18	6	0.44
9 57	1½	24 30	9	35 25	18½	7	0.56
10 0	1½	25 10	9½	35 55	19	8	0.74
10 40	1¾	25 50	10	36 25	19½	9	0.94
11 29	2	26 30	10½	36 55	20	10	1.16
12 50	2½	27 10	11	37 20	20½	11	1.40
13 30	2¾	27 45	11½	37 50	21	12	1.66
14 4	3	28 20	12	38 15	21½	13	1.92
15 12	3½	28 55	12½	38 45	22	14	2.24
15 45	3¾	29 30	13	39 15	22½	15	2.61
16 16	4	30 5	13½	39 40	23	16	2.99
17 15	4½	30 40	14	40 5	23½	17	3.39
18 12	5	31 15	14½			18	3.76
19 30	5¾	31 45	15			19	4.23
						20	4.64

Note.—By this table every chain's length measured on the side of a hill must be shortened by the number of links opposite the angle of acclivity or declivity, or as near that angle as may be.

TABLE XIII.

TABLE of Square Links in any number of Acres, Roods, and Perches.

ACRES.		ROODS.		PERCHES.			
No.	Square links.	No.	Square links.	No.	Square links.	No.	Square links.
1	100000	1	25000	1	625	21	13125
2	200000	2	50000	2	1250	22	13750
3	300000	3	75000	3	1875	23	14375
4	400000			4	2500	24	15000
5	500000			5	3125	25	15625
6	600000			6	3750	26	16250
7	700000			7	4375	27	16875
8	800000			8	5000	28	17500
9	900000			9	5625	29	18125
10	1000000			10	6250	30	18750
20	2000000			11	6875	31	19375
30	3000000			12	7500	32	20000
40	4000000			13	8125	33	20605
50	5000000			14	8750	34	21250
60	6000000			15	9375	35	21875
70	7000000			16	10000	36	22500
80	8000000			17	10628	37	23125
90	9000000			18	11250	38	23750
100	10000000			19	11875	39	24375
				20	12500		

TABLE XIV.

TABLE of Multipliers for Polygons from Three to Twelve Sides. See Polygons, p. 39.

No. of sides.	Names.	1 Angles at Centres. ° ′	2 Angles at Circum. ° ′	3 Areas.	4 Radius of Circum.	5 Sides.	6 Radius.	No. of sides.
3	Trigon	120 00	60 00	0.43301	0.57735	1.73205	2.	3
4	Square	90 00	90 00	1.00000	0.70710	1.41421	1.41	4
5	Pentagon	72 00	108 00	1.72048	0.85065	1.17557	1.238	5
6	Hexagon	60 00	120 00	2.59807	1.00000	1.00000	1.156	6
7	Heptagon......	51 25$\frac{5}{7}$	128 34$\frac{2}{7}$	3.63391	1.15238	0.86776	1.11	7
8	Octagon	45 00	135 00	4.82843	1.30656	0.76536	1.08	8
9	Nonagon.......	40 00	140 00	6.18182	1.46190	0.68404	1.062	9
10	Decagon.......	36 00	144 00	7.69421	1.61803	0.61803	1.05	10
11	Undecagon....	32 43$\frac{7}{11}$	147 16$\frac{4}{11}$	9.36564	1.77473	0.56346	1.04	11
12	Duodecagon...	30 00	150 00	11.19615	1.93185	0.51763	1.037	12

TABLE XX.

TABLE showing the Weight of a Lineal Foot of Square and Round Bar-Iron.

Square in pounds.	Side and diameter in inches.	Round in pounds.	Square in pounds.	Side and diameter in inches.	Round in pounds.
.208	¼	.163	30.	3	23.56
.468	⅜	.368	32.55	3⅛	25.56
.833	½	.654	35.2	3¼	27.65
1.33	⅝	1.02	37.96	3⅜	29.82
1.87	¾	1.47	40.8	3½	32.07
2.55	⅞	2.	43.8	3⅝	34.4
3.33	1	2.61	46.87	3¾	36.81
4.21	1⅛	3.31	50.05	3⅞	39.31
5.2	1¼	4.09	53.33	4	41.88
6.3	1⅜	4.94	56.71	4⅛	44.54
7.5	1½	5.89	60.2	4¼	47.28
8.8	1⅝	6.91	63.8	4⅜	50.11
10.2	1¾	8.01	67.5	4½	53.01
11.71	1⅞	9.2	71.3	4⅝	56.0
13.33	2	10.47	75.2	4¾	59.06
15.05	2⅛	11.82	79.21	4⅞	62.21
16.87	2¼	13.25	83.33	5	65.45
18.8	2⅜	14.76	92.46	5¼	72.30
20.8	2½	16.36	101.03	5½	79.35
22.96	2⅝	18.03	114.43	5¾	86.73
25.2	2¾	19.79	120.24	6	94.43
27.55	2⅞	21.63			

TABLE XXI.

TABLE showing the Weight in Pounds of a Lineal Foot of Flat Bar-Iron closely hammered.

Breadth in inches.	Thickness in parts of an inch.							Breadth in inches.
	¼	⅜	½	⅝	¾	⅞	1	
1	.83	1.25	1.66	2.08	2.5	2.91	3.33	1
1⅛	.93	1.4	1.87	2.34	2.81	3.28	3.75	1⅛
1¼	1.04	1.56	2.08	2.6	3.12	3.64	4.16	1¼
1⅜	1.14	1.71	2.29	2.86	3.4	4.01	4.58	1⅜
1½	1.25	1.87	2.5	3.12	3.75	4.37	5.	1½
1¾	1.45	2.18	2.91	3.64	4.37	5.1	5.83	1¾
2	1.66	2.5	3.33	4.16	5.	5.83	6.66	2
2¼	1.87	2.81	3.75	4.68	5.62	6.56	7.5	2¼
2½	2.08	3.12	4.16	5.2	6.25	7.29	8.33	2½
2¾	2.29	3.43	4.58	5.72	6.87	8.02	9.16	2¾
3	2.5	3.75	5.	6.25	7.5	8.75	10.	3
3¼	2.7	4.06	5.41	6.77	8.12	9.47	10.83	3¼
3½	2.91	4.37	5.83	7.29	8.75	10.2	11.66	3½
3¾	3.11	4.68	6.25	7.8	9.37	10.93	12.5	3¾
4	3.33	5.	6.66	8.32	10.	11.66	13.33	4
4¼	3.53	5.31	7.08	8.85	10.62	12.39	14.16	4¼
4½	3.74	5.62	7.5	9.36	11.25	13.12	15.	4½
4¾	3.95	5.93	7.91	9.88	11.87	13.85	15.83	4¾
5	4.16	6.25	8.32	10.4	12.5	14.58	16.66	5
5¼	4.37	6.55	8.75	10.92	13.12	15.31	17.49	5¼
5½	4.58	6.86	9.16	11.45	13.75	16.04	18.32	5½
5¾	4.79	7.18	9.58	11.97	14.37	16.77	19.16	5¾
6	5.	7.5	10.	12.5	15.	17.5	20.	6
6¼	5.2	7.81	10.41	13.02	15.62	18.22	20.83	6¼

Weight of ordinary angle iron in lbs. per lineal foot:

Breadth in inches..........	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½
Weight, per foot, in lbs...	1.5	2.45	3.64	4.09	5.8	7.11	8.2	9.18	11.12	13.12

TABLE XXII.

Weight of Hexagon Head and Nut, Square Head and Nut, Rose Head and Nut; will equal that of Rod-Iron of the same Diameter as the Bolts, and of the following Lengths.

Diameter of bolt.	Hexagon head & nut.	Square head & nut.	Rose head & nut.	Diameter of bolt.	Hexagon head & nut.	Square head & nut.	Rose head & nut.
	inches.	inches.	inches.		inches.	inches.	inches.
¼	1¼	1½	1	1⅜	6⅞	8¼	5½
⅜	1⅞	2¼	1½	1½	7½	9	6
½	2½	3	2	1⅝	8⅛	9¾	6½
⅝	3⅛	3¾	2½	1¾	8¾	10½	7
¾	3¾	4½	3	1⅞	9⅜	11¼	7½
⅞	4⅜	5¼	3½	2	10	12	8
1	5	6	4	2¼	11¼	13½	9
1⅛	5⅝	6¾	4½	2½	12½	15	10
1¼	6¼	7½	5				

Thickness of nut = diameter of bolt.
„ head = ½ diameter of bolt.

Whitworth's pitch of thread: inclination of sides of thread is an angle of 55° for all diameters, rounded top and bottom.

Every yard run 1 inch square = 10 lbs.
„ foot super. 1 inch thick = 40 lbs.
Cast-iron is 5 per cent., or 100 lbs. weight, less than wrought-iron.
5 per cent. for rivets on the whole amount of tonnage.
A cube foot of wrought-iron = 480 lbs.

TABLE XXIII.

Table for calculating Girders on the Skew.

Rule: Multiply the width of the Girder on the Square by the Numbers opposite the given Angle.

Deg. min.	Multiply.	Deg. min.	Multiply.	Deg. min.	Multiply.	Deg. min.	Multiply.
19.00	3.0715	24.00	2.4586	29.00	2.0626	34.00	1.7883
.15	3.0331	.15	2.4347	.15	2.0466	.15	1.7768
.30	2.9958	.30	2.4114	.30	2.0380	.30	1.7655
.45	2.9593	.45	2.3885	.45	2.0152	.45	1.7544
20.00	2.9238	25.00	2.3662	30.00	2.0000	35.00	1.7434
.15	2.8892	.15	2.3443	.15	1.1950	.15	1.7326
.30	2.8554	.30	2.3228	.30	1.9703	.30	1.7221
.45	2.8225	.45	2.3018	.45	1.9558	.45	1.7116
21.00	2.7904	26.00	2.2812	31.00	1.9416	36.00	1.7013
.15	2.7591	.15	2.2609	.15	1.9277	.15	1.6912
.30	2.7285	.30	2.2412	.30	1.9139	.30	1.6812
.45	2.6986	.45	2.2217	.45	1.9004	.45	1.6713
22.00	2.6694	27.00	2.2027	32.00	1.8871	37.00	1.6616
.15	2.6410	.15	2.1840	.15	1.8740	.15	1.6521
.30	2.6132	.30	2.1657	.30	1.8611	.30	1.6426
.45	2.5859	.45	2.1477	.45	1.8485	.45	1.6334
23.00	2.5593	28.00	2.1300	33.00	1.8361	38.00	1.6242
.15	2.5333	.15	2.1127	.15	1.8238	.15	1.6153
.30	2.5079	.30	2.0957	.30	1.8118	.30	1.6064
.45	2.4830	.45	2.0790	.45	1.8000	.45	1.5976

(continued)

TABLE XXIII.—*continued.*

Deg. min.	Multiply.	Deg. min.	Multiply.	Deg. min.	Multiply.	Deg. min.	Multiply.
39.00	1.5890	50.15	1.3006	61.30	1.1379	72.45	1.0471
.15	1.5805	.30	1.2957	.45	1.1353	73.00	1.0457
.30	1.5721	.45	1.2913	62.00	1.1326	.15	1.0443
.45	1.5639	51.00	1.2867	.15	1.1300	.30	1.0430
40.00	1.5547	.15	1.2823	.30	1.1274	.45	1.0416
.15	1.5477	.30	1.2778	.45	1.1249	74.00	1.0403
.30	1.5398	.45	1.2734	63.00	1.1224	.15	1.0390
.45	1.5320	52.00	1.2690	.15	1.1199	.30	1.0377
41.00	1.5243	.15	1.2647	.30	1.1174	.45	1.0365
.15	1.5167	.30	1.2605	.45	1.1150	75.00	1.0353
.30	1.5092	.45	1.2563	64.00	1.1126	.15	1.0341
.45	1.5017	53.00	1.2521	.15	1.1102	.30	1.0329
42.00	1.4945	.15	1.2480	.30	1.1080	.45	1.0318
.15	1.4873	.30	1.2440	.45	1.1056	76.00	1.0306
.30	1.4801	.45	1.2420	65.00	1.1034	.15	1.0295
.45	1.4732	54.00	1.2361	.15	1.1012	.30	1.0284
43.00	1.4662	.15	1.2322	.30	1.0990	.45	1.0274
.15	1.4594	.30	1.2283	.45	1.0968	77.00	1.0263
.30	1.4527	.45	1.2245	66.00	1.0947	.15	1.0253
.45	1.4461	55.00	1.2208	.15	1.0925	.30	1.0243
44.00	1.4396	.15	1.2171	.30	1.0904	.45	1.0234
.15	1.4331	.30	1.2134	.45	1.0884	78.00	1.0224
.30	1.4267	.45	1.2098	67.00	1.0864	.15	1.0214
.45	1.4204	56.00	1.2062	.15	1.0844	.30	1.0205
45.00	1.4142	.15	1.2027	.30	1.0824	.45	1.0196
.15	1.4049	.30	1.1992	.45	1.0805	79.00	1.0187
.30	1.4020	.45	1.1958	68.00	1.0785	.15	1.0176
.45	1.3961	57.00	1.1924	.15	1.0766	.30	1.0170
46.00	1.3902	.15	1.1890	.30	1.0748	80.00	1.0154
.15	1.3843	.30	1.1857	.45	1.0730	.30	1.0139
.30	1.3786	.45	1.1824	69.00	1.0711	81.00	1.0124
.45	1.3729	58.00	1.1792	.15	1.0694	.30	1.0111
47.00	1.3673	.15	1.1760	.30	1.0676	82.00	1.0108
.15	1.3618	.30	1.1728	.45	1.0659	.30	1.0086
.30	1.3564	.45	1.1697	70.00	1.0642	83.00	1.0075
.45	1.3510	59.00	1.1666	.15	1.0625	.30	1.0065
48.00	1.3457	.15	1.1636	.30	1.0608	84.00	1.0055
.15	1.3404	.30	1.1606	.45	1.0592	85.00	1.0048
.30	1.3352	.45	1.1576	71.00	1.0576	86.00	1.0039
.45	1.3301	60.00	1.1547	.15	1.0561	87.00	1.0013
49.00	1.3251	.15	1.1518	.30	1.0545	88.00	1.0006
.15	1.3200	.30	1.1490	.45	1.0530	89.00	1.0001
.30	1.3151	.45	1.1461	72.00	1.0514	90.00	1.0000
.45	1.3102	61.00	1.1434	.15	1.0500		
50.00	1.3054	.15	1.1406	.30	1.0485		

TABLE XXIV.

TABLE of the Weight of a Lineal Foot of Cast-Iron Pipes, in Pounds.

Dia. in inches.	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
2	8.8	12.3	16.1	20.3				
2½	10.6	14.7	19.2	23.9				
3	12.4	17.2	22.2	27.6	33.3	39.3	45.6	
3½	14.2	19.6	25.3	31.3	37.6	44.2	51.1	
4	16.1	22.1	28.4	35.0	41.9	49.1	56.6	64.4
4½	18.0	24.5	31.4	38.7	46.2	54.0	62.1	70.6
5	19.8	27.0	34.5	42.3	50.5	58.9	67.6	76.7
5½	21.6	29.5	37.6	46.0	54.8	63.8	73.2	82.8
6	23.5	31.9	40.7	49.7	59.1	68.7	78.7	88.8
6½	25.3	34.4	43.7	53.4	63.4	73.4	84.2	95.1
7	27.2	36.8	46.8	56.8	67.7	78.5	89.7	101.2
7½	29.0	39.1	49.9	60.7	72.0	83.5	95.3	107.4
8	30.8	41.7	52.9	64.4	76.2	88.4	100.8	113.5
8½	32.9	44.4	56.2	68.3	80.8	93.5	106.5	119.9
9	34.5	46.6	59.1	71.8	84.8	98.2	111.8	125.8
9½	36.3	49.1	62.1	75.5	89.1	103.1	117.4	131.9
10	38.2	51.5	65.2	79.2	93.4	108.0	122.8	138.1
10½	...	54.0	68.2	82.8	97.7	112.9	128.4	144.2
11	...	56.4	71.3	86.5	102.0	117.8	133.9	150.3
11½	...	58.9	74.3	90.1	106.3	122.7	139.4	156.4
12	...	61.3	77.4	93.6	110.6	127.6	145.0	162.6
13	...	...	82.7	101.2	118.2	137.4	154.1	173.5
14	...	...	89.3	108.2	126.5	146.2	165.3	185.2
15	...	...	95.2	115.7	135.3	156.2	176.2	198.1
16	...	...	...	123.3	143.1	166.1	187.5	211.3
17	...	...	...	130.2	152.5	178.5	198.2	223.4
18	...	...	...	137.0	161.2	185.3	209.1	235.6
19	...	...	...	...	169.2	195.7	222.3	247.1
20	...	...	...	...	178.1	205.2	233.2	259.0
21	...	...	...	...	...	214.1	243.5	273.2
22	...	...	...	...	...	223.0	254.8	285.4
23	...	...	...	...	...	233.4	265.5	298.3
24	...	...	...	...	...	245.2	277.5	310.6

Note.—The weight of two flanges of a pipe are generally reckoned equal to the weight of one foot in length.

TABLE XXV.

TABLE for purchasing Leases, Annuities, or Estates.

	YEARS' PURCHASE.							
Years of lease, &c.	3 p. c.	4 p. c.	5 p. c.	6 p. c.	7 p. c.	8 p. c.	9 p. c.	10 p. c.
1	0.97	0.96	0.95	0.94	0.93	0.93	0.92	0.91
2	1.91	1.89	1.86	1.83	1.81	1.78	1.76	1.74
3	2.83	2.78	2.72	2.67	2.62	2.58	2.53	2.49
4	3.72	3.63	3.55	3.47	3.39	3.31	3.24	3.17
5	4.58	4.45	4.33	4.21	4.10	3.99	3.89	3.79
6	5.42	5.24	5.08	4.92	4.77	4.62	4.49	4.36
7	6.23	6.00	5.79	5.58	5.39	5.21	5.03	4.87
8	7.02	6.73	6.46	6.21	5.97	5.75	5.53	5.34
9	7.79	7.44	7.11	6.80	6.52	6.25	6.00	5.76

(*continued*)

Table XXV.—*continued.*

YEARS' PURCHASE.

Years of lease, &c.	3 p. c.	4 p. c.	5 p. c.	6 p. c.	7 p. c.	8 p. c.	9 p. c.	10 p. c.
10	8.53	8.11	7.72	7.36	7.02	6.71	6.42	6.14
11	9.25	8.76	8.31	7.89	7.50	7.14	6.81	6.50
12	9.95	9.39	8.86	8.38	7.94	7.54	7.16	6.81
13	10.64	9.99	9.39	8.85	8.36	7.90	7.49	7.10
14	11.30	10.56	9.90	9.30	8.75	8.24	7.79	7.37
15	11.94	11.12	10.38	9.71	9.11	8.56	8.06	7.61
16	12.56	11.65	10.84	10.11	9.45	8.85	8.31	7.82
17	13.17	12.17	11.27	10.48	9.76	9.12	8.54	8.02
18	13.75	12.66	11.69	10.83	10.06	9.37	8.76	8.20
19	14.32	13.13	12.09	11.16	10.34	9.60	8.95	8.37
20	14.88	13.59	12.46	11.47	10.59	9.82	9.13	8.51
21	15.42	14.03	12.82	11.76	10.84	10.02	9.29	8.65
22	15.94	14.45	13.16	12.04	11.06	10.20	9.44	8.77
23	16.44	14.86	13.49	12.30	11.27	10.37	9.58	8.88
24	16.94	15.25	13.80	12.55	11.47	10.53	9.71	8.99
25	17.41	15.62	14.09	12.78	11.65	10.67	9.82	9.08
26	17.88	15.98	14.38	13.00	11.83	10.81	9.93	9.16
27	18.33	16.33	14.64	13.21	11.99	10.94	10.03	9.24
28	18.76	16.66	14.90	13.41	12.14	11.05	10.12	9.31
29	19.19	16.98	15.14	13.59	12.28	11.16	10.20	9.37
30	19.60	17.29	15.37	13.77	12.41	11.26	10.27	9.43
31	20.00	17.59	15.59	13.93	12.53	11.35	10.34	9.48
32	20.39	17.87	15.80	14.08	12.65	11.43	10.41	9.53
33	20.77	18.15	16.00	14.23	12.75	11.51	10.46	9.57
34	21.13	18.41	16.19	14.37	12.85	11.59	10.52	9.61
35	21.49	18.67	16.37	14.50	12.95	11.65	10.57	9.64
36	21.83	18.91	16.55	14.62	13.04	11.72	10.61	9.68
37	22.17	19.14	16.71	14.74	13.12	11.78	10.65	9.71
38	22.49	19.37	16.87	14.85	13.19	11.83	10.69	9.73
39	22.80	19.58	17.02	14.95	13.26	11.88	10.73	9.76
40	23.12	19.79	17.16	15.05	13.33	11.93	10.76	9.78
41	23.41	19.99	17.29	15.14	13.39	11.97	10.79	9.80
42	23.70	20.19	17.42	15.23	13.45	12.01	10.81	9.82
43	23.98	20.37	17.55	15.31	13.51	12.04	10.84	9.83
44	24.25	20.55	17.66	15.38	13.56	12.08	10.86	9.85
45	24.52	20.72	17.77	15.46	13.61	12.11	10.88	9.86
46	24.78	20.89	17.88	15.52	13.65	12.14	10.90	9.88
47	25.03	21.04	17.98	15.59	13.69	12.16	10.92	9.89
48	25.27	21.20	18.08	15.65	13.73	12.19	10.93	9.90
49	25.50	21.34	18.17	15.71	13.77	12.21	10.95	9.91
50	25.73	21.48	18.26	15.76	13.80	12.23	10.96	9.92
55	26.77	22.11	18.63	15.99	13.94	12.32	11.01	9.95
60	27.68	22.62	18.93	16.16	14.04	12.33	11.05	9.97
65	28.45	23.05	19.16	16.29	14.10	12.42	11.07	9.98
70	29.12	23.40	19.34	16.39	14.16	12.44	11.08	9.99
75	29.70	23.68	19.49	16.46	14.20	12.46	11.09	9.99
80	30.20	23.92	19.60	16.51	14.22	12.47	11.10	10.00
85	30.63	24.11	19.68	16.55	14.24	12.48	11.10	10.00
90	31.00	24.27	19.75	16.58	14.25	12.49	11.11	10.00
95	31.32	24.40	19.81	16.60	14.26	12.49	11.11	10.00
100	31.60	24.51	19.85	16.62	14.27	12.49	11.11	10.00
Perp.	33.33	25.00	20.00	16.67	14.29	12.50	11.11	10.00

TABLE XXVI.

TABLE for purchasing Leases, Estates, or Annuities, held upon a Single Life.

Years of Age.	YEARS' PURCHASE.					
	3 p. c.	4 p. c.	5 p. c.	6 p. c.	7 p. c.	8 p. c.
1	16.02	13.47	11.56	10.11	8.96	8.05
2	18.60	15.63	13.42	11.72	10.39	9.32
3	19.58	16.46	14.14	12.35	10.94	9.81
4	20.21	17.01	14.61	12.77	11.32	10.15
5	20.47	17.25	14.83	12.96	11.49	10.30
6	20.73	17.48	15.04	13.16	11.67	10.47
7	20.85	17.61	15.17	13.28	11.78	10.57
8	20.89	17.66	15.23	13.34	11.84	10.63
9	20.81	17.63	15.21	13.34	11.85	10.64
10	20.66	17.52	15.14	13.28	11.81	10.61
11	20.48	17.39	15.04	13.21	11.75	10.57
12	20.28	17.25	14.94	13.13	11.69	10.52
13	20.08	17.10	14.83	13.04	11.62	10.46
14	19.87	16.95	14.71	12.95	11.55	10.40
15	19.66	16.79	14.59	12.86	11.47	10.34
16	19.44	16.63	14.46	12.76	11.38	10.27
17	19.22	16.46	14.33	12.66	11.30	10.20
18	19.01	16.31	14.22	12.56	11.23	10.14
19	18.82	16.17	14.11	12.48	11.16	10.08
20	18.64	16.03	14.01	12.40	11.09	10.03
21	18.47	15.91	13.92	12.33	11.04	9.99
22	18.31	15.80	13.83	12.27	10.99	9.95
23	18.15	15.68	13.75	12.20	10.94	9.91
24	17.98	15.56	13.66	12.13	10.89	9.87
25	17.81	15.44	13.57	12.06	10.84	9.82
26	17.64	15.31	13.47	11.99	10.78	9.78
27	17.47	15.18	13.38	11.92	10.72	9.73
28	17.29	15.05	13.28	11.48	10.66	9.69
29	17.11	14.92	13.18	11.76	10.60	9.64
30	16.92	14.78	13.07	11.68	10.54	9.58
31	16.73	14.64	12.97	11.60	10.47	9.53
32	16.54	14.50	12.85	11.51	10.40	9.48
33	16.34	14.35	12.74	11.42	10.33	9.42
34	16.14	14.20	12.62	11.33	10.26	9.36
35	15.94	14.04	12.50	11.24	10.18	9.30
36	15.73	13.88	12.38	11.14	10.10	9.23
37	15.52	13.72	12.25	11.04	10.02	9.16
38	15.30	13.55	12.12	10.93	9.94	9.09
39	15.08	13.38	11.98	10.82	9.85	9.02
40	14.85	13.20	11.84	10.71	9.75	8.94
41	14.62	13.02	11.70	10.59	9.66	8.86
42	14.39	12.84	11.55	10.47	9.56	8.78
43	14.16	12.66	11.41	10.36	9.47	8.70
44	13.93	12.47	11.26	10.24	9.37	8.62
45	13.69	12.28	11.11	10.11	9.26	8.53
46	13.45	12.09	10.95	9.98	9.15	8.44
47	13.20	11.89	10.78	9.85	9.04	8.35
48	12.95	11.69	10.62	9.71	8.93	8.25
49	12.69	11.48	10.44	9.56	8.80	8.15

(*continued*)

TABLE XXVI.—*continued.*

	YEARS' PURCHASE.					
Years of age.	3 p. c.	4 p. c.	5 p. c.	6 p. c.	7 p. c.	8 p. c.
50	12.44	11.26	10.27	9.42	8.68	8.04
51	12.18	11.06	10.10	9.27	8.56	7.94
52	11.93	10.85	9.23	9.13	8.44	7.83
53	11.67	10.64	9.75	8.98	8.31	7.73
54	11.41	10.42	9.57	8.83	8.18	7.61
55	11.15	10.20	9.38	8.67	8.05	7.50
56	10.88	9.98	9.19	8.51	7.91	7.38
57	10.61	9.75	9.00	8.34	7.77	7.26
58	10.34	9.52	8.80	8.17	7.62	7.13
59	10.06	9.28	8.60	8.00	7.47	7.00
60	9.78	9.04	8.39	7.82	7.31	6.86
61	9.19	8.80	8.18	7.64	7.15	6.72
62	9.21	8.55	7.97	7.45	6.99	6.57
63	8.91	8.29	7.74	7.25	6.82	6.42
64	8.61	8.03	7.51	7.05	6.64	6.26
65	8.30	7.76	7.28	6.84	6.45	6.10
66	7.99	7.49	7.03	6.63	6.26	5.92
67	7.68	7.21	6.79	6.41	6.06	5.74
68	7.37	6.93	6.54	6.18	5.86	5.56
69	7.05	6.65	6.28	5.95	5.65	5.37
70	6.73	6.36	6.02	5.72	5.43	5.18
71	6.42	6.08	5.76	5.48	5.22	4.98
72	6.10	5.79	5.50	5.24	5.00	4.78
73	5.79	5.57	5.25	5.00	4.78	4.58
74	5.49	5.23	4.99	4.77	4.57	4.38
75	5.20	4.96	4.74	4.54	4.35	4.18
76	4.93	4.71	4.51	4.32	4.15	3.99
77	4.65	4.46	4.28	4.11	3.95	3.81
78	4.37	4.20	4.04	3.88	3.74	3.61
79	4.08	3.92	3.78	3.64	3.51	3.39
80	3.78	3.64	3.52	3.39	3.28	3.17
81	3.50	3.38	3.26	3.16	3.06	2.96
82	3.23	3.12	3.02	2.93	2.84	2.75
83	2.98	2.89	2.80	2.71	2.63	2.56
84	2.79	2.71	2.63	2.55	2.48	2.41
85	2.62	2.54	2.47	2.40	2.34	2.28
86	2.46	2.39	2.33	2.27	2.21	2.15
87	2.31	2.25	2.19	2.14	2.09	2.04
88	2.19	2.13	2.08	2.03	1.98	1.94
89	2.01	1.97	1.92	1.88	1.84	1.80
90	1.79	1.76	1.72	1.69	1.66	1.63
91	1.50	1.47	1.45	1.42	1.40	1.37
92	1.19	1.17	1.15	1.37	1.12	1.10
93	0.84	0.83	0.82	0.81	0.80	0.79
94	0.54	0.53	0.52	0.52	0.51	0.51
95	0.24	0.24	0.24	0.24	0.23	0.23

TABLE XXVII.

Table of Lineal Measure.

Inches.	Links.	Feet.	Yards.	Poles.	Chains.	Furlongs.	Miles.
7.92	1						
12	1.515	1					
36	4.545	3	1				
198	25	16.5	5.5	1			
792	100	66	22	4	1		
7920	1000	660	220	40	10	1	
63360	8000	5280	1760	320	80	8	1

TABLE XXVIII.

Table of Square Measure.

Sq. links.	Sq. feet.	Sq. yards.	Sq. poles or perches.	Sq. chains.	Sq. Roods.	Sq. acres.	Sq. mile.
2.296	1						
20.661	9	1					
625	272.25	30.25	1				
10000	4356	484	16	1			
25000	10890	1210	40	2.5	1		
100000	43560	4840	160	10	4	1	
64000000	27878400	3097600	102400	6400	2560	640	1

The French acre, or arpent, contains 54540 English square feet.

TABLE XXIX.

Table of Solid Measure.

1728 cubic inches	1 foot.
27 „ feet	1 yard.
50 „ „	1 load of squared timber.
40 „ „	1 load of unhewn timber.

An imperial standard gallon contains 277.274 cubic inches.

TABLE XXX.

Table of Avoirdupois Weight.

Drachms.	Ounces.	Pounds.	Quarters.	Cwts.	Ton.
16	1				
256	16	1			
7168	448	28	1		
28672	1792	112	4	1	
573440	35840	2240	80	20	1

175 lbs. troy = 144 lbs. avoirdupois.

TABLE XXXI.

Equivalents of Avoirdupois Weight, 112 lbs. the Integer.

Pounds.	Decimals.	Ounces.	Decimals.	Quarters.	Decimals.
1	.008928	1	.000558	1	.25
2	.017857	2	.001116	2	.5
3	.026786	3	.001674	3	.75
4	.035714	4	.002232		
5	.044643	5	.002790		
6	.053571	6	.003348		
7	.0625	7	.003906		
8	.071428	8	.004464		
9	.080357				
10	.089286				
11	.098214				
12	.107143				
13	.116071				
14	.125				

Note.—The decimals for any quantity greater than in the annexed table, take out the different parts and add them together, thus: 5 + .142857 + .002790 = 2 cwts. 16 lbs. 5 ozs.

TABLE XXXII.

Equivalents of English Coin, £1 the Integer.

Shillings.	Decimals.	Pence.	Decimals.	Farthings.	Decimals.
1	.05	1	.004166	1	.0010416
2	.1	2	.008333	2	.0020833
3	.15	3	.0125	3	.003125
4	.2	4	.016666		
5	.25	5	.020833		
6	.3	6	.025		
7	.35				
8	.4				
9	.45				
10	.5				
11	.55				
12	.6				
13	.65				
14	.7				
15	.75				
16	.8				
17	.85				
18	.9				
19	.95				

TABLE XXXIII.

Equivalents of Feet and Yards, Integer One Foot and One Yard.

Inches.	Feet.	Yards.	Inches.	Feet.	Yards.
1	.083333	.027777	7	.583333	.194444
2	.166666	.055555	8	.666666	.222222
3	.25	.083333	9	.75	.25
4	.333333	.111111	10	.833333	.277777
5	.416666	.138888	11	.916666	.305555
6	.5	.166666	12	1.000000	.333333

TABLE XXXIV.

Weight of Materials per Cubic Foot, and Number of Feet to a Ton.

	Weight of cubic feet in lbs.	Cube foot per ton.
Brick, red	135	17
„ common	110	20⅜
„ London stock	115	19¾
Cement, Portland	84	26⅝
„ Roman	60	37⅓
Chalk	140	17⅓
Earth	95	23½
Gravel	112	20
Lime stone	53	42¼
Ditto, chalk	44	51
Sand	90	25
Yorkshire stone	143	14½
Portland	131	15
Bath	112	16
Granite	164	13½
Deals	44	66
Fir timber	37	64
Elm	37	60
Oak	58	50
Water, rain	62½	35¾

A cubic yard of earth before digging will occupy 1⅓ cubic yards when dug.

TABLE XXXV.

An English Foot (Long Measure) compared with a Foot of different Countries.

	English inches.		English inches.
Bologna foot	15.534	Grecian foot	12.0875
Dantzic	11.328	Nuremberg	12
Danish	12.504	Paris	12.816
Rhineland	12.396	Swedish	11.733
Strasburg	11.424	Venetian	13.944

TABLE XXXVI.

Length of a Mile in different Countries, given in English Yards.

	Yards.		Yards.
English mile	1760	German mile	5826
Russian ditto	1110	Swedish ditto	7233
Italian ditto	1467	Danish ditto	7233
Scotch and Irish ditto	2200	Small degree, in France	2933
Polish ditto	4400	Mean ditto	3666
Spanish ditto	5028	Large ditto	4400

TABLE XXXVII.

Foreign Money compared with English Money.

Countries.	Name of coin.	Number in £1 English.	Countries.	Name of coin.	Number in £1 English.
England . . .	Shilling .	20	Holland . . .	Florin . .	11.97
America . . .	Dollar . .	4.8	Portugal . .	Milreis. .	4.285
Austria . . .	Florin . .	9.83	Prussia . . .	Dollar . .	6.9
Denmark . .	Dollar . .	9.104	Russia . . .	Rouble .	6.4
France . . .	Franc . .	25.57	Spain . . .	Dollar . .	6.31

TABLE XXXVIII.

Of French Weights and Measures.

The French system of weights and measures at present is made easy of comprehension, it being only necessary to bear in mind the following, namely: The Are, Mètre, Stère, Litre, and Gramme; the others all being related in a tenfold proportion, the preceding part of the word making the distinction, the terminations all being the same.

Thus: Decamètre, Hectomètre, Kilomètre, &c., signifying 10, 100, 1000, &c., times the mètre; and Decimètre, Centimètre, Millimètre, &c., signifying 10, 100, 1000 parts of the mètre; the others in all cases being the same, as Décigramme, Centigramme, Milligramme, &c.

1. The Are is used in superficial measure, and is equal to 119.6046 English square inches.
2. The Mètre is used for lineal measure, or measures of length, and is equal to 39.371 English inches.
3. The Stère is used in solid measure, and is equal to 35.317 English cubic feet.
4. The Litre is used in measures of capacity, and is equal to 61.028 English inches.
5. The Gramme is used in all cases of weights, and is equal to 15.444 English grains.

TABLE XXXIX.

Approximate Rules for various purposes in Decimals.

1. To reduce square inches to square feet, in place of dividing by 144, multiply by .007.

2. To reduce cubic inches to cubic feet, in place of dividing by 1728, multiply by .00058.

3. To reduce cubic feet to ale gallons, in place of dividing by 282, multiply by 6.128; and to reduce cubic inches to ale gallons, multiply by .003546.

4. To reduce cubic feet to wine gallons, in place of dividing by 231, multiply by 7.48; and to reduce cubic inches to wine gallons, multiply by .00433.

5. To reduce cubic feet to imperial gallons, in place of dividing by 277.274, multiply by 6.232; and to reduce cubic inches to imperial gallons, multiply by .003607.

6. To reduce pounds to cwts., in place of dividing by 112, multiply by .00893; and to reduce pounds to tons, in place of dividing by 112 and by 12, multiply by .00044743; or for common purposes, by .00045.

7. To reduce old measure to new:

Corn measure	multiply	by .96943
Wine measure	„	by .83311
Ale measure	„	by 1.01704

8. To reduce new measure to old:

Corn measure	multiply	by 1.03153
Wine measure	„	by 1.20032
Ale measure	„	by .98324

9. To reduce avoirdupois to troy pounds, multiply by 1.21527; and to reduce avoirdupois ounces to troy ounces, multiply by .9115.

10. To reduce troy pounds to avoirdupois pounds, multiply by .823; and to reduce troy ounces to avoirdupois ounces, multiply by 1.1; and to reduce troy grains to avoirdupois drachms, multiply by .03657.

11. To reduce cube inches of cast-iron, wrought-iron, brass, and lead to cwts.:

Cast-iron	multiply	by .00235
Wrought-iron	„	by .002464
Brass	„	by .002553
Lead	„	by .00367

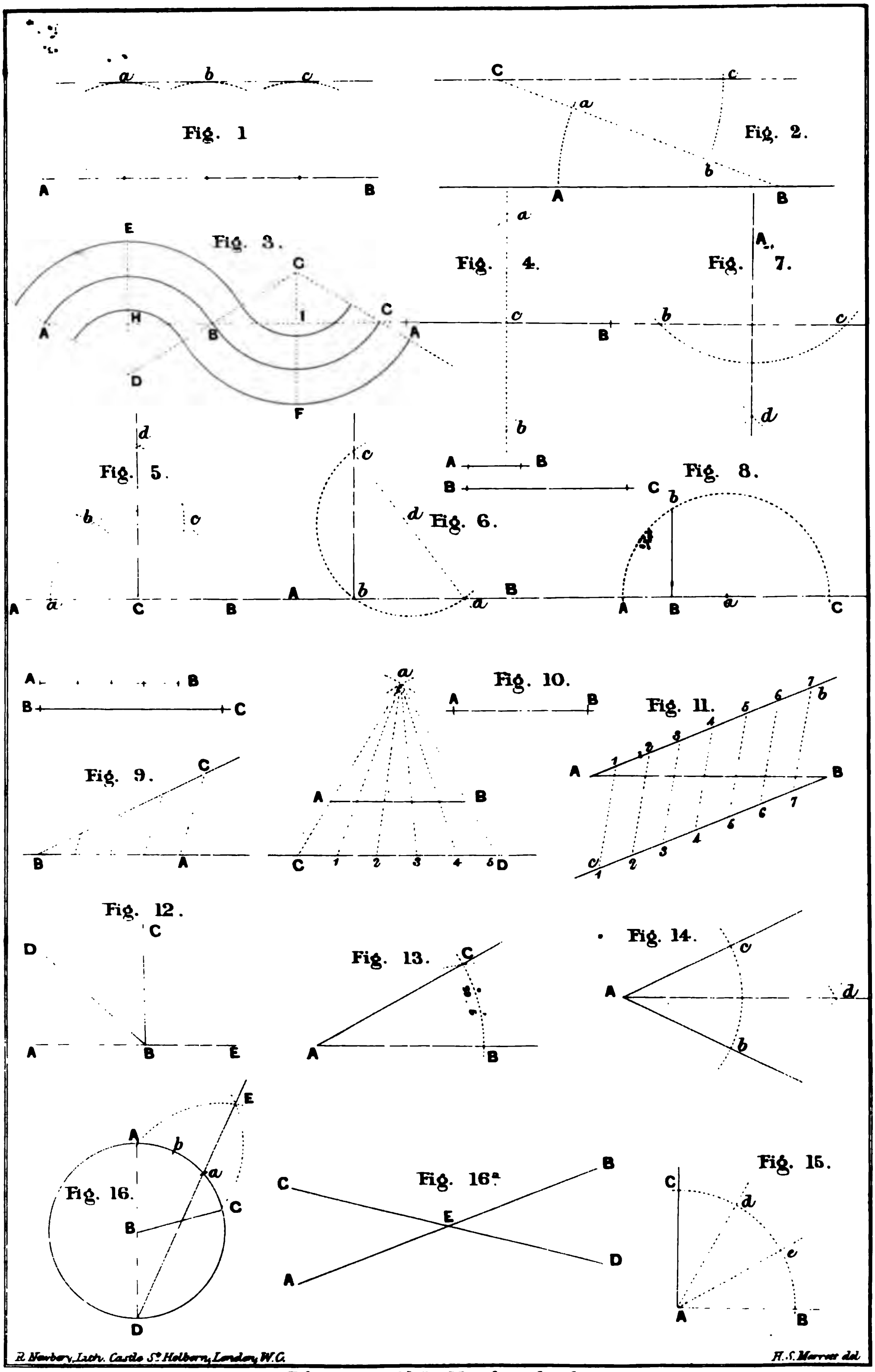
TABLE XL.

Sizes of Drawing Paper.

	Ft. in. Ft. in.		Ft. in. Ft. in.
Antiquarian	4. 4 × 2. 7	Double crown	3. 6 × 1.8
Ditto, extra size	4. 8 × 3. 4	Imperial	2. 6 × 1.9
Double elephant	3. 4 × 2. 2	Super royal	2. 3 × 1.7
Atlas	2. 9 × 2. 2	Royal	2. 0 × 1.7
Columbia	2.10 × 1.11	Medium	1.10 × 1.5
Elephant	2. 3¾ × 1.10¼	Demy	1. 8 × 1.3

THE END.

C. WHITING, BEAUFORT HOUSE, STRAND.



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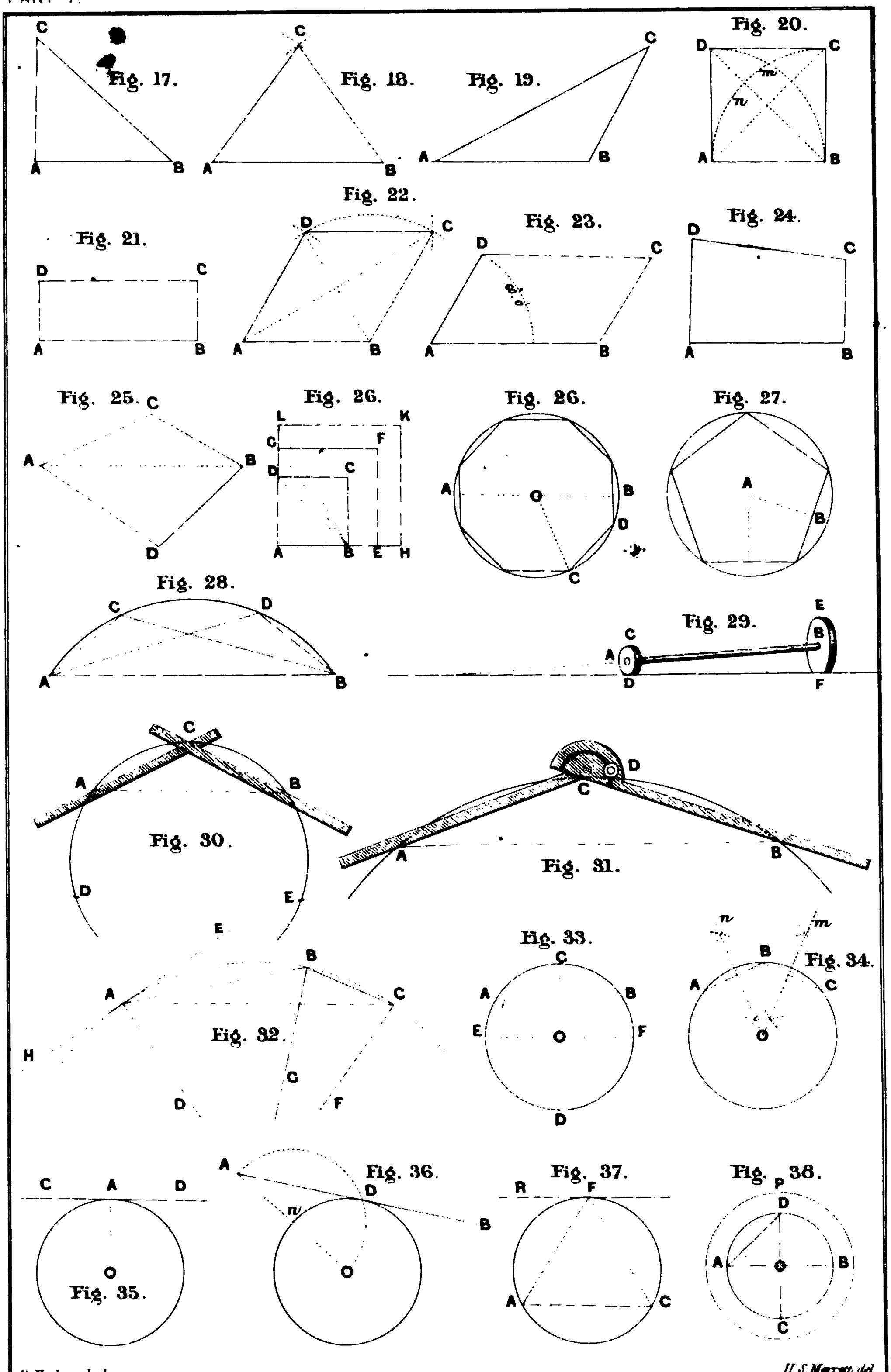

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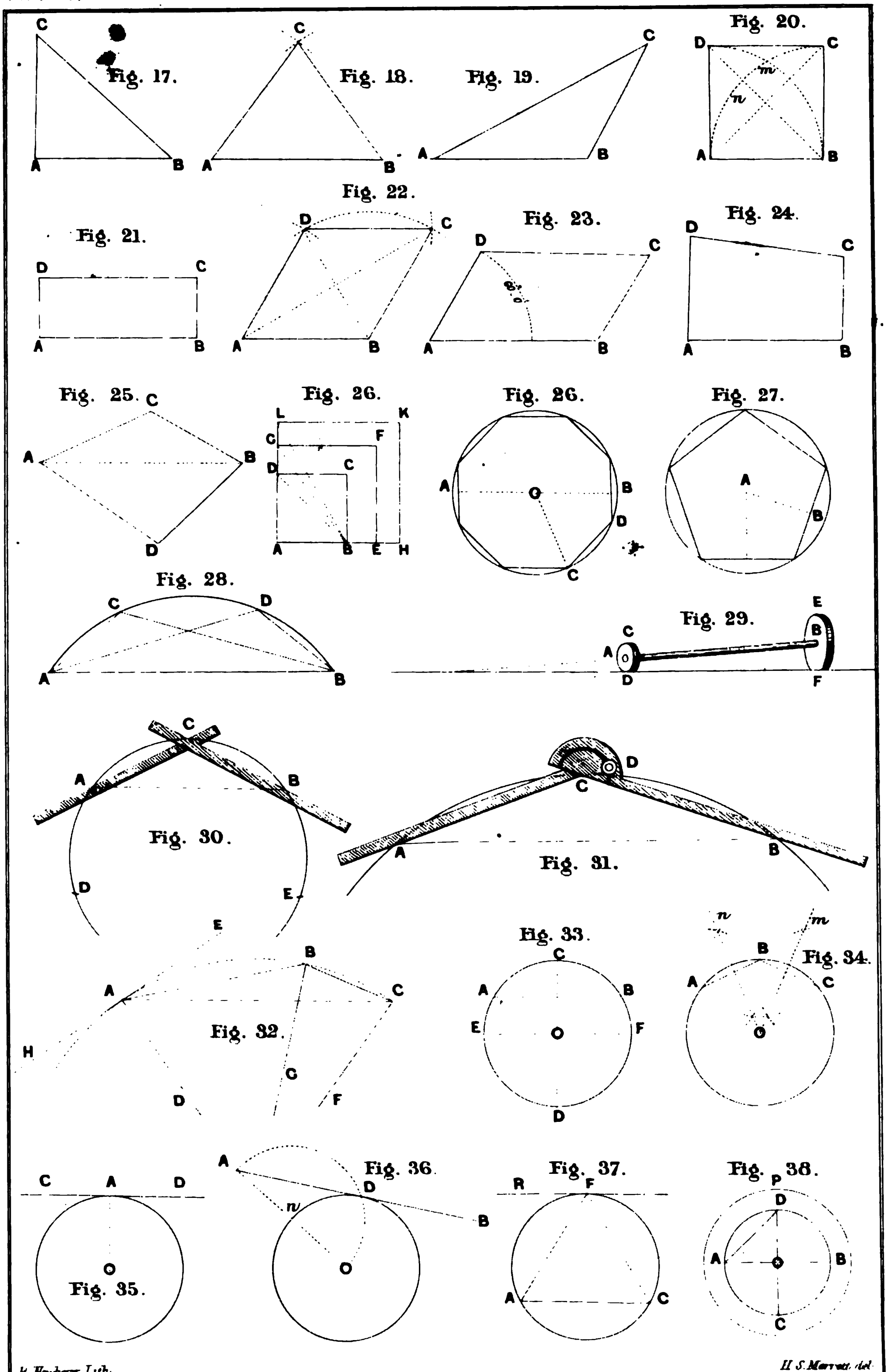

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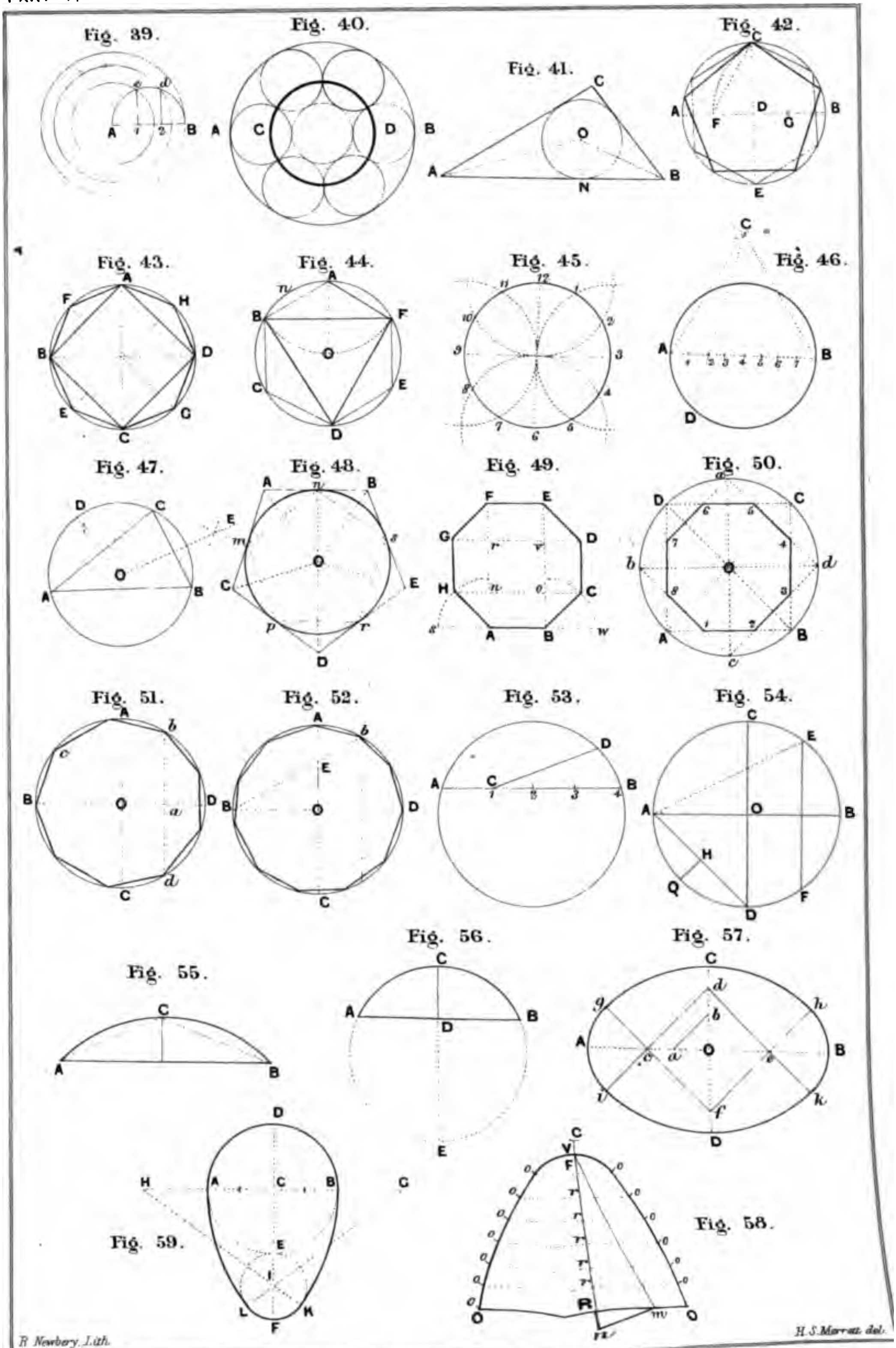

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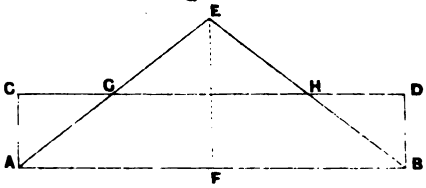
Fig. 61.



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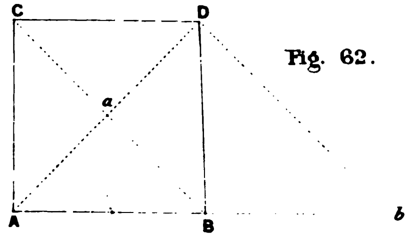

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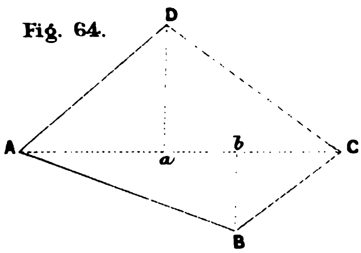
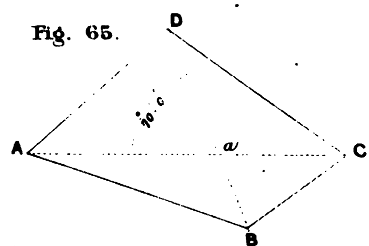


Fig. 65.



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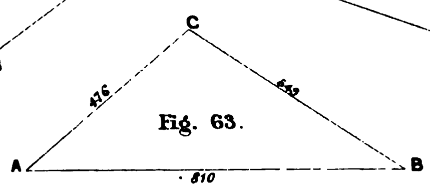

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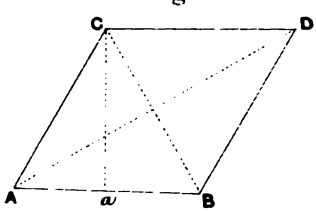

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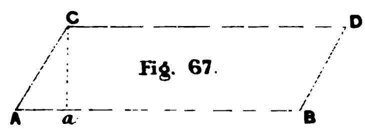

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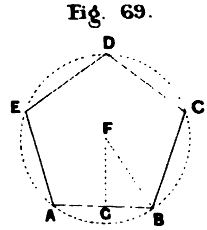

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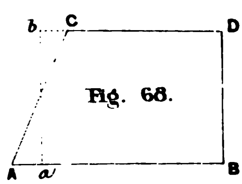

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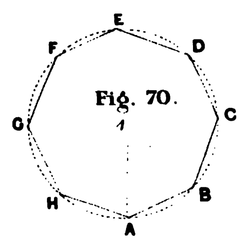

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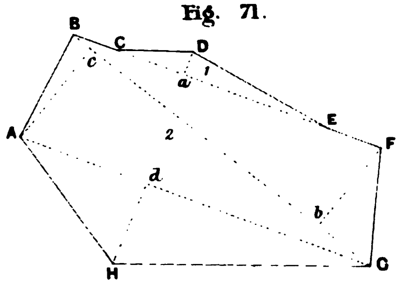

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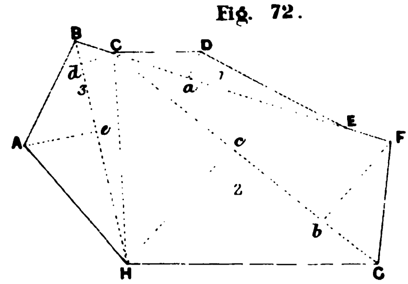

R Newbery Lith.

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E & F. N Spon, 16, Bucklersbury, London

Fig. 73.

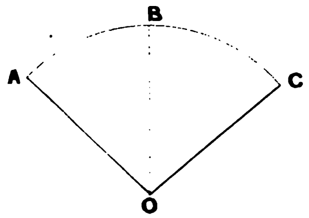


Fig. 74.

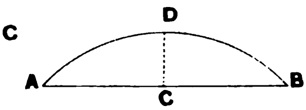


Fig. 75.

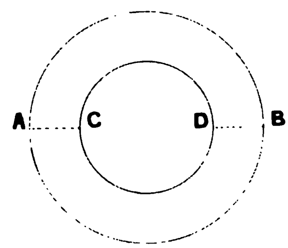


Fig. 77.

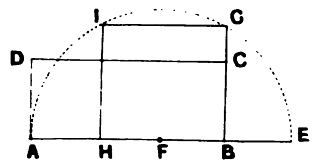


Fig. 76.

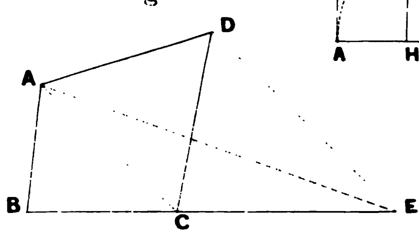
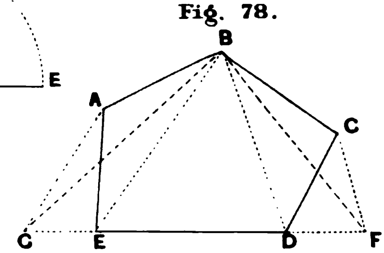


Fig. 78.



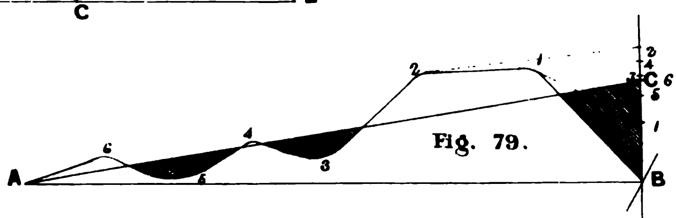
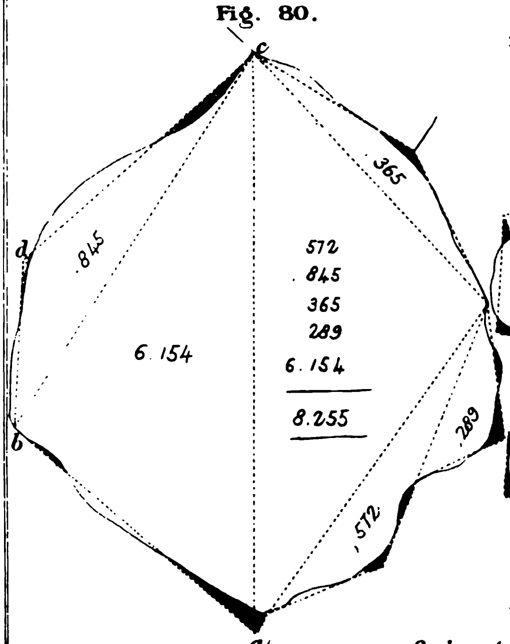
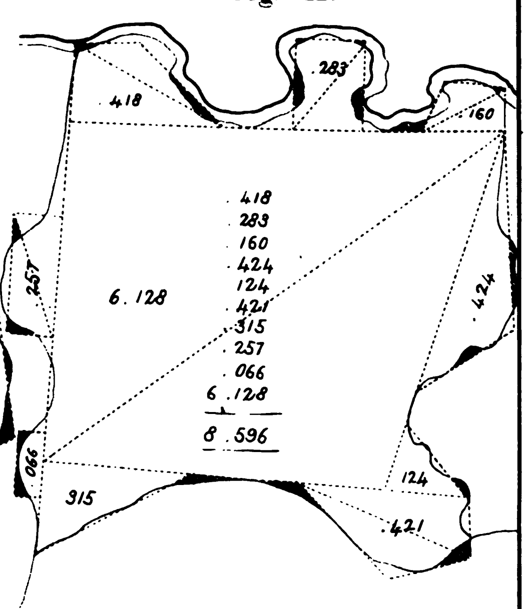


Fig. 79.

Fig. 81.

Fig. 80.





Scale 4 Chains - 1 Inch.

R Newbery, Litho. H.S. Merrett del.

E & F. N. Spon, 16, Bucklersbury, London.

Fig. 82.

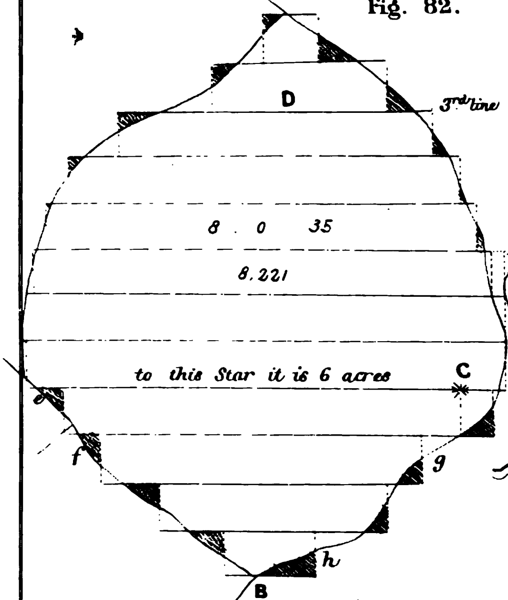


Fig. 83.

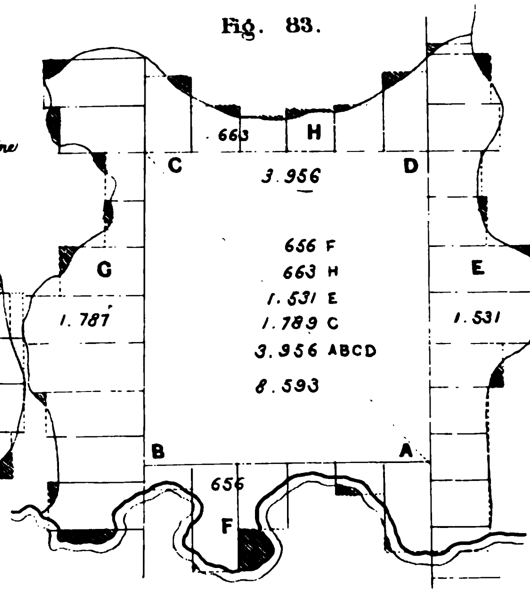


Fig. 84.

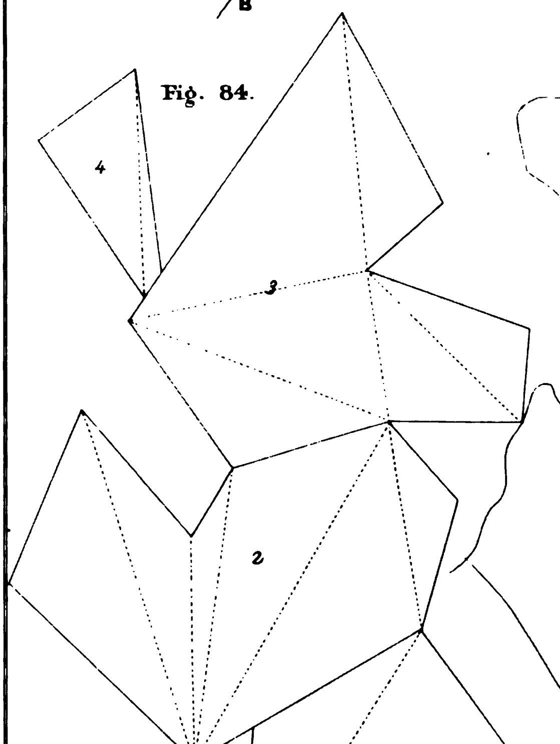
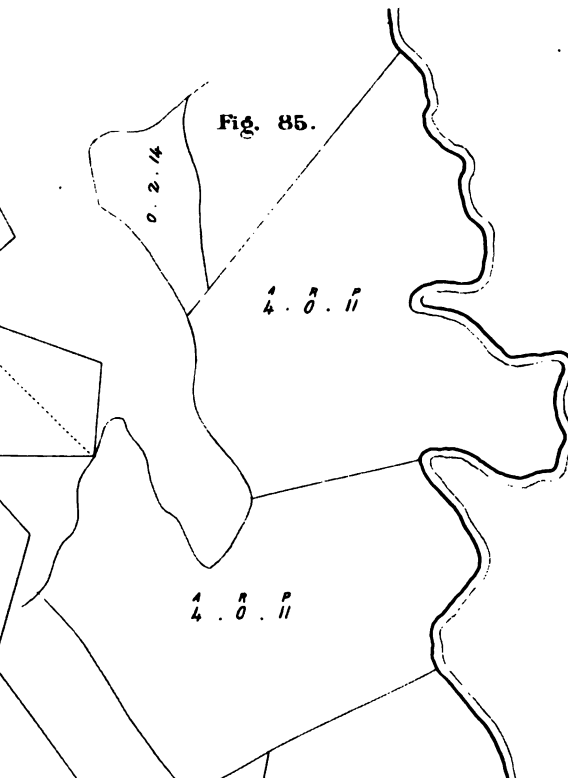


Fig. 85.



SCALE
4 Chains = 1 Inch

R Newbery, Lith.   H. S. Merrett, del.

E & F N. Spon, 16, Bucklersbury, London.

Fig. 1.

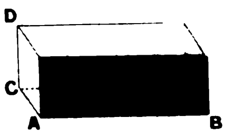


Fig. 2.

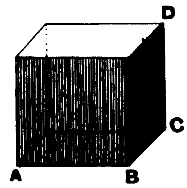


Fig. 3.

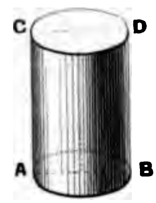


Fig. 4.

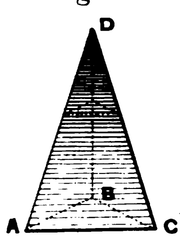


Fig. 5.

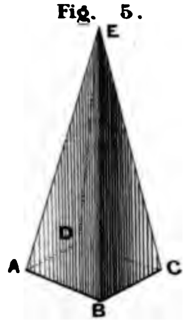


Fig. 6.

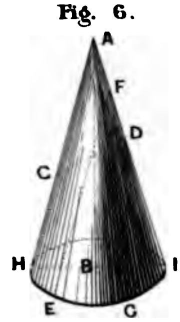


Fig. 7.

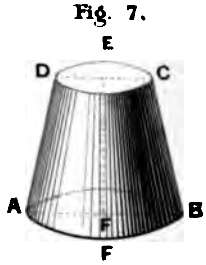


Fig. 8.

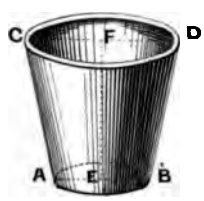


Fig. 9.

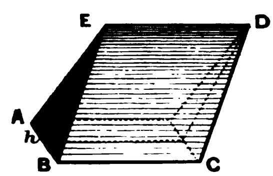


Fig. 10.

Fig. 11.

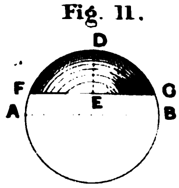
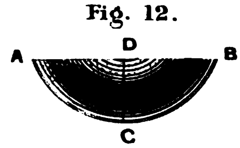


Fig. 12.



R Newbery, Litho
H. S. Merrett del.
E & F. N. Spon, 16, Bucklersbury, London.

Fig. 1.

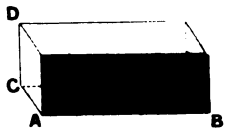


Fig. 2.

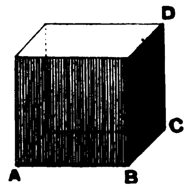


Fig. 3.

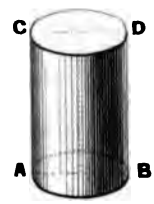


Fig. 4.

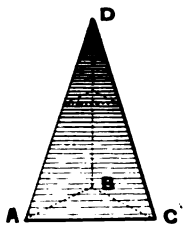


Fig. 5.

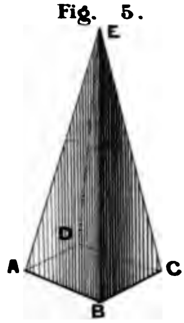


Fig. 6.

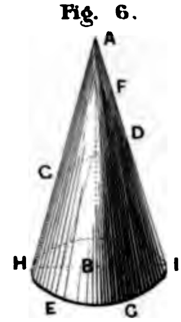


Fig. 7.

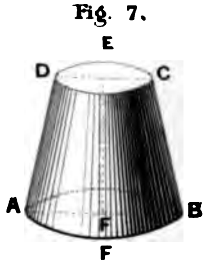


Fig. 8.

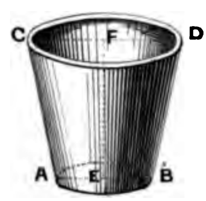


Fig. 9.

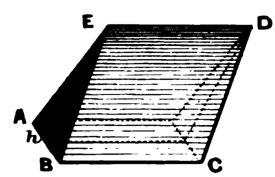


Fig. 10.

Fig. 11.

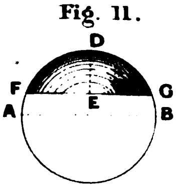
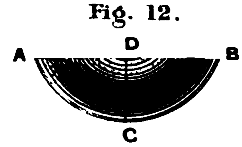


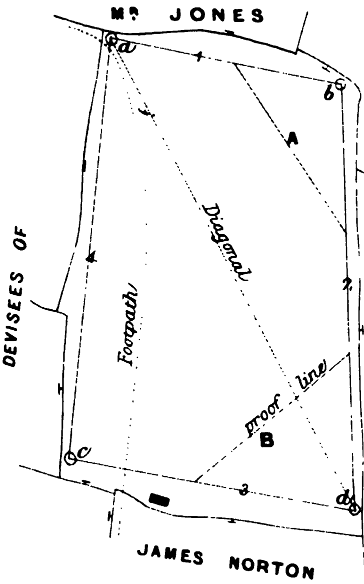
Fig. 12.



R Newbery, Litho H S. Merrett del

E & F. N. Spon, 16, Bucklersbury, London.

Fig. 3.



Scale 4 Chains to the Inch.

Fig. 1.

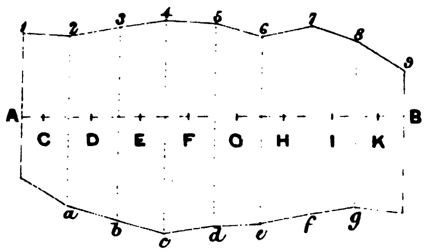
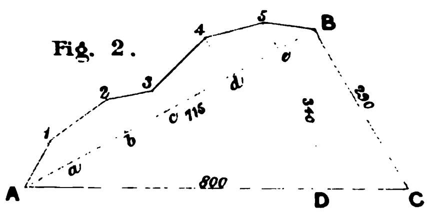
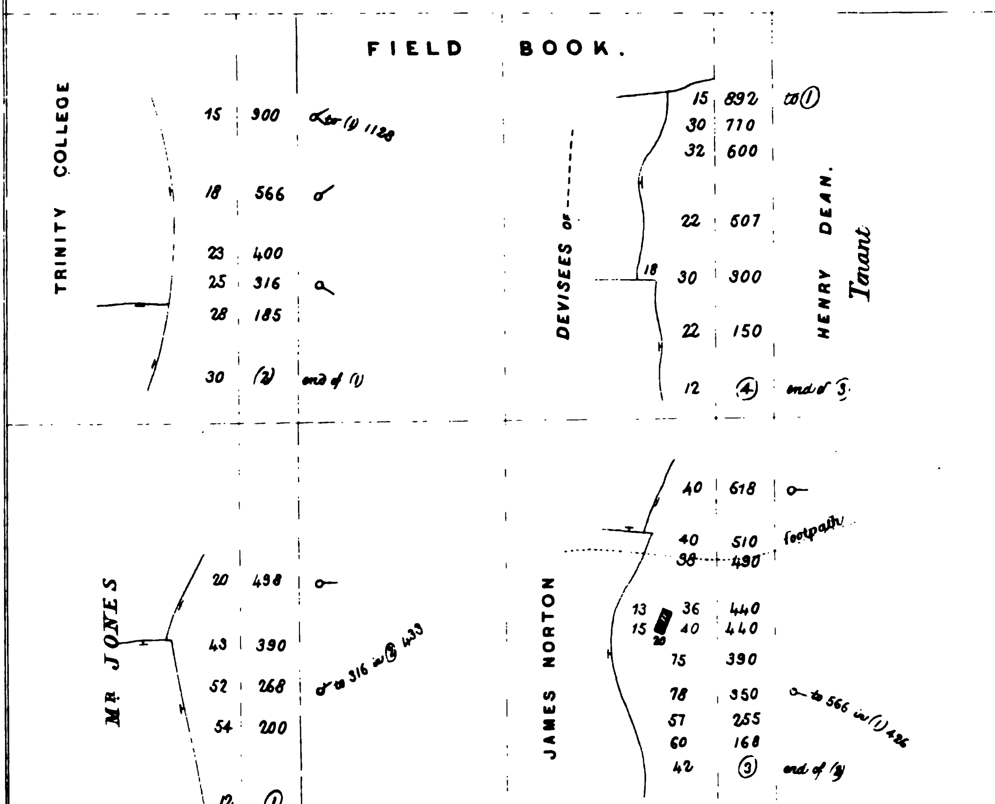


Fig. 2.

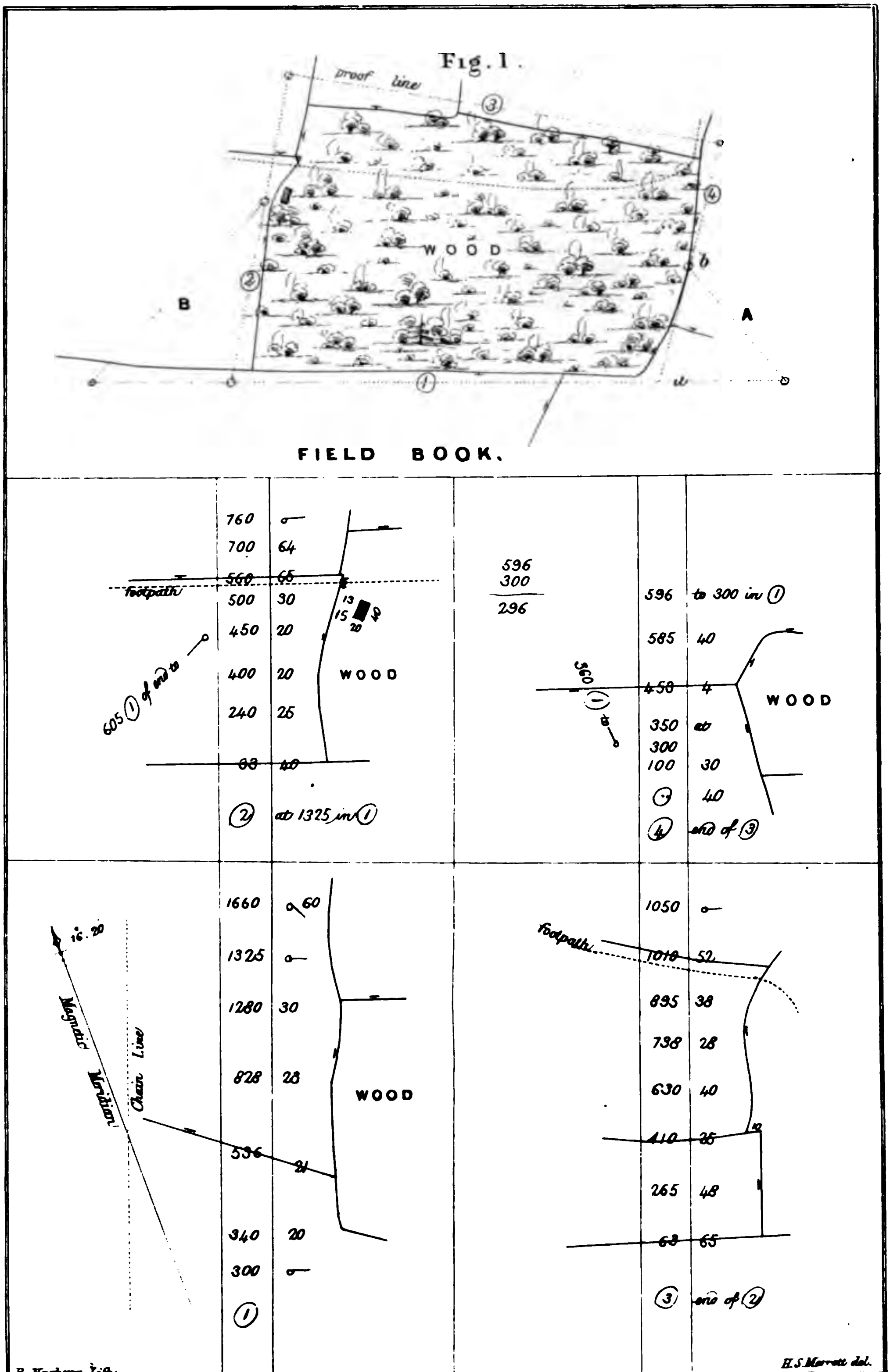


FIELD BOOK.



P. Norbury Lith. H. J. Marriott del.

E & F. N. Spon, 16, Bucklersbury, London



E & F. N. Spon, 16, Bucklersbury, London.

Fig. 82.

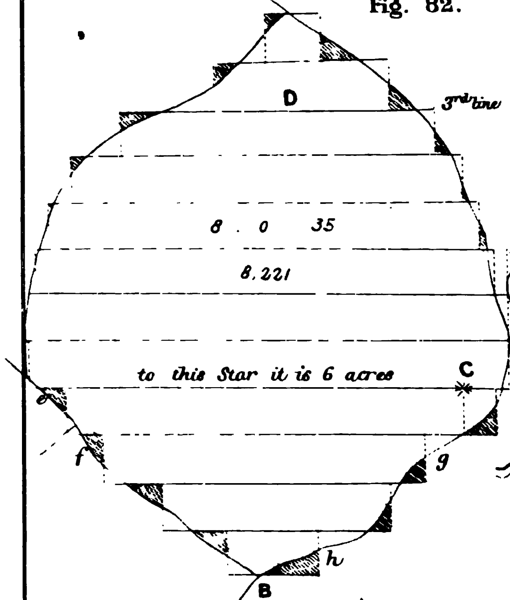


Fig. 83.

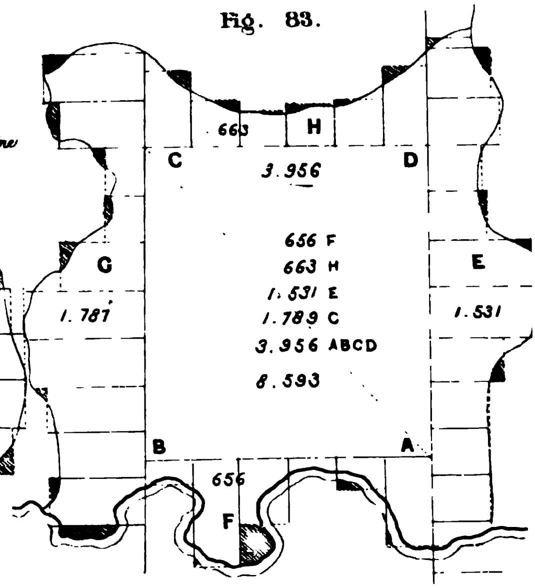


Fig. 84.

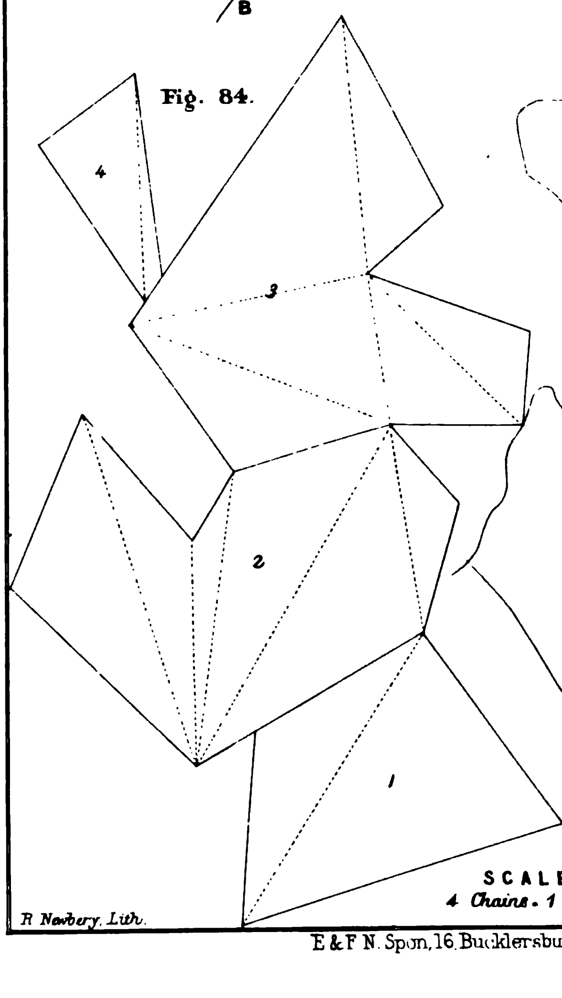
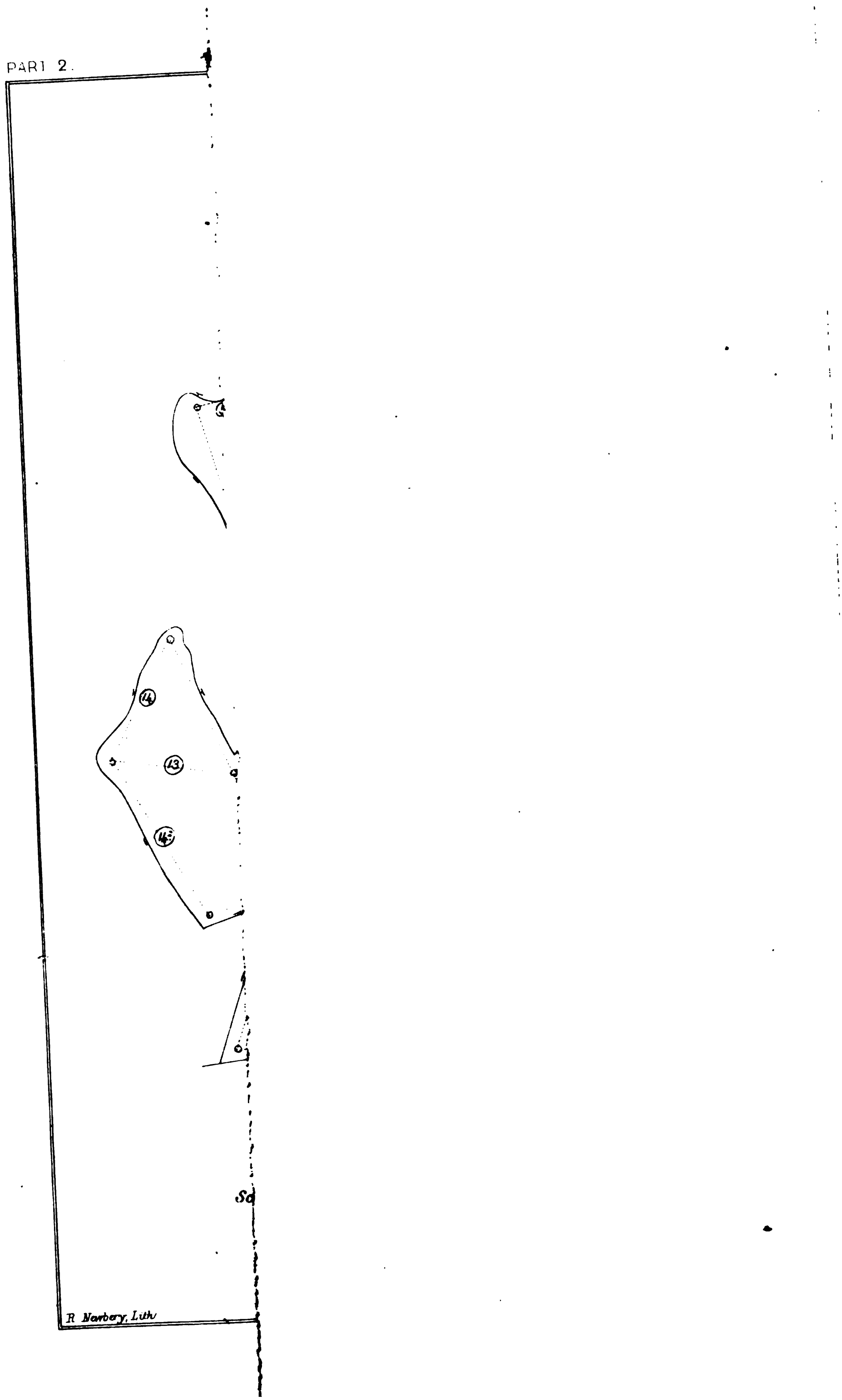


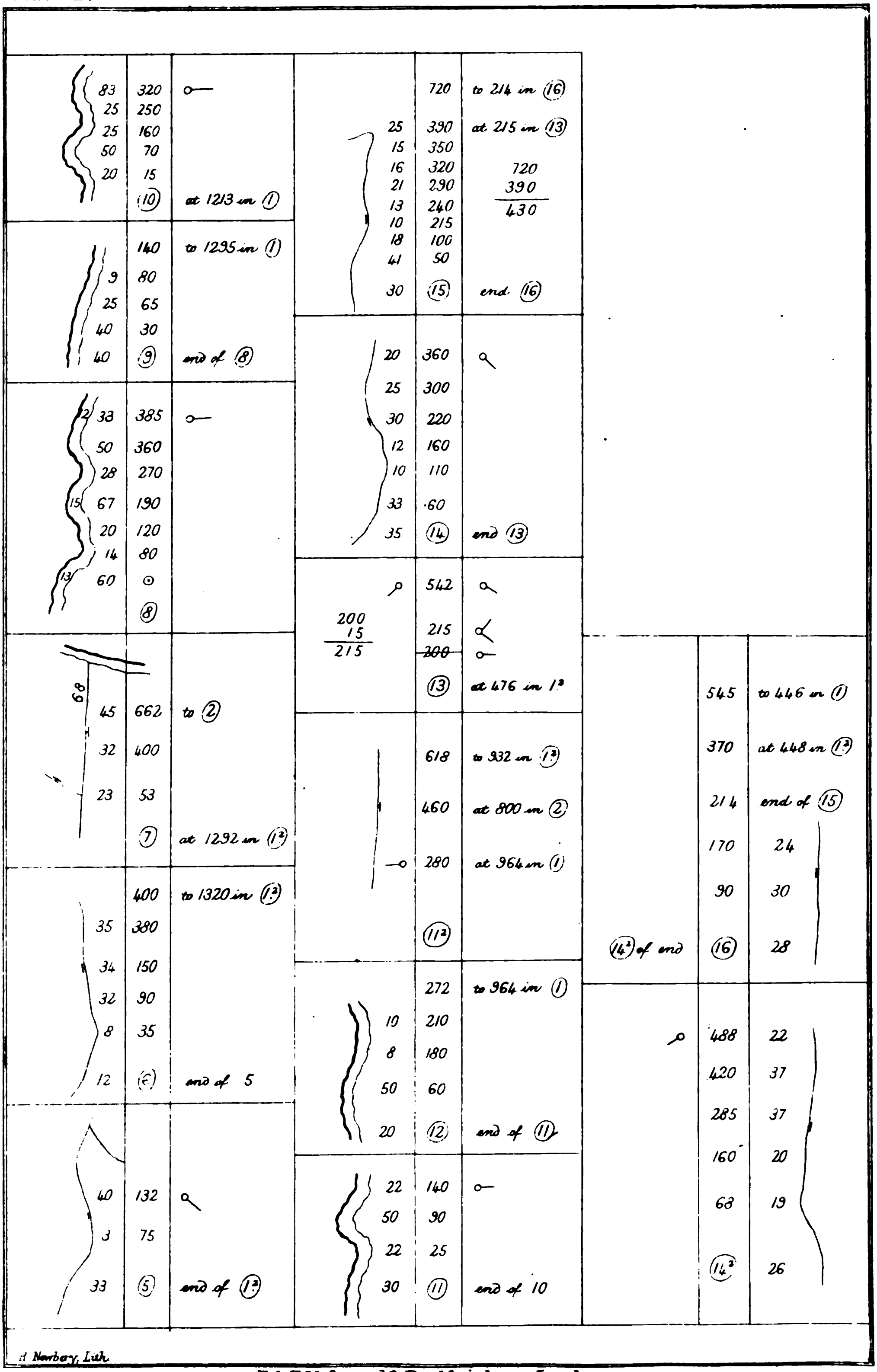
Fig. 85.

SCALE
4 Chains. 1 Inch

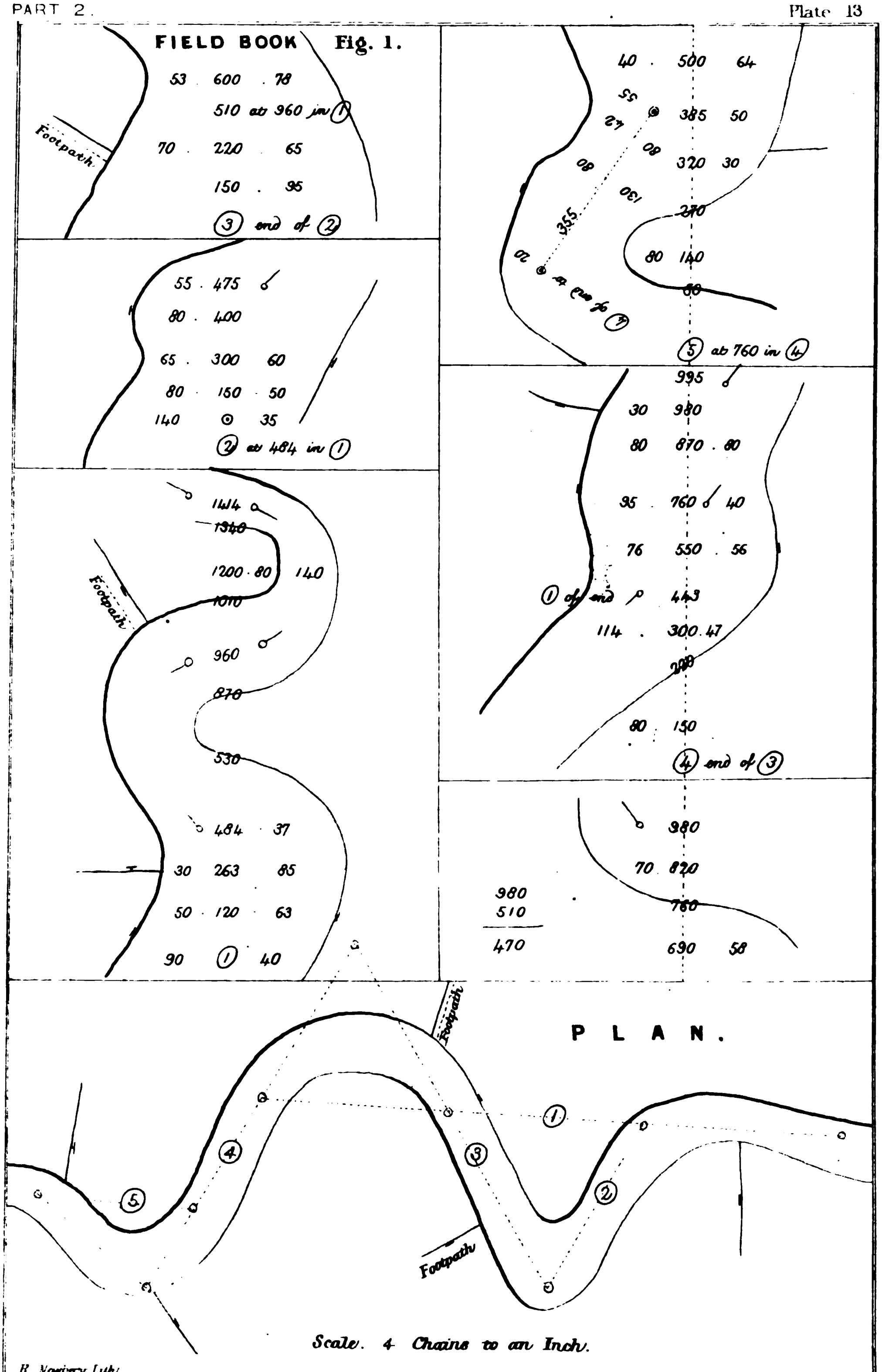
R Newbery, Lith. H.S. Merrett del.
E & F N. Spon, 16, Bucklersbury London.

PART 2.





E. & F. N. Spon, 16, Bucklersbury, London.



E & F.N. Spon, 16, Bucklersbury, London.

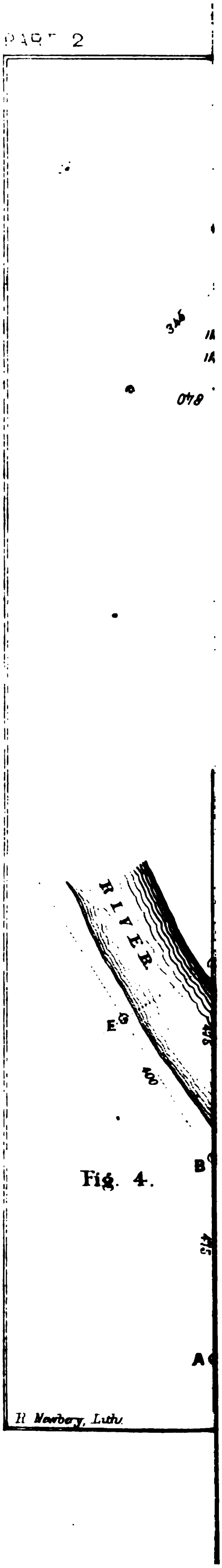
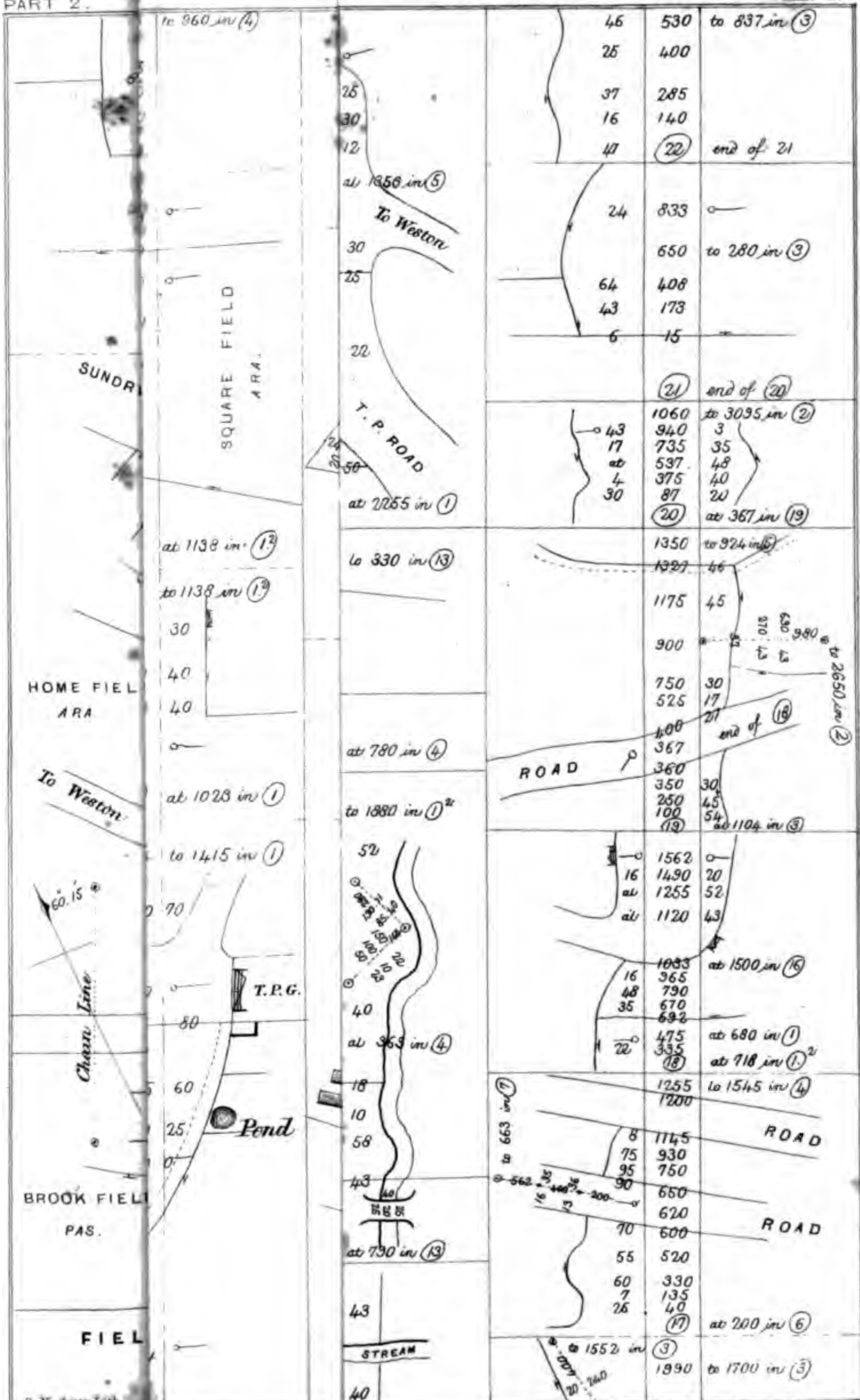


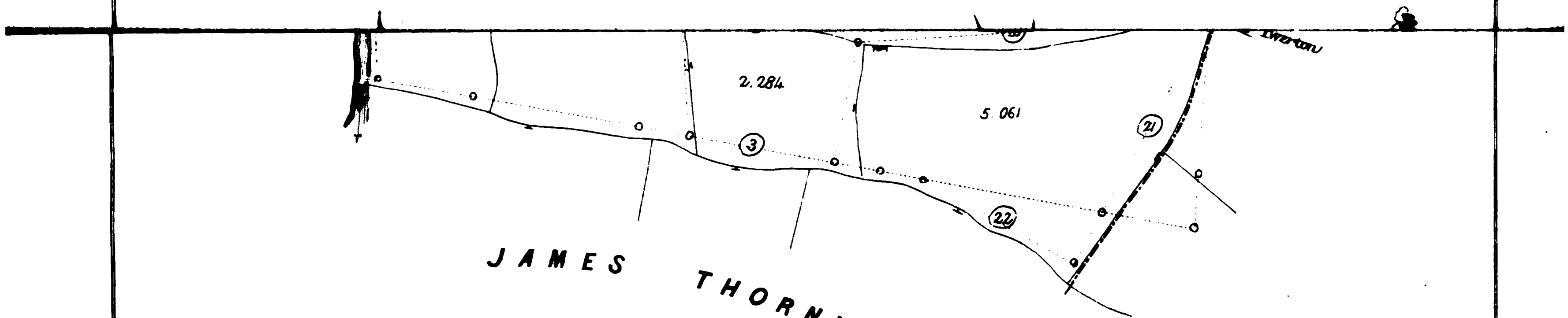
Fig. 4.

R Newbery, Lith.


SUNDRY
SQUARE FIELD
ARA.
HOME FIEL
ARA
To Weston
Chain Line
T.P.G.
Pond
BROOK FIEL
PAS.
FIEL
To Weston
T. P. ROAD
STREAM
ROAD
ROAD
ROAD
end of 21
end of (20)
end of (18)
R Newbery, Lith

PLAN OF AN ESTATE OR PARISH

BY THE CHAIN ONLY.



JAMES THORNHILL, ESQ.

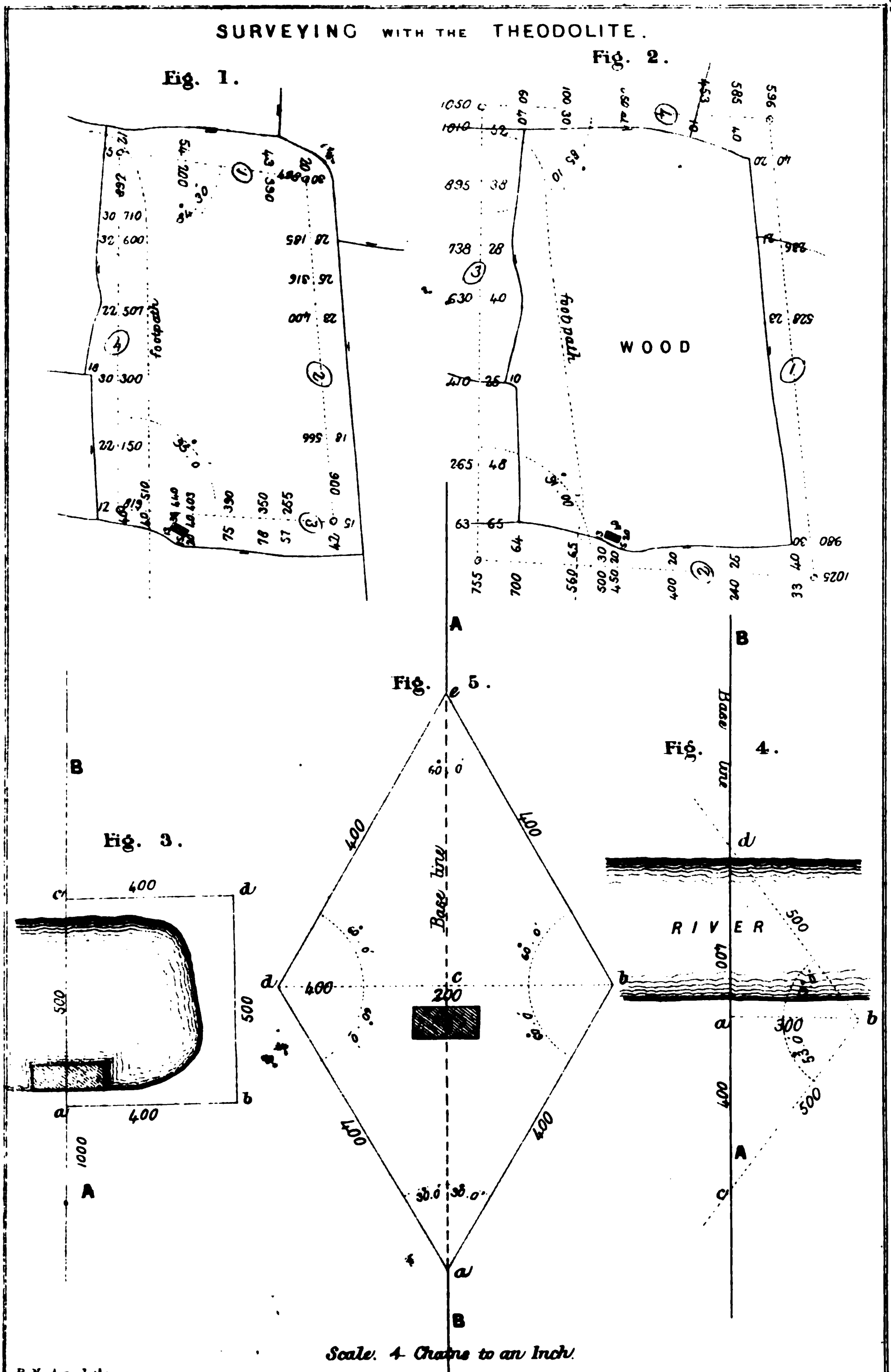
Total $\overset{A}{68}.\ \overset{r}{3}.\ \overset{p}{16}$

Scale. 5 Chains 1 Inch.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

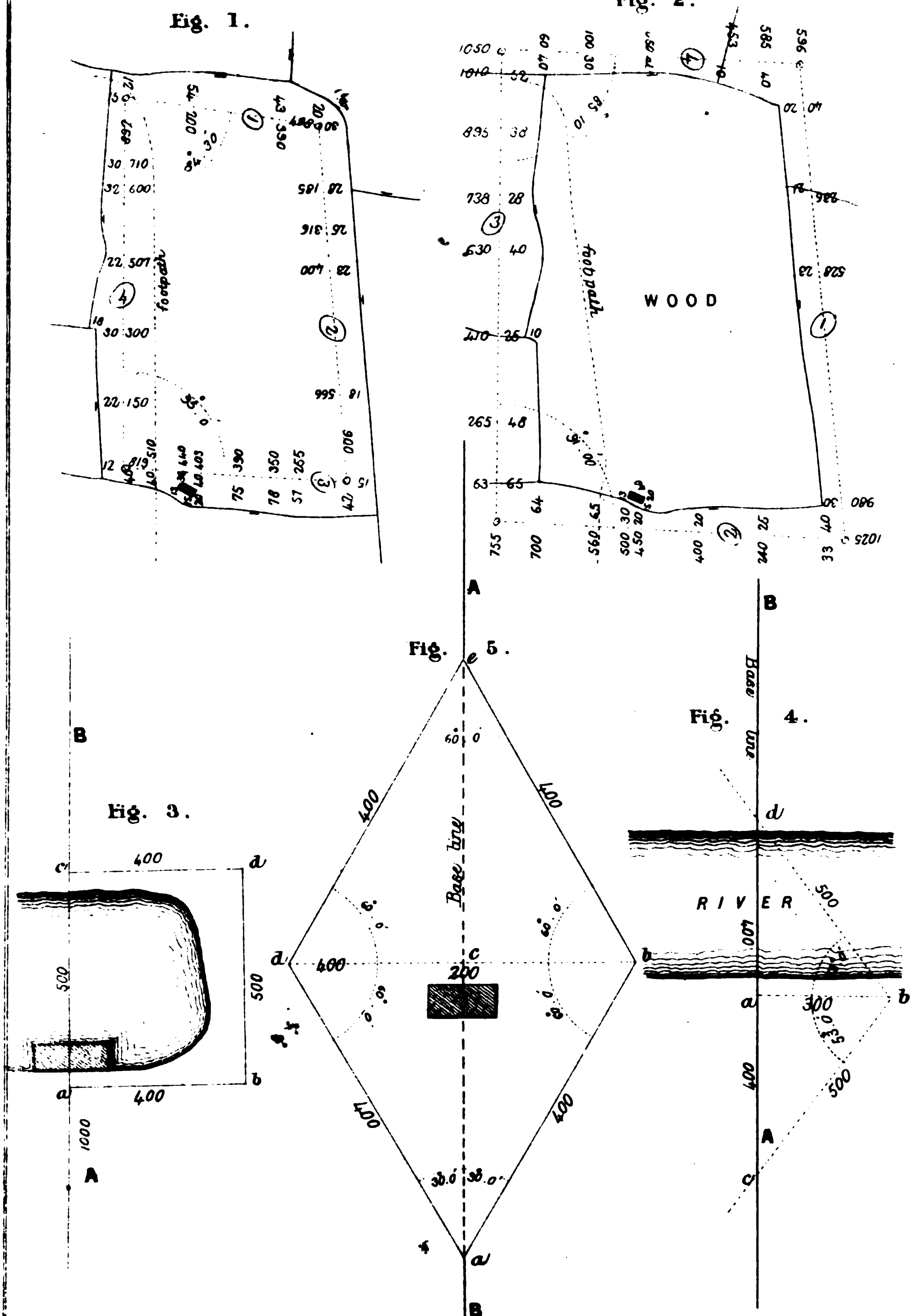
R. Newbery Lith.

E & F N. Spon, 16, Bucklersbury, London



E. & F. N. Spon, 16, Bucklersbury, London.

SURVEYING WITH THE THEODOLITE.

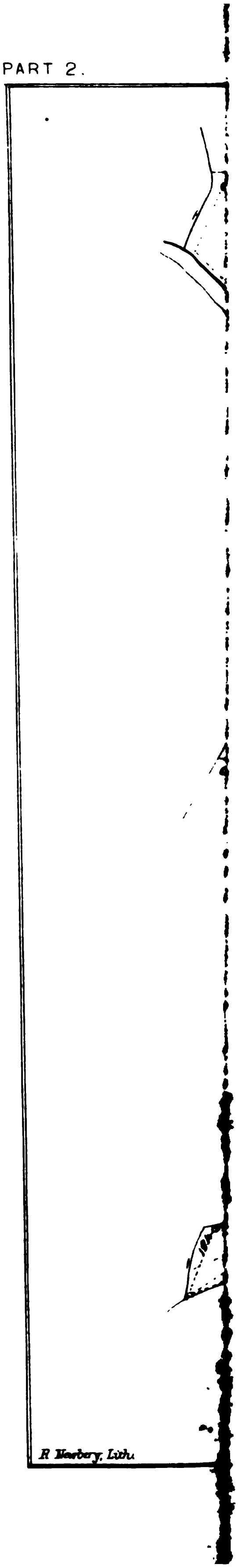


Scale. 4 Chains to an Inch.

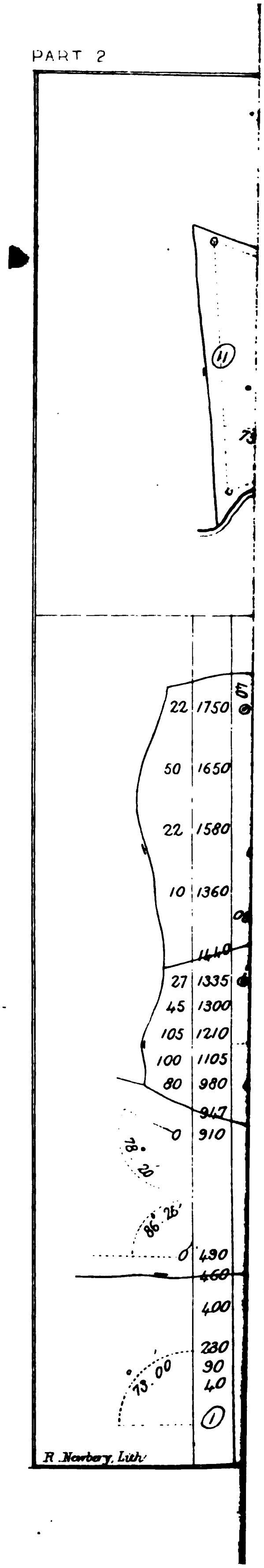
H. Newbery, Lith.

E & F.N. Spon, 16, Bucklersbury, London.

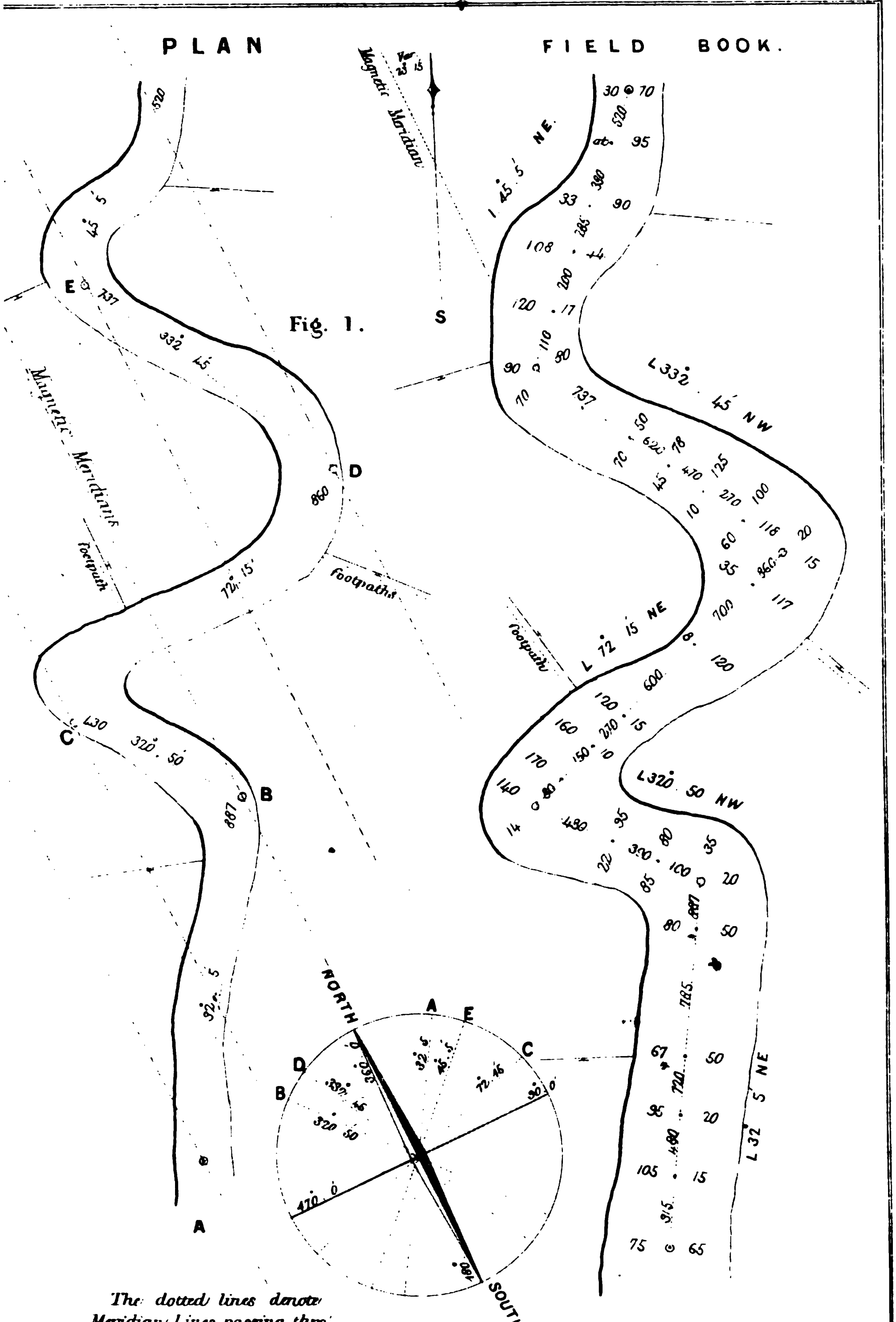
PART 2.



PART 2



11
73
40
22 1750
50 1650
22 1580
10 1360
1440
27 1335
45 1300
105 1210
100 1105
80 980
947
0 910
78·20
86·26'
0 430
460
400
230
90
40
73·00
1
R. Newbery, Lith.

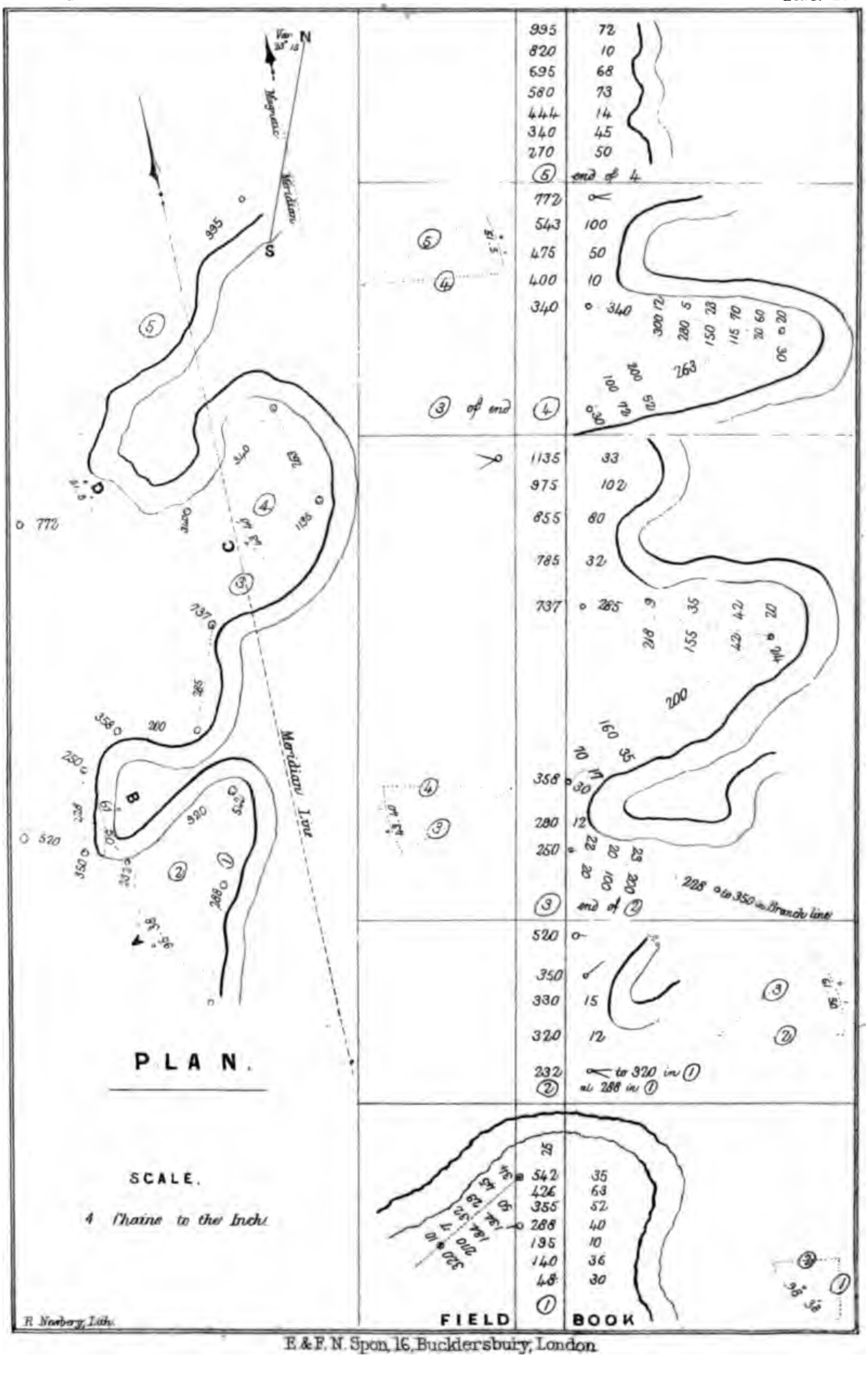


The dotted lines denote Meridian Lines passing thro' each Station point.

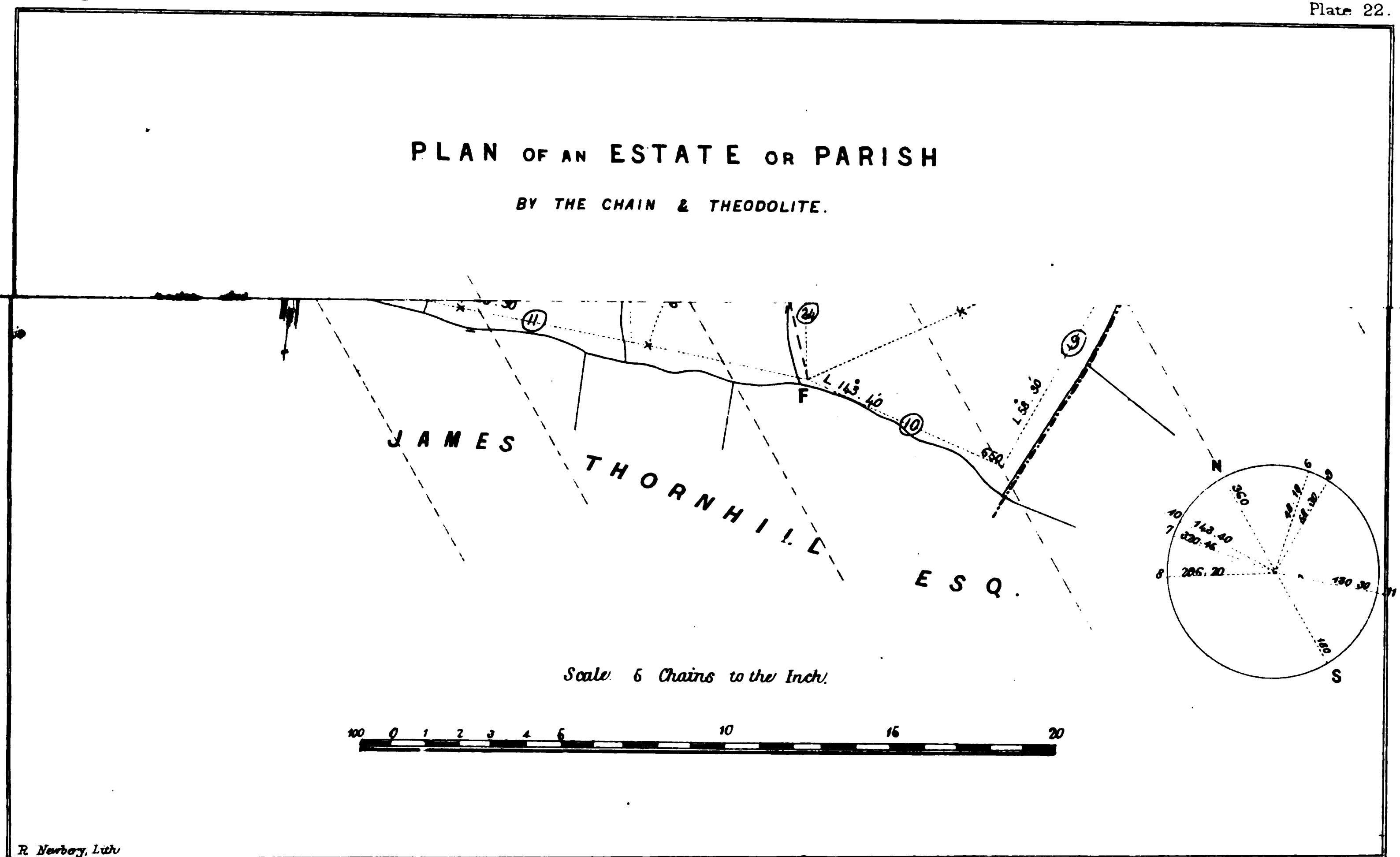
Scale 4 Chains to an Inch.

R. Newbery Lith.

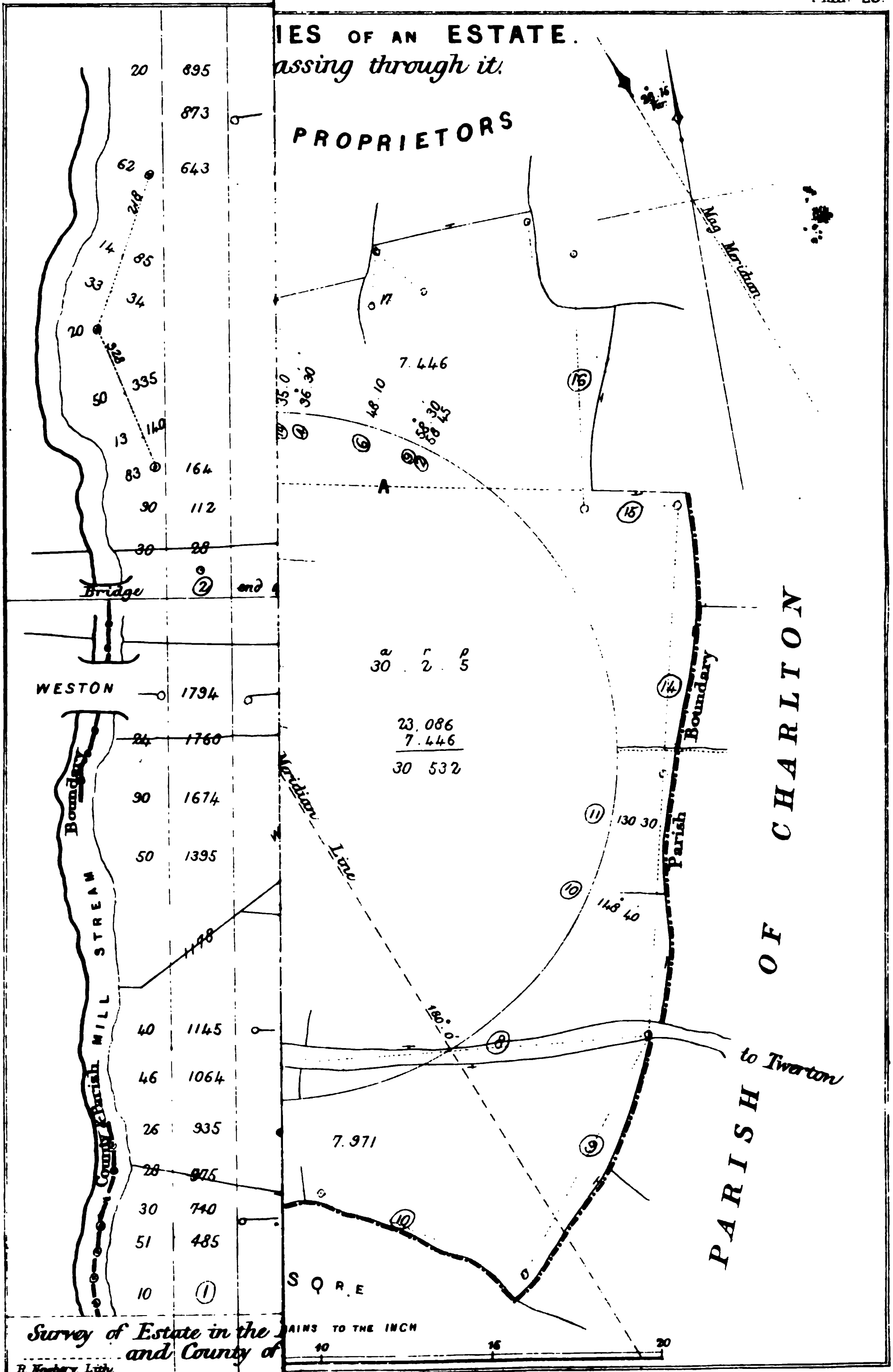
E & F N. Spon. 16, Bucklersbury, London.



E & F. N. Spon, 16, Bucklersbury, London



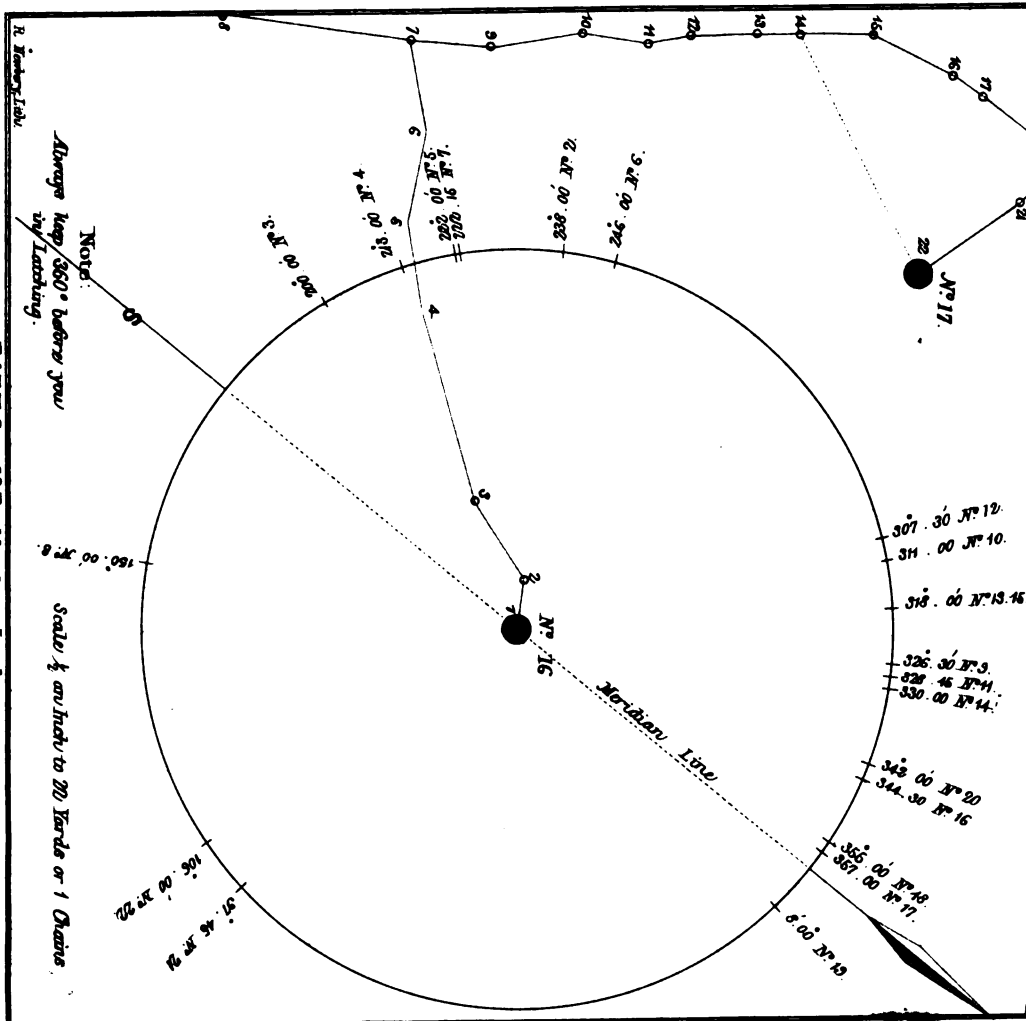
E & F. N. Spon, 16, Bucklersbury, London.


IES OF AN ESTATE.
assing through it.
PROPRIETORS
Mag. Meridian
PARISH OF CHARLTON
Parish Boundary
to Twerton
WESTON
Bridge and
MILL STREAM
County & Parish Boundary
Meridian Line
SQ R. E
Survey of Estate in the
and County of
AINS TO THE INCH
R. Norbury, Lith.

LATCHING BOOK

Fungous Coal. No 16 & 17 Pits latched beginning at Centre of Shaft No 16. from thence to face of work (or Coal) & along the face of Coal to No 17 Pit.

Points	Degrees	Links	
1 2	238°. 0′	62	
3	200 . 0	100	
4	213 . 0	213	
5	222 . 0	30	
6	246 . 0	110	
×7	282 . 15	38	Face of Coal
7 8	150 . 0	200	End of Wall S.W.
7 9	326 30	80	
10	311 . 0	30	
11	328 45	72	
12	307 30	40	
13	318 . 0	72	
14	330 . 0	60	
15	318 . 0	67	
16	344 30	100	
17	357 . 0	38	
18	355 . 0	60	
19	8 . 0′	43	
20	342 . 0	53	End Wall of Coal N.W.
19 21	37 . 45	55	
22	106 . 0	134	to Pit No 17.



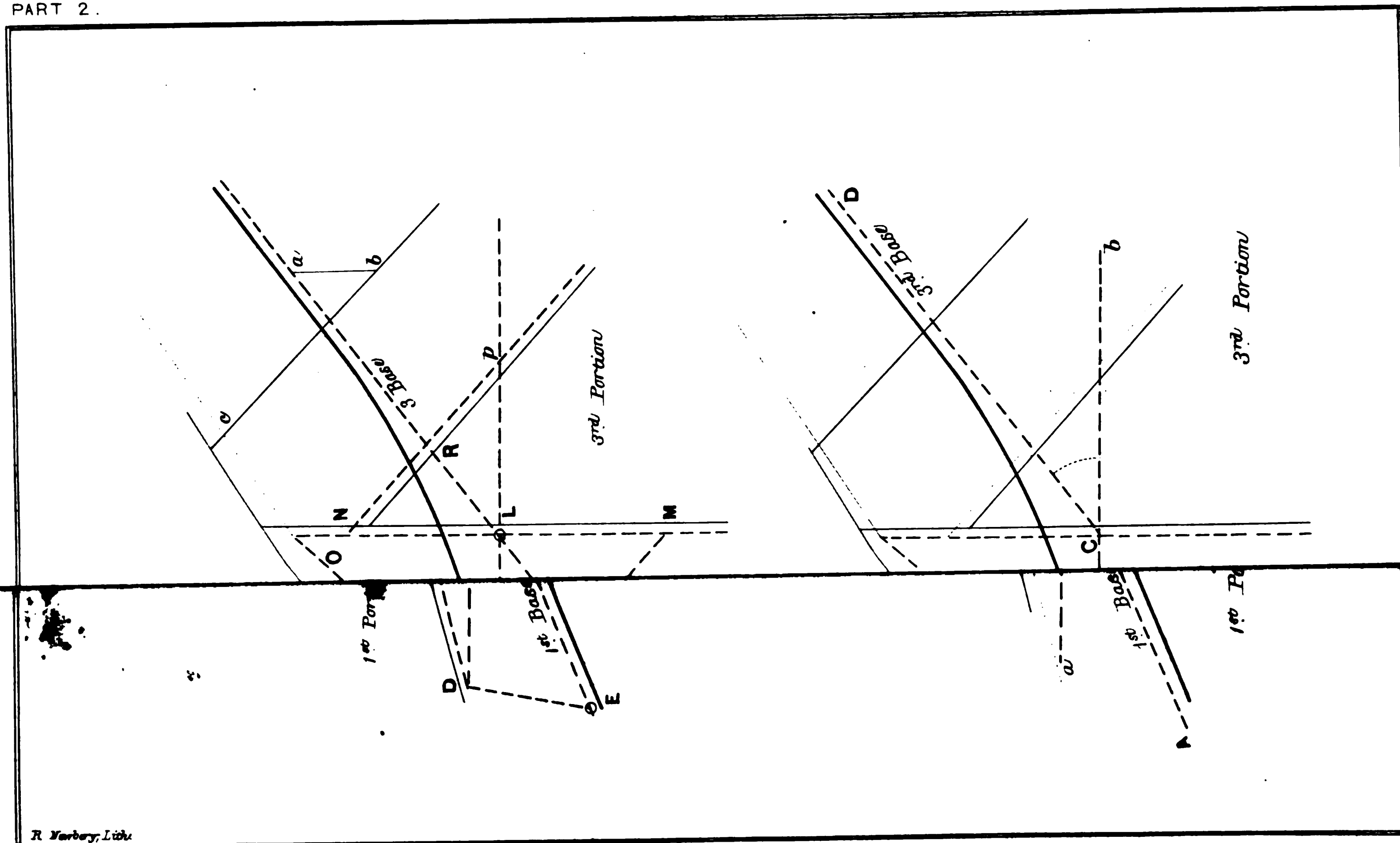
Note: *Always keep 360° before you in Latching.*

Scale ½ an Inch to 22 Yards or 1 Chains.

E. & F. N. Spon, 16, Bucklersbury, London.

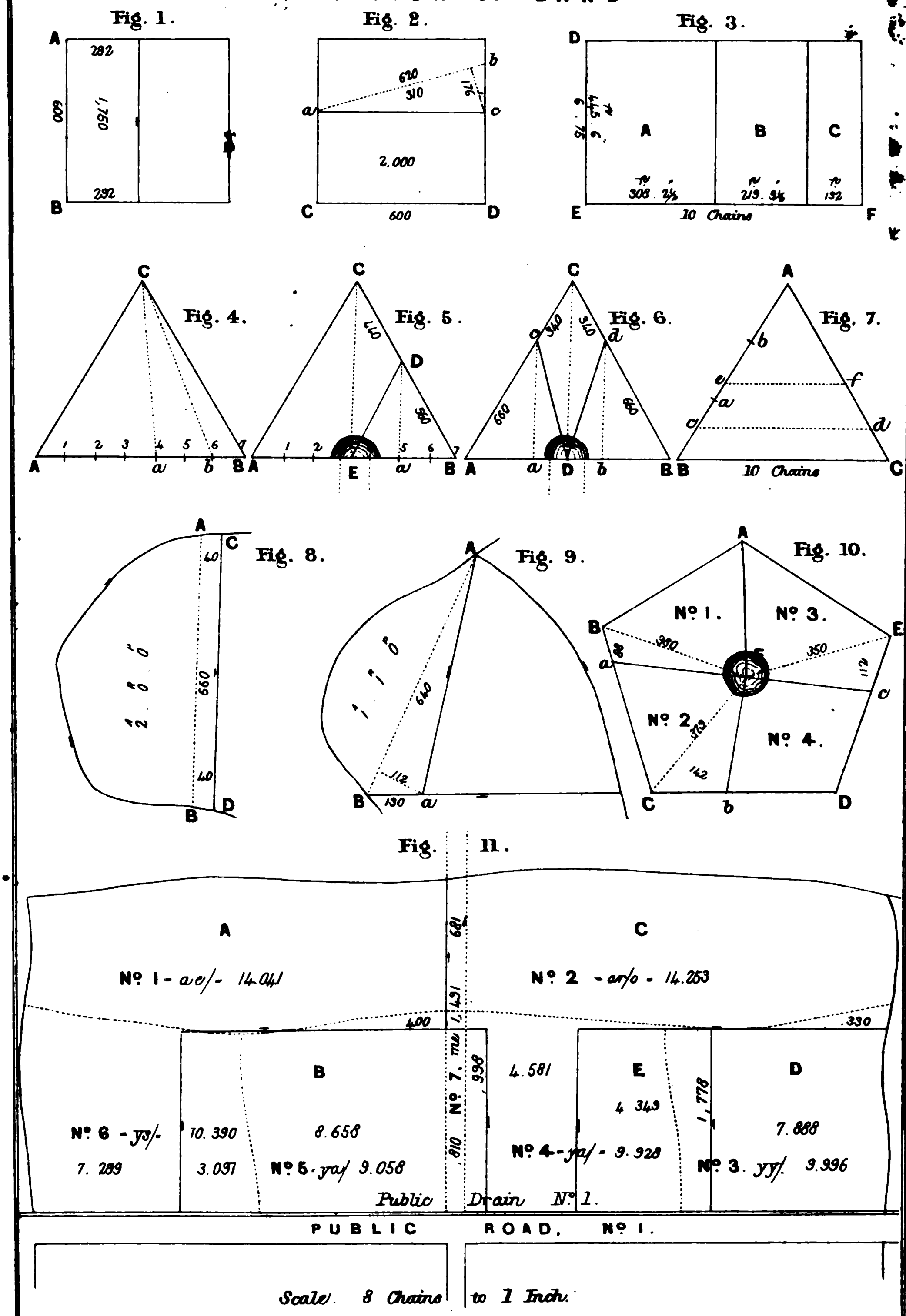
PART 2.

R. Newbery, Lith.



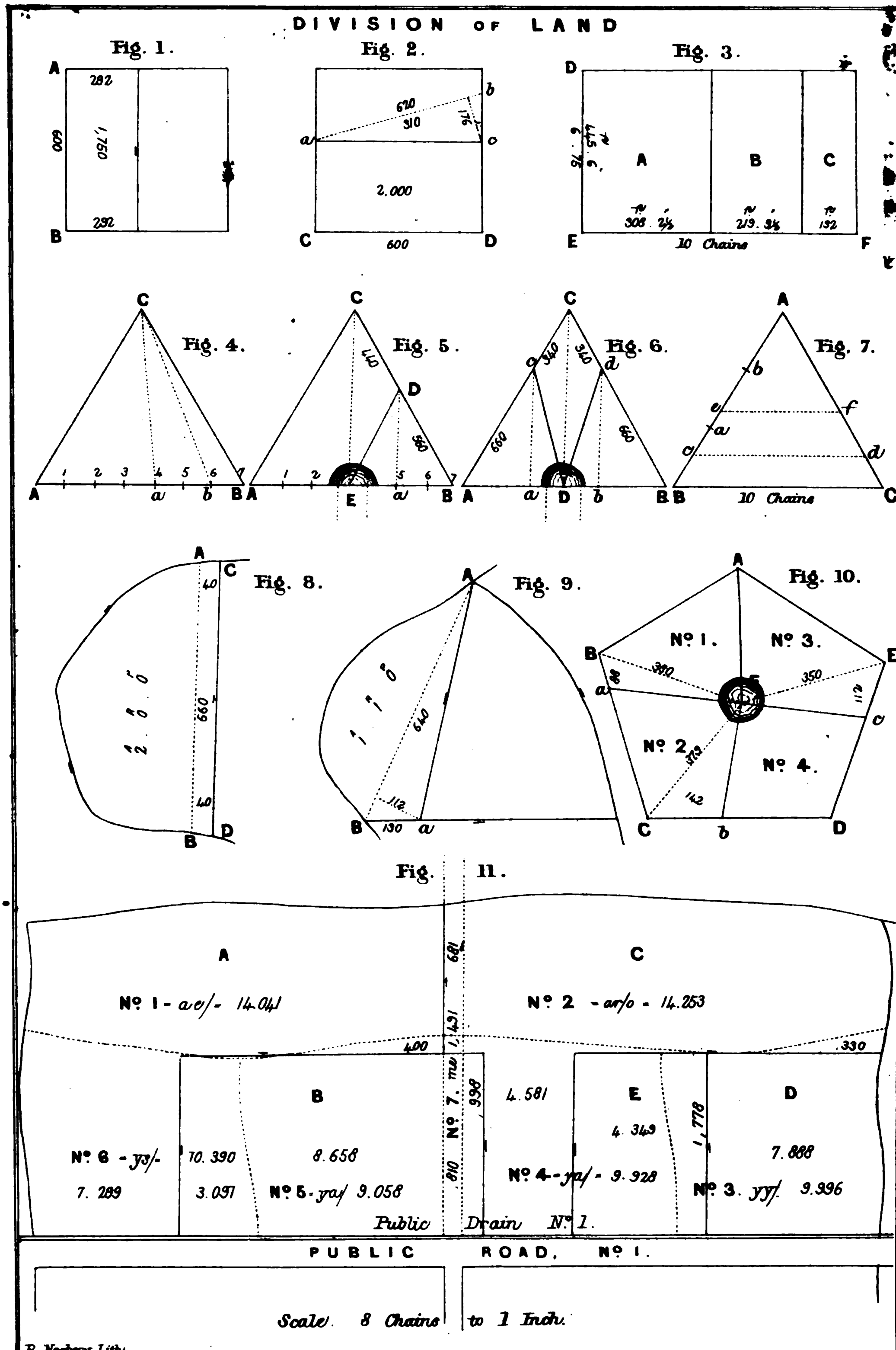
E & F N. Spon, 16, Bucklersbury, London.

DIVISION OF LAND

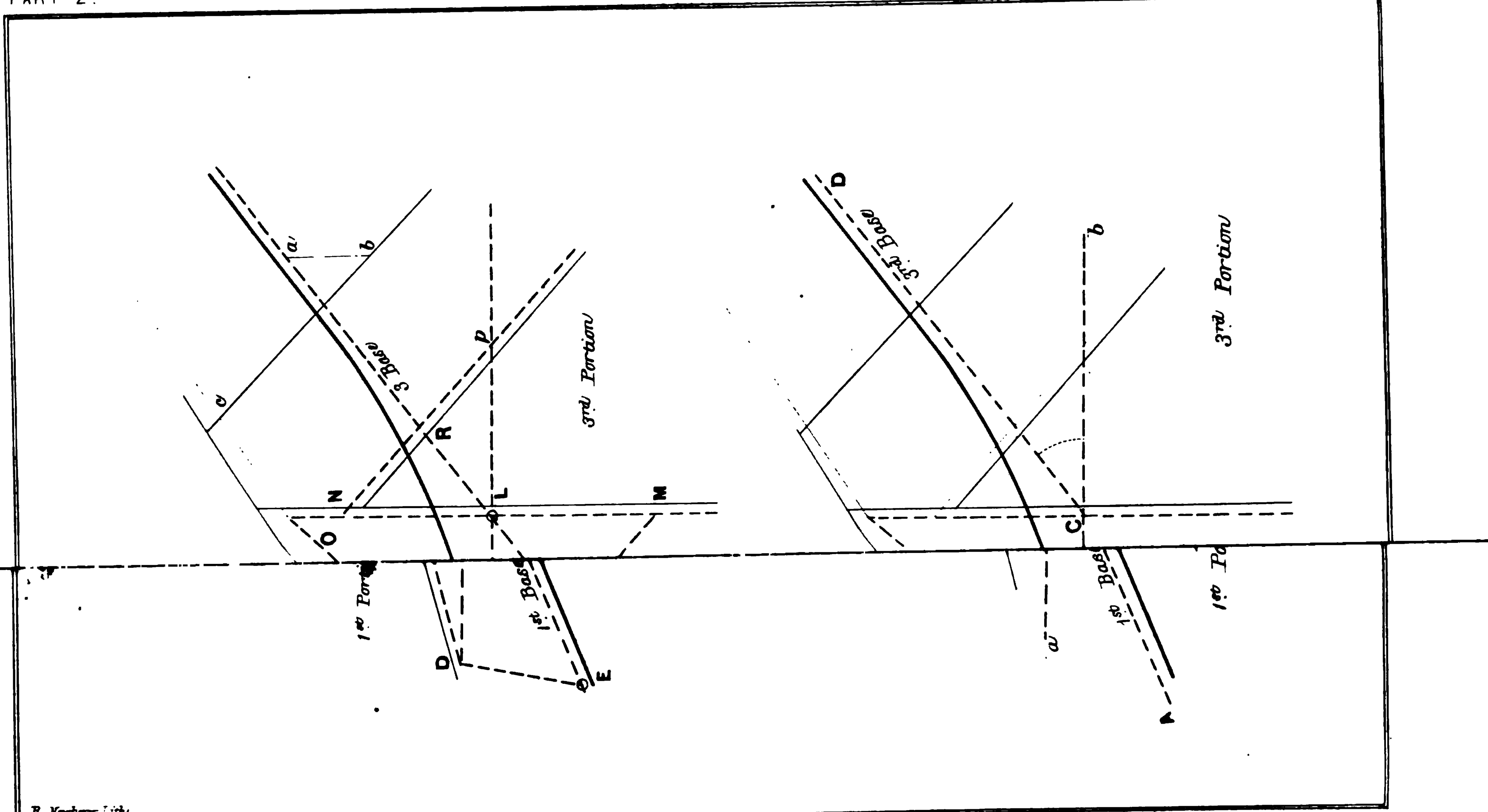


R. Newbery. Lith.

E. & F. N. Spon, 16, Bucklersbury, London.



E.&F.N. Spon, 16, Bucklersbury, London.



R Newbery Lith

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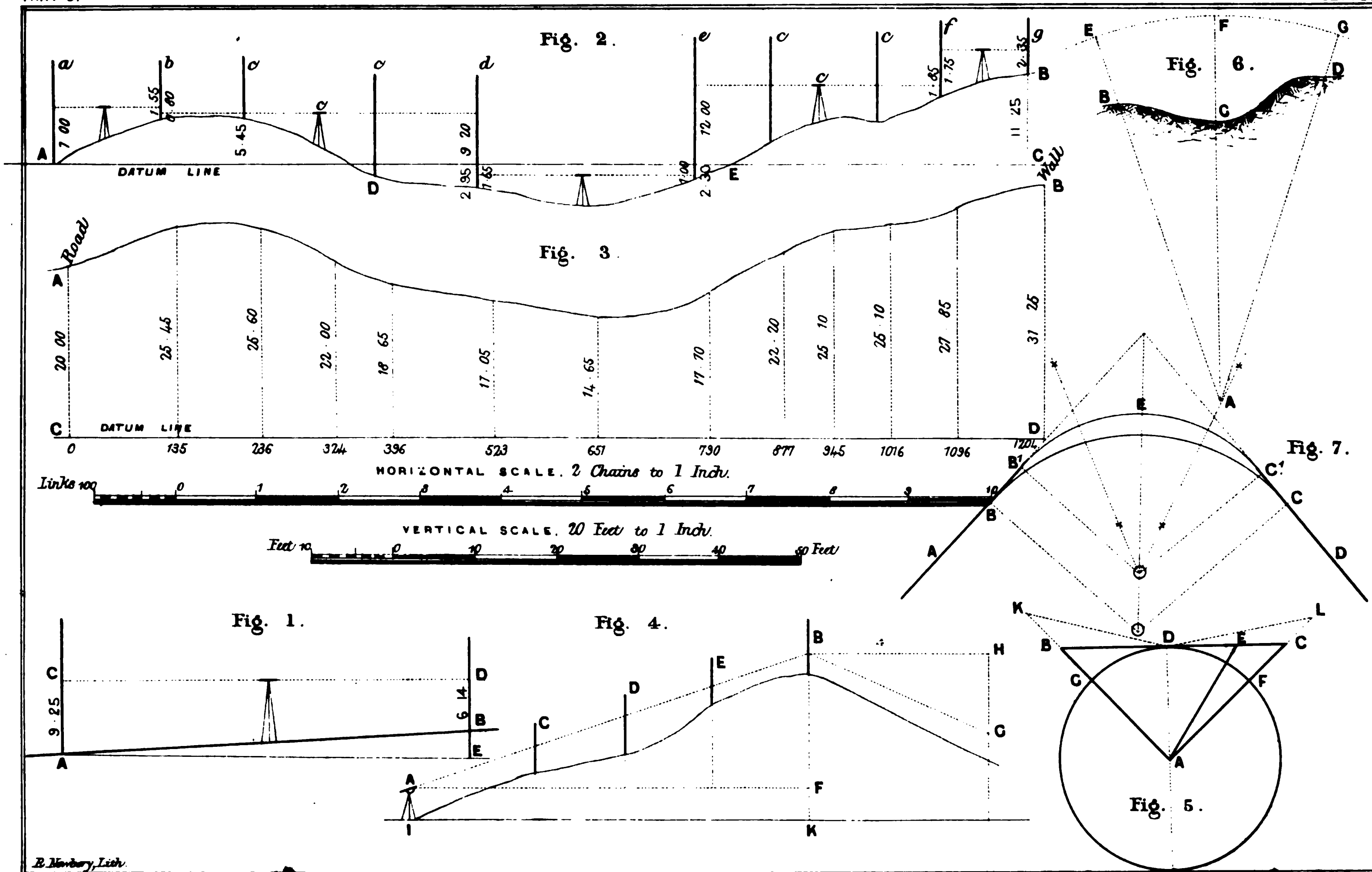

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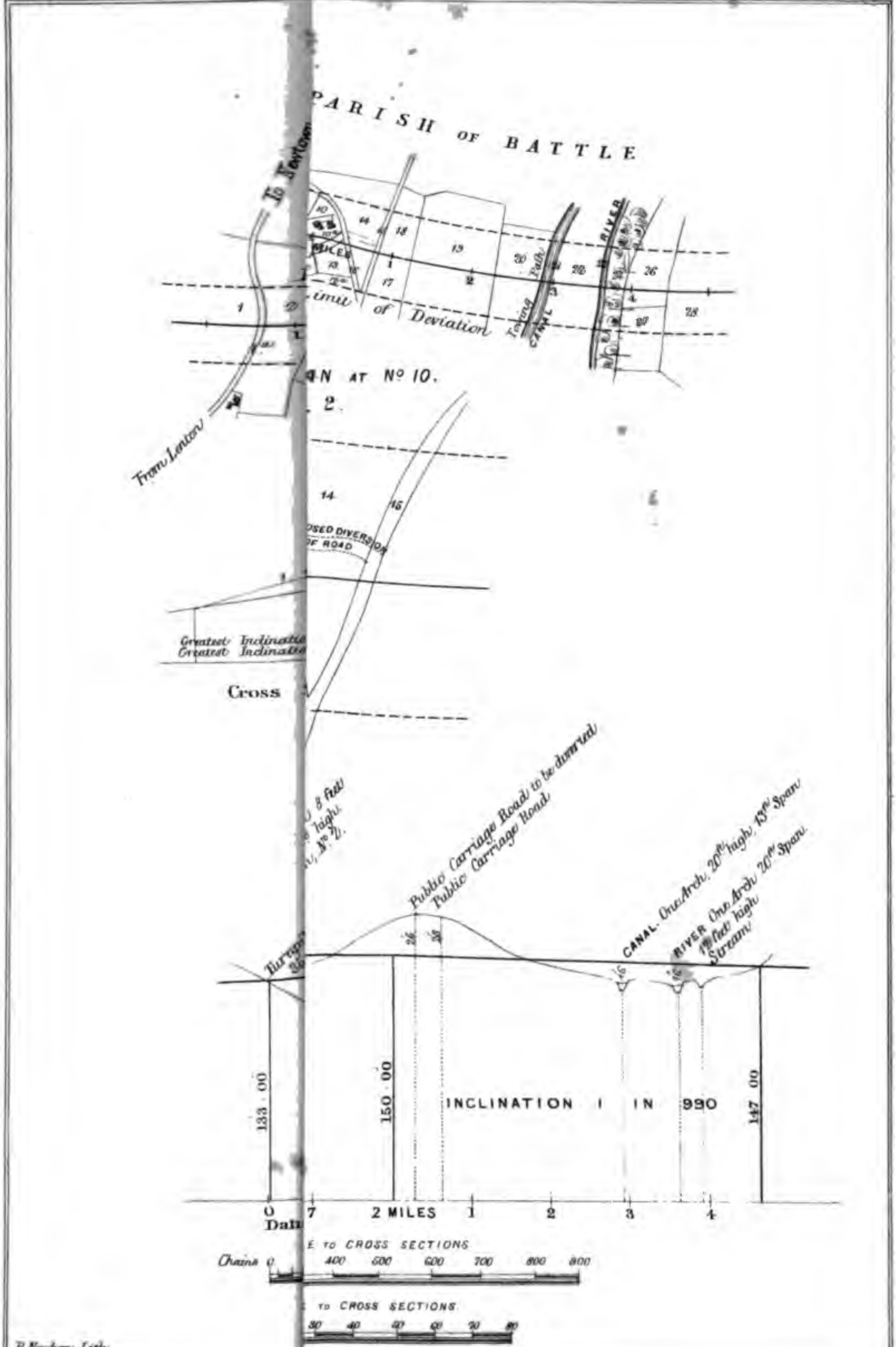

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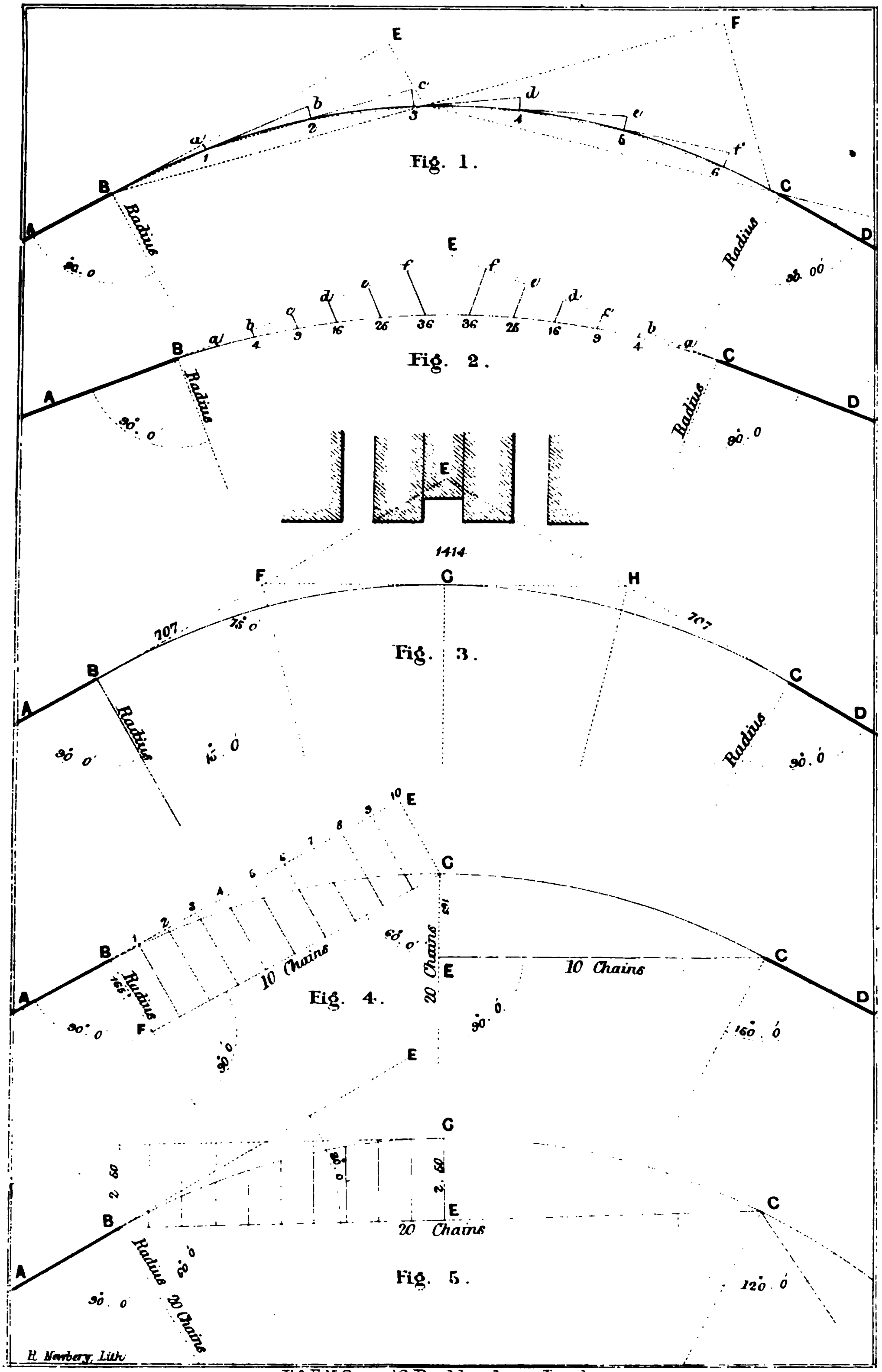

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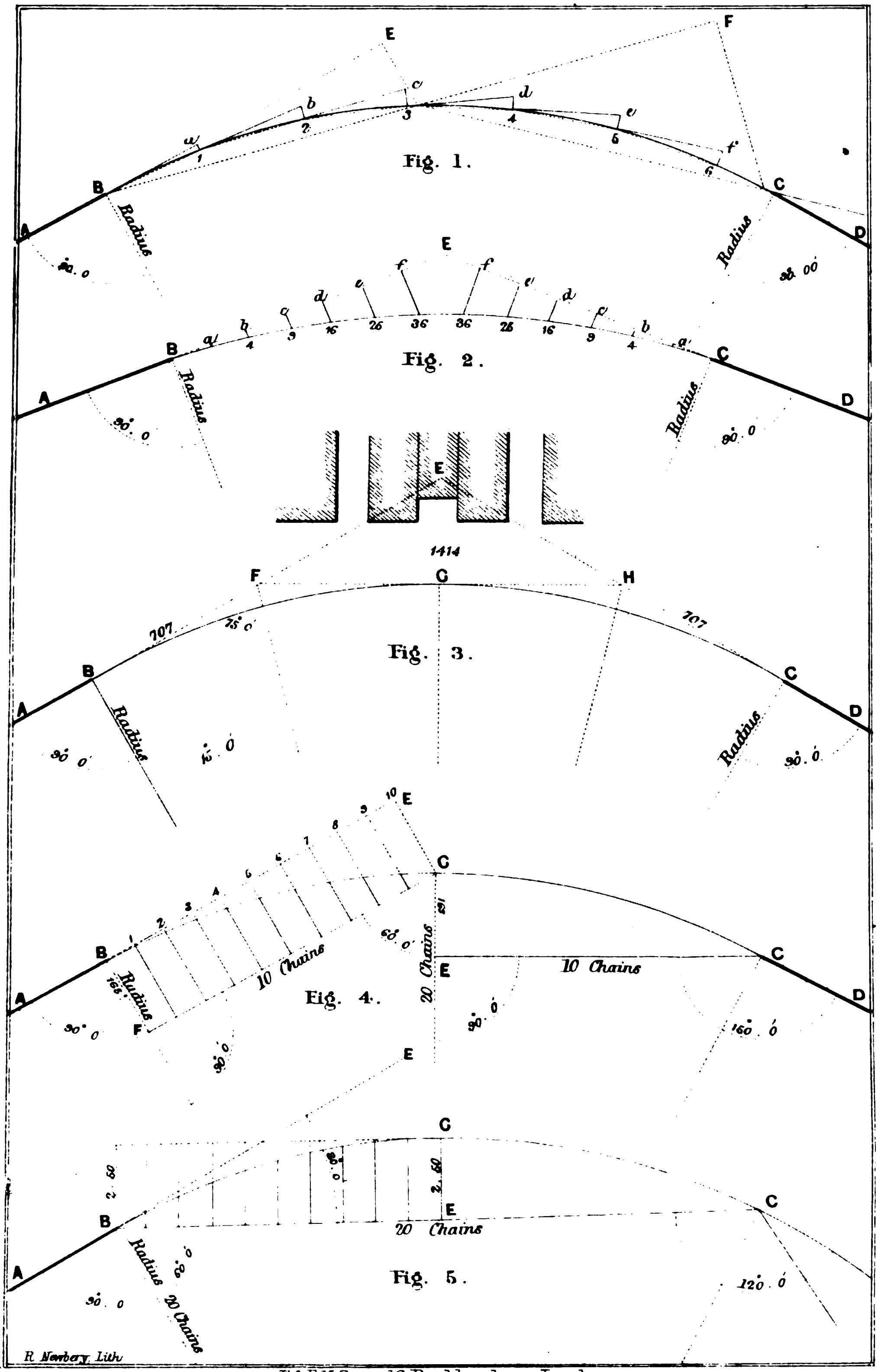

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

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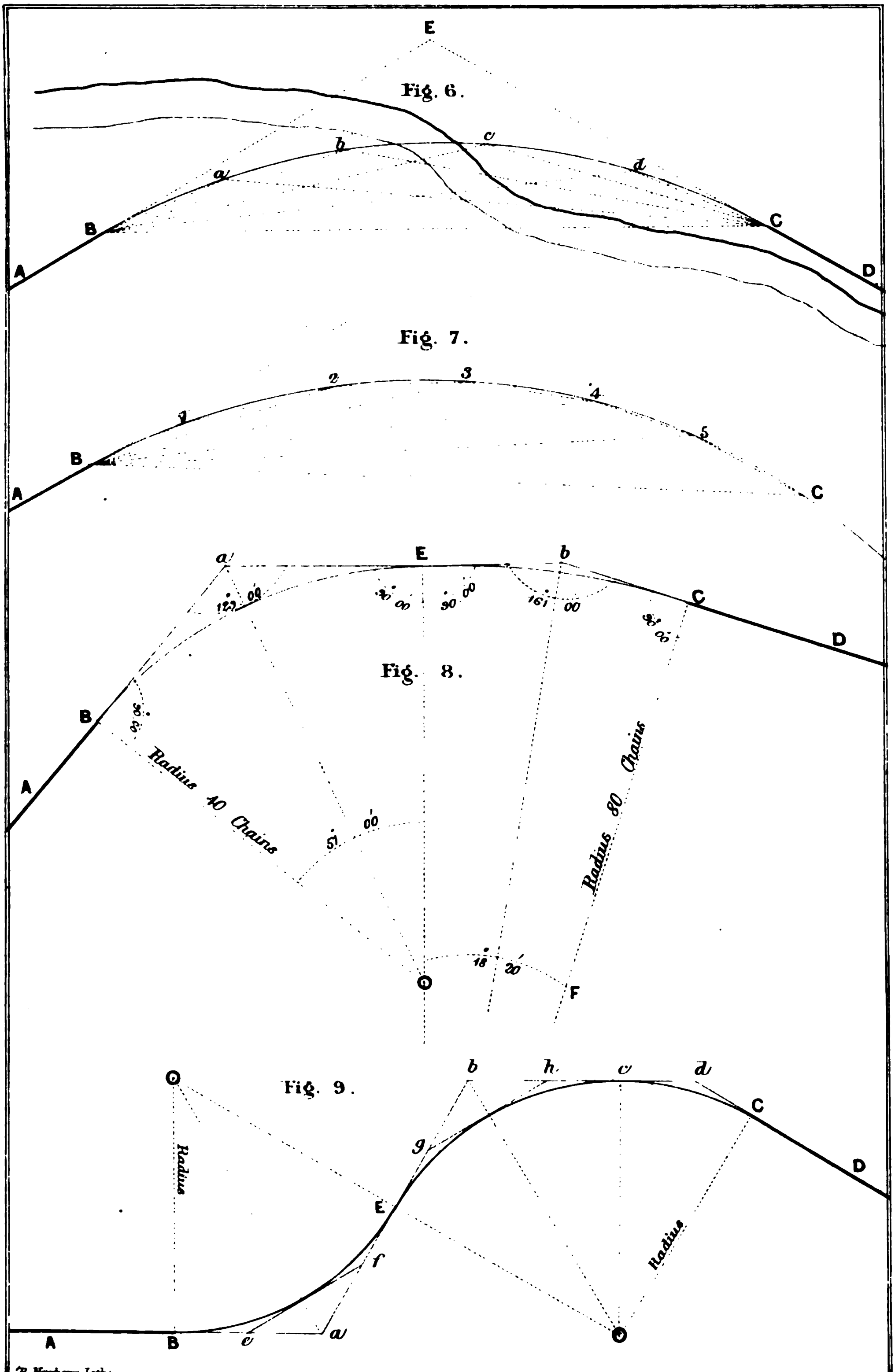

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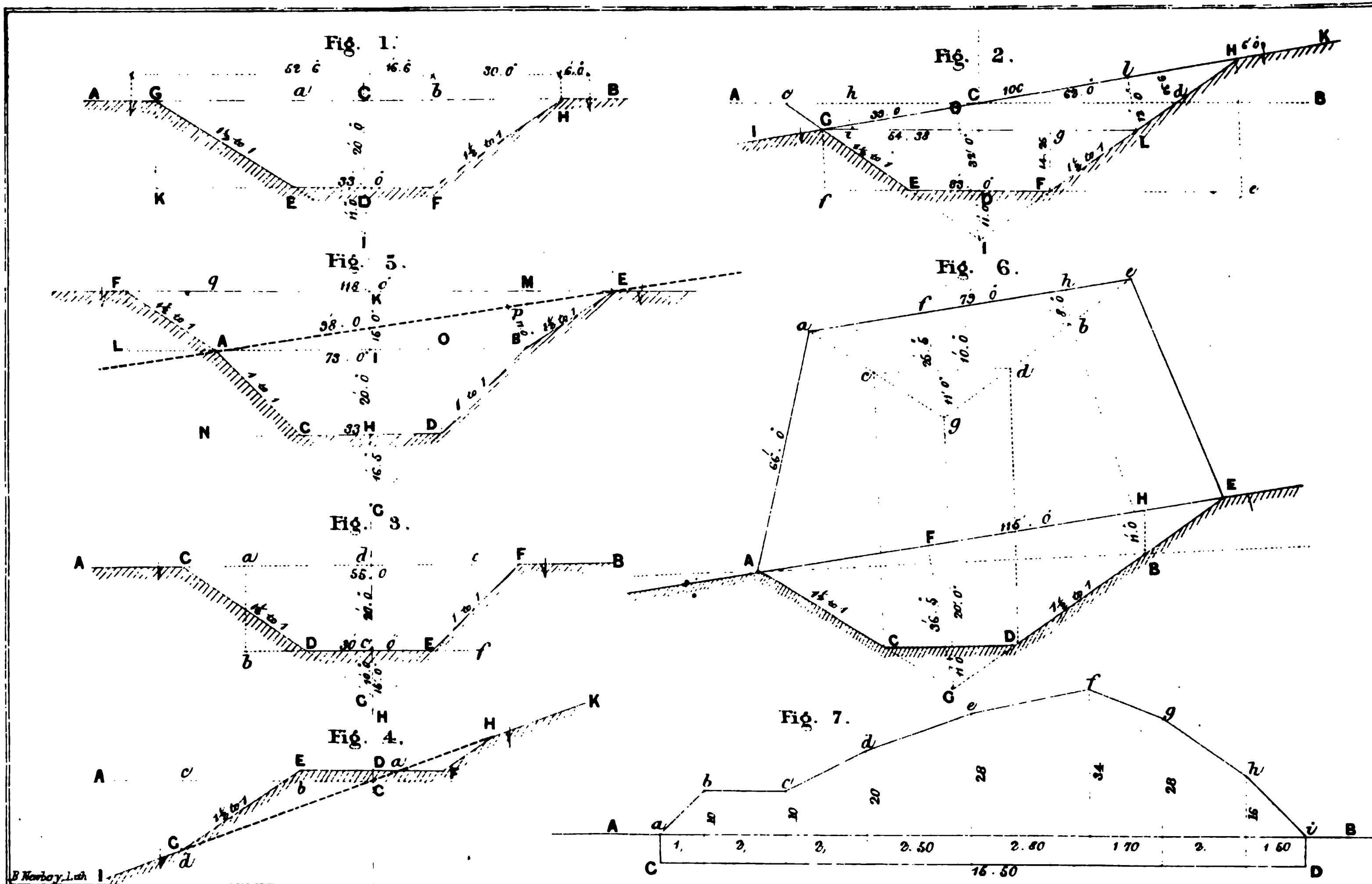

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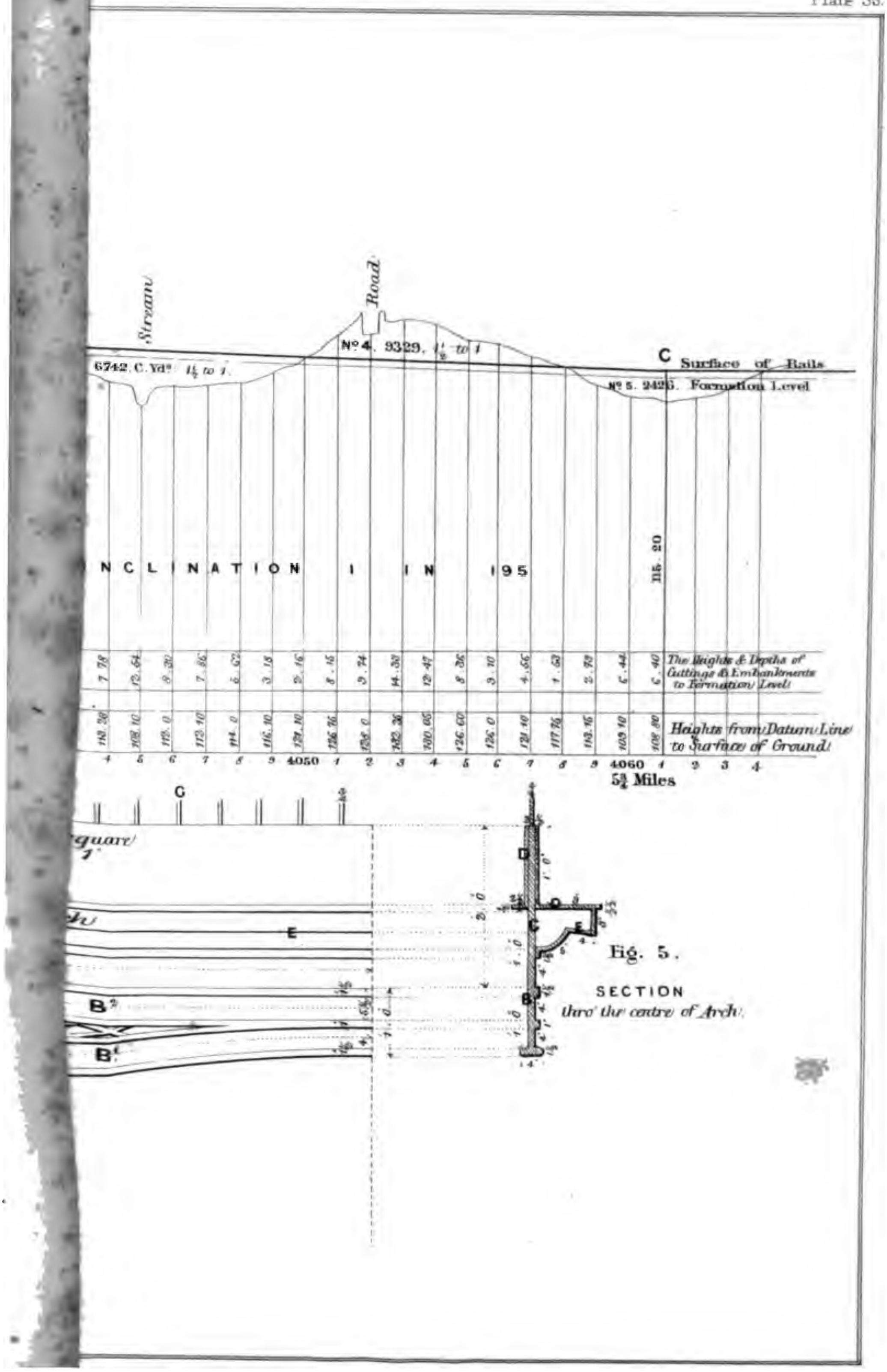

Fig. 5.
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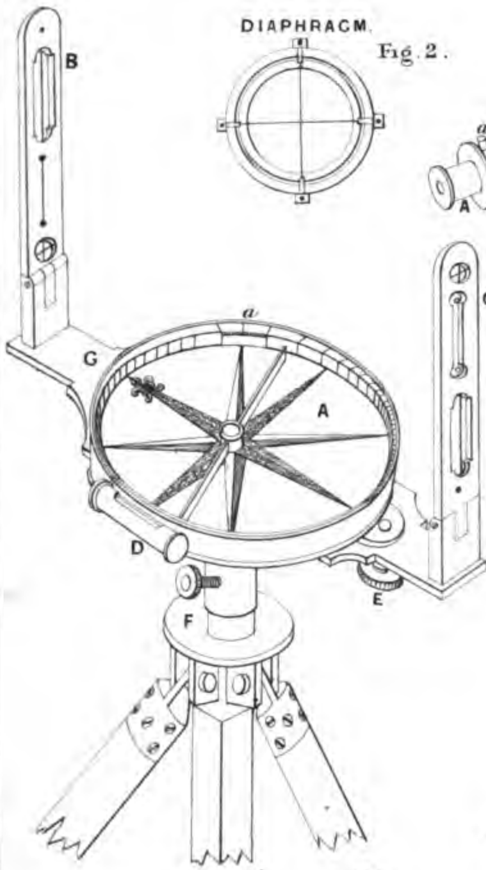

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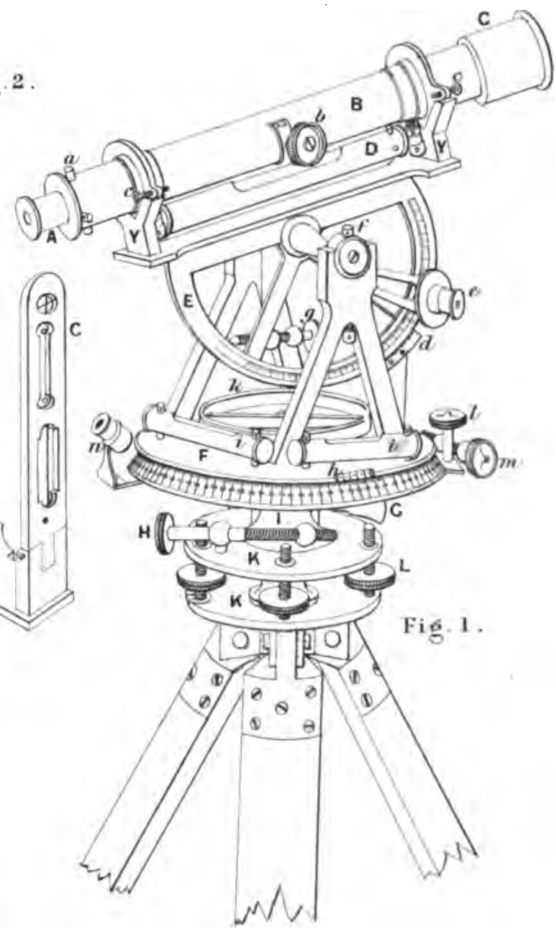
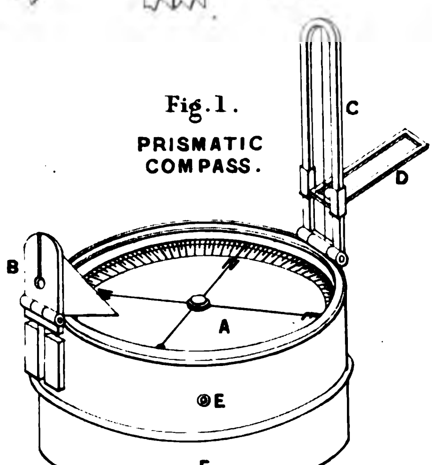


Fig. 1.

Fig. 1.

PRISMATIC COMPASS.



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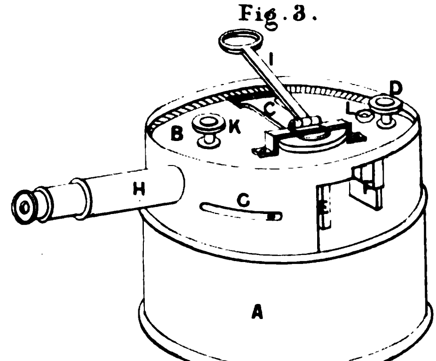
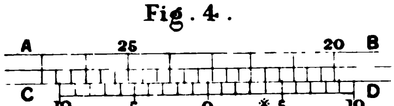


BOX SEXTANT.

Fig. 4.



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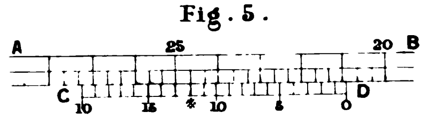

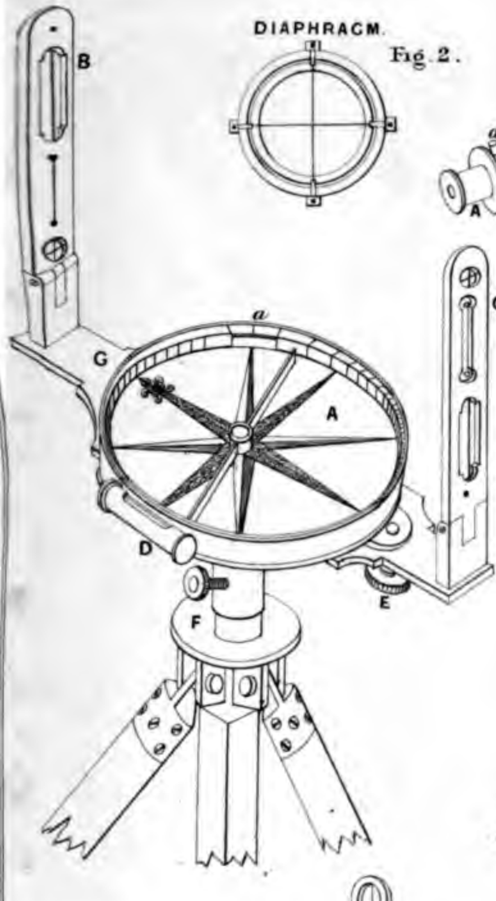

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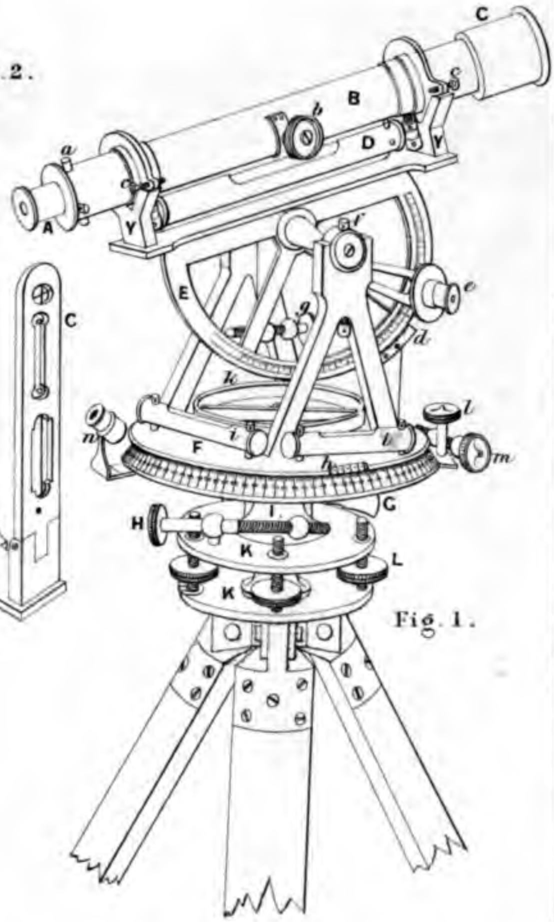

Fig. 1.

Fig. 1.

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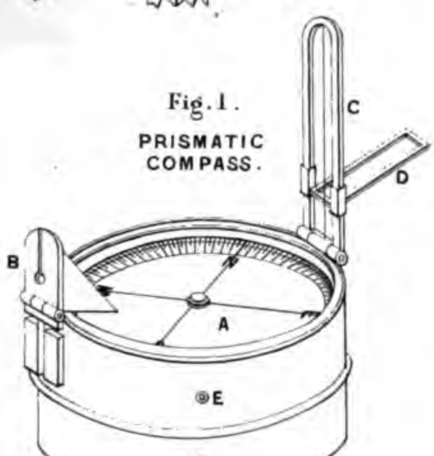

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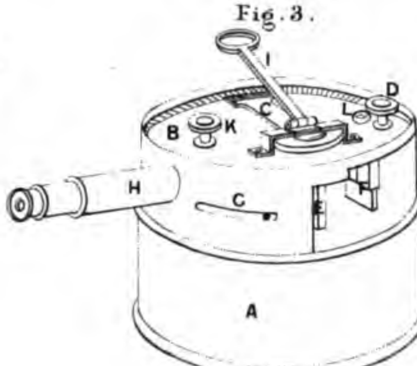

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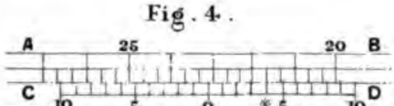

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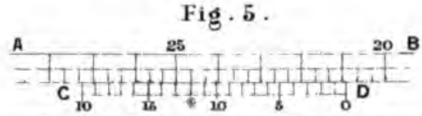

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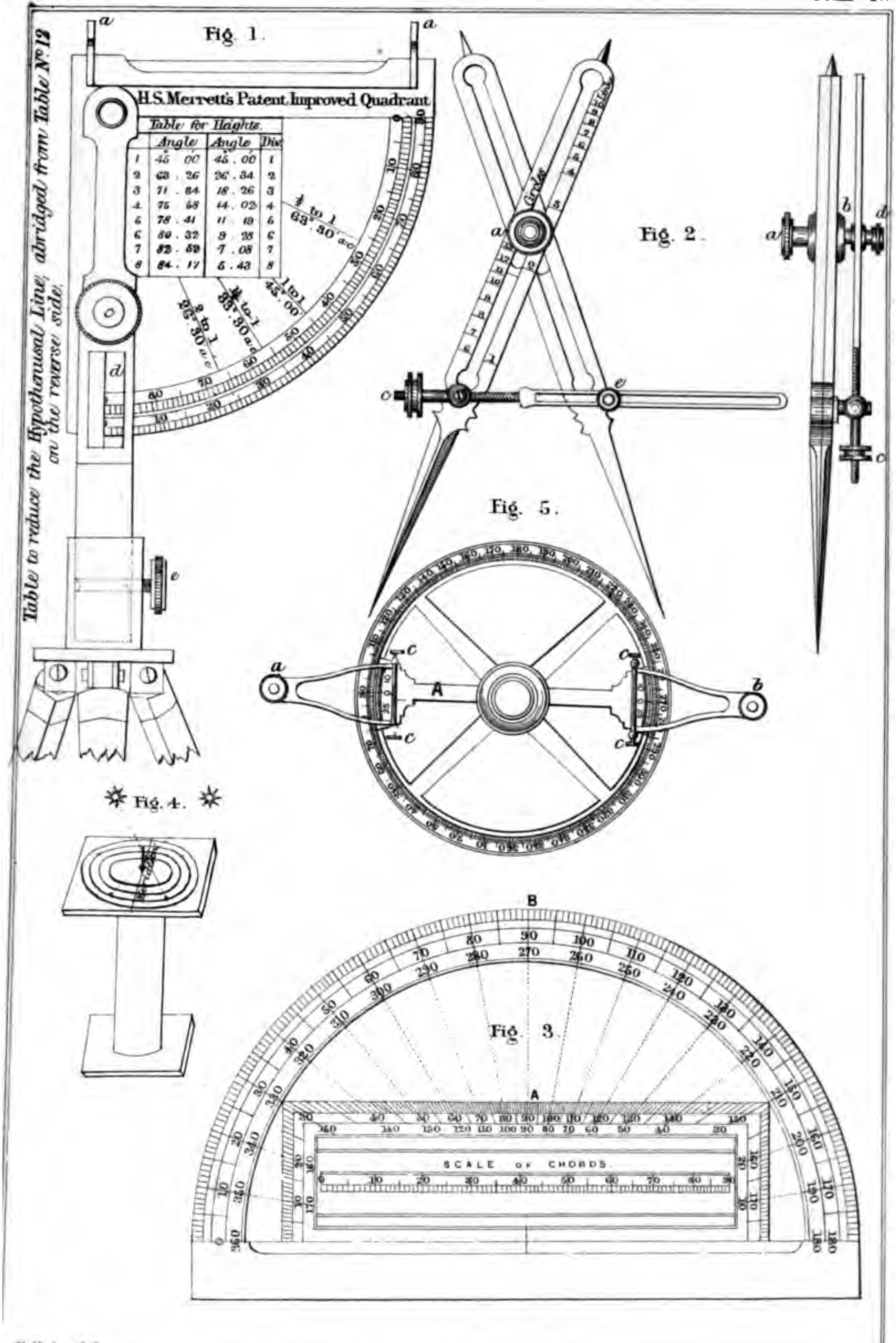

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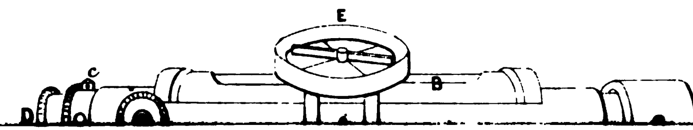

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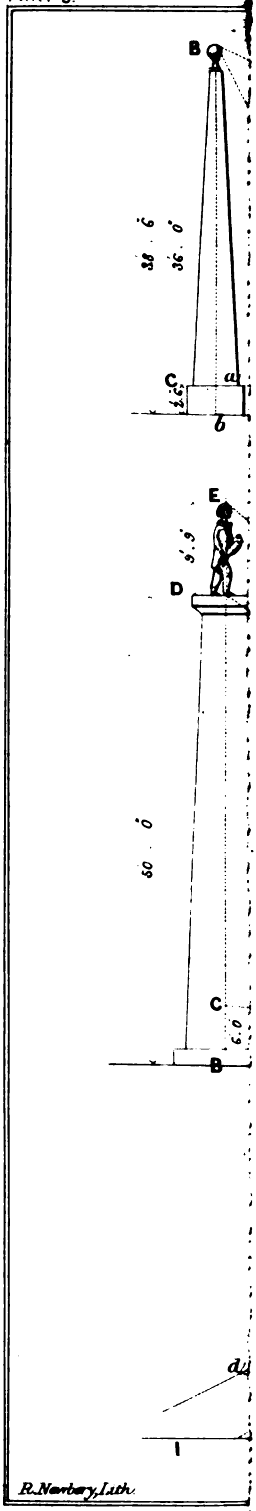

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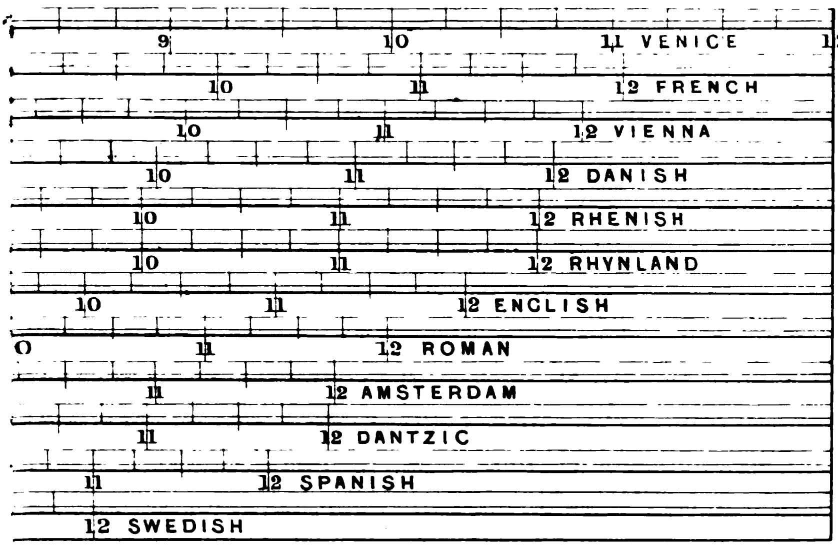

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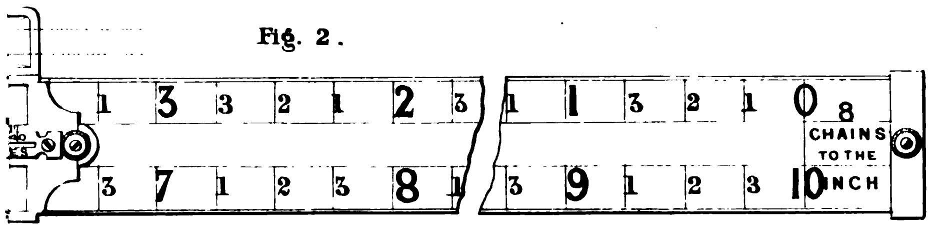


Fig. 4.

DIAGONAL SCALE OF EQUAL PARTS 20 AND 40 TO THE INCH.

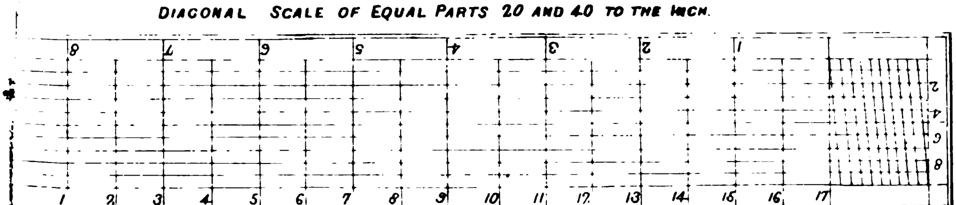
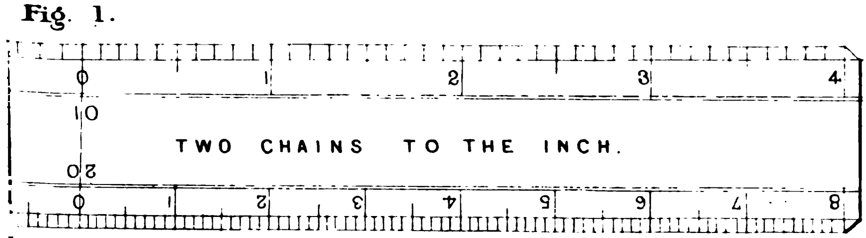


Fig. 1.



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